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ASSESSMENT OF AEROELASTIC CHARACTERISTICS OF CANTILEVER WINGS

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ABSTRACT. *The present work deals with the determination of the aeroelastic characteristics of cantilever wings. The structural representation is made using the classical finite element method of analysis. The aerodynamic representation is made using the strip theory and the doublet-lattice method. The aeroelastic solution is obtained in the modal base representation using the k and the pk methods of solution. The results obtained using the various methods are discussed in terms of computational efficiency and accuracy of the solutions. Further, the present results are compared with previous analytical solutions and wind tunnel experimental findings.*

1. INTRODUCTION

Aeroelasticity is the branch of applied mechanics dealing with stability and response problems of a flexible structures in the presence of incremental nonstationary external airloads. The interest in the subject started early in this century with the advent of high speed flexible aircrafts, where several fatal structural failures, attributed to aeroelastic problems, were observed. Since then the subject has been extensively investigated analytically, experimentally and in wind tunnel tests. Several text-books dealing exclusively on the subject are availables. Among them one can cite Scalan and Rosenbaum (1951), Bisplinghoff and Ashley (1962), Fung (1969), Dowell et al. (1978) and Bismarck-Nasr (1999). In the present work a study of the aeroelastic stability characteristics of cantilever wings is presented. The structural representation is made using the finite element method of analysis which is the state-of-art representation of complex structural dynamic configurations. The aerodynamic representation is made using the strip theory and the doublet-lattice method. The aeroelastic solution is obtained in the modal base representation using the *k* and the *p-k* methods of solution. The results obtained using the various methods are discussed in terms of computational efficiency and accuracy of the solutions. Further, the present results are compared with previous analytical solutions and wind tunnel experimental findings.

2. PROBLEM FORMULATION

Consider an elastic flight vehicle flying, in a cartesian frame of reference x,y,z, in a state of equilibrium under the action of a set of external loads. Due to an external disturbance, e.g., a gust or a sudden application of a pilot induced control action, the structure will strart to perform a perturbuted motion with normalwash displacements $w(x,y,z,t)$. Due to this incremental motion a distribution of incremental nonstationary airloads, with a pressure distribution $\Delta p(x,y,z,t)$, will be introduced. In aeroelasticity we are first interested, in the determination of the stability of the motion of the elastic structure, and second in evaluation of the response of the structure due to these external disturbances. The present paper deals only with the stability part of the problem. Following Bismarck-Nasr (1993 and 1999), we can formulate the problem using Hamilton's variational principle to obtain the related equation of motions of the flexible structural, and we can write,

$$[M]\, q'' + [B]\, q' + [K]\, q = F \tag{1}$$

where q is the displacement of the structure, primes stand for time derivatives, M, B and K represent the mass, structural damping and the stiffness matrices of the flexible structure, and F is the incremental nonstationary airload. The finite element method represent nowadays the state of art, and is the most accurate way of formulation of the elastic-mechanical properties of complex structures, the method will thus be used exclusively to represent the left hand side of Eq. (1). The right hand side of Eq. (1) is the vector of the incremental nonstationary airloads written in the nodal degrees of freedom of the finite element model. Equation (1) represents a highly coupled system of simultaneous equations with an excessive number of degrees of freedom, rendering the dynamic stability study numerically excessive, even with the use of high speed modern computational devices, and at the same time would furnish results that are difficult to be interpreted physically. Both problems can be resolved using a modal transformation base on Eq. (1), thus writing,

$$\{q\} = [Q]\{\eta\} \tag{2}$$

where $[Q]$ is the matrix of modal transformation and $\{\eta\}$ is the vector of modal amplitude, the set of equations (1) is transformed to,

$$[\mu]\{\eta''\} + [\beta]\{\eta'\} + [\gamma]\{\eta\} = \{\phi\} \tag{3}$$

where $[\mu]$ and $[\gamma]$ are the generalized mass and stiffness matrices, and are diagonal matrices, $[\beta]$ is the damping matrix, and for slightly damped structures, can be assumed as a diagonal matrix in the modal representation, and $\{\phi\}$ is the generalized incremental unsteady aerodynamic loads vector. For incompressible subsonic flows, the right hand side of Eq. (3) can be formulated using the strip theory or the doublet-lattice method. Theodorsen and Garrick in (1940) developed the bases of the strip theory and Albano and Rodden in (1969) originated the formulation of the doublet-lattice method. The details of both methods are given in Bismarck-Nasr (1999). In either cases, the equations of motion can be written as,

$$[\mu]\{\eta''\} + [\beta]\{\eta'\} + [\gamma]\{\eta\} = [A_1]\{\eta'\} + [A_2]\{\eta\} \tag{4}$$

where $[A_1]$ and $[A_2]$ are the generalized aerodynamic matrices written in the modal base and are proportional to the modal velocity and modal displacement amplitudes respectively. The set of equations (4) represent a parametric eigenvalue problem with the air density ρ, the unperturbed Mach number M and the reduced frequency k, which are implicit in the aerodynamic matrices, are considered as the parameter of the problem. The solution of the parametric eigenvalue problem will determine the stability borderlines of the problem. Basically three groups of methods of solution are avaible in the literature for the determination of the stability parameters of the problem. These are grouped in the p methods, the k methods and the p-k methods. In the p methods the motion is assumed exponential with p (in general a complex number) consired as the exponent of the motion and therefore the eigenvalue of the problem. The p methods are limited to applications were the aerodynamic loads can be written in explicit forms directly proportional to the displacements and the velocities. In the k methods of solution the motion is assumed harmonic with ω (a real value) being the frequency of motion and a fictitious structural damping g is introduced in the system of equations in order to compensate the out of phase airloads. The solution of the parametric eigenvalue problem in the complex plane will determine the ω and g values at the corresponding airspeed velocity V. The method has the defect that the g parameter obtained is not the physical structural damping and therefore the solution is correct only at the critical velocity, where the damping is physically null, on the other hand these are the points of interest in the solution of the problem. A further defect of the method is that the airloads are calculated at a fixed Mach number, (to represent the compressibility effects), which does not corresponds to the critical velocities and therefore the method will need a matching numerical evaluation between the velocity and Mach number for the correct determination of the critical conditions. In spite of these defects the method bears its popularity due to its simplicity and its direct evaluation of the critical velocities. In the p-k methods, the motion is assumed exponentional with p (in general complex) considered as the exponent of the motion. The solution strarts by fixing a value for the airspeed velocity, say V, and the corresponding Mach number M, and an initial guess of the the first modal oscillation frequency ω. The eigenvalue problem is solved and the first p value is determined. This is compared with the initial guess and an iteration process is started to improve the solution. The iterations are stopped when a desired accuracy will be achieved. The process is then repeated for all the modes. The complete iteration process is then repeated for other airspeed velocities. The great advantage of the method is that the damping obtained is a real physical damping value and can be compared with wind tunnel results and flight tests. Further, the process does not need a matching study between the velocity and the Mach number, since the aerodynamic matrices are calculated for the correct Mach number values. However, the iterations of the numerical process are extremely time consuming as compared to the k methods of solution. The complete details on the theoretical formulation of the methods of solution can be found in Scalan and Rosenbaum (1951), Bisplinghoff and Ashley (1962), Fung (1969), Dowell et al. (1978) and Bismarck-Nasr (1999).

3. NUMERICAL RESULTS

In this section several numerical results are presented and the results obtained using the various methods of formulation and solution are discussed in terms of computational efficiency and accuracy of the solutions. Further, the present results are compared with previous analytical solutions and wind tunnel experimental findings. In present study five cantilever wing models were investigated. Table 1 below gives the identification used in the present study, the reference of previous analytical and experimental studies and the main characteristics of the structures considered. The wings studied have all trapezoidal planform, with a constant chord *c*, and span *l*. Table 2 gives the dimensions used for the different models analyzed.

TABLE 1 – MODELS DESCRIPTION

MODEL IDENTIFICATION	REFERENCCE	MODEL CHARACTERISTICS
1	NASA TN D-1824	15° sweptback, untapered, aspect ratio 5.34, aluminum-made
2	NASA TN D-1824	30° sweptback, untapered, aspect ratio 4.16, aluminum-made
3	NACA RM L55E11	45° sweptback, untapered, aspect ratio 2.76, aluminum-made
4	NACA RM L55E11	60° sweptback, untapered, aspect ratio 1.39, aluminum-made
5	NACA TR 685	unswept, untapered, aspect ratio 13.5, aluminum-made

TABLE 2 – MODELS DIMENSIONS

MODEL	CHORD, TAKEN PERPENDICULAR TO LEADING EDGE	SPAN, TAKEN ALONG LEADING EDGE	ANGLE OF SWEEP
	c , inches	l , inches	Λ , degrees
1	*2.00*	*5.72*	*15*
2	*2.00*	*5.55*	*30*
3	*2.00*	*5.51*	*45*
4	*2.00*	*5.55*	*60*
5	*12.00*	*81.00*	*0*

The process of analysis for each model was performed in three main steps, listed below,

1. Adjustment of the structural finite element model in order to reproduce the referenced physical properties of the model, such as weight, center of gravity (c.g.) location, natural frequencies and elastic axis (e.a.) location since not all the input data were availables in the cited references;

2. Aerodynamic modeling in order to obtain the flutter velocity and frequency (critical point), using various aerodynamic theories;

3. Repetition of the previous step about the critical point, in order to verify the computational efficiency.

For the finite element representation, plate elements were used for the structural models. The number of elements in the spanwise and chordwise directions are given in Table 3. The boundary conditions applied in models 1 to 4 restrain all degrees of freedom of nodes located at 12.5% and 87.5% of the chord length at the wing root. These boundary conditions were applied in order to correlate the present structural dynamic results with the results given in the “NASTRAN Version 68 Aeroelastic Analysis Manual (1994). In model 5, all degrees of freedom at the wing root were constrained.

TABLE 3 – STRUCTURAL FINITE ELEMENT MODEL REFINEMENT

MODEL	NUMBER OF PLATE ELEMENTS IN STRUCTURAL FINITE ELEMENT MODEL	
	SPANWISE DIRECTION	CHORDWISE DIRECTION
1	7	4
2	7	4
3	7	4
4	7	4
5	12	3

The following method was used in adjusting the weight: with a typical value of the material density, the total weight of the model was obtained in a first run, and since the physical models were all solid models, the density was adjusted by multiplying that

typical value by the ratio between the measured weight and the calculated weight. The C.G. location was checked only for model 5, since models 1 to 4 have a very simple cross section, while model 5 has an airfoil-shape cross section. The method used was as follows: after the first run, the calculated C.G. location was ahead of the actual location; a heavier material was then used in the rear portion of the wing (from 70% of chord to trailing edge), and a second run performed. The results of these two runs were then interpolated (C.G. location versus density of the rear portion of the wing), and a new value for the density found. A new run was performed only to confirm the C.G. location and to re-scale the weight, as previously explained.
After adjusting the densities to provide correct weight and C.G. location for all models, the stiffnesses were adjusted to exhibit the modal characteristics (frequencies and mode shapes) of the referenced models. The method in adjusting stiffness was the modification of the mechanical properties (Young's and shear moduli, and Poisson's ratio). The Young's and shear moduli are multiplied by the square of the frequency ratios (i.e., the ratio of referred experimental values and the present calculated frequencies). A new run was performed, the frequency values checked again, and the process repeated if there was no agreement among calculated and experimental values. In general, the method converged in two or three runs. With the stiffnesses so adjusted, the modal shapes were also verified, but in a qualitative sense, for models 1 to 4, through the comparison of node lines of the finite element model and those reported by Tuovila (1955) and Yates (1963). The aerodynamic models used in the present study are given in Table 4. The number of panels or strips in each model is indicated in the table.

TABLE 4 – AERODYNAMIC MODEL REFINEMENT

MODEL	DOUBLET-LATTICE METHOD		STRIP THEORY
	SPANWISE DIVISIONS	CHORDWIISE DIVISIONS	SPANWISE DIVISIONS
1	6	4	6
2	6	4	6
3	6	4	6
4	6	4	6
5	12	3	6

The results obtained in the present investigation are summarized in Tables 5 for the frequencies and normal modes. Table 6 gives the flutter velocities results. Table 7 provides the flutter frequencies, and Table 8 flutter reduced frequencies. The present results are compared in these tables with the experimental findings of the cited references.

TABLE 5 – FREQUENCY RESULTS

MODEL	FREQUENCY, Hz							
	MODE # 1		MODE # 2		MODE # 3		MODE # 4	
	CALCULATED	TESTED	CALCULATED	TESTED	CALCULATED	TESTED	CALCULATED	TESTED
1	34.3	36	210..0	210	260.4	242	-------	-------
2	40.1	35	207.7	210	310.5	270	-------	-------
3	46.7	35	200.9	198	365.2	294	-------	-------
4	54.8	48	212.8	210	392.6	396	-------	-------
5	1.22	1.31	7.7	7.7	17.8	17.8	22.0	20.8

TABLE 6 – FLUTTER VELOCITY RESULTS

MODEL	FLUTTER VELOCITY, ft/s				
	DOUBLET-LATTICE METHOD		STRIP THEORY		TESTED
	K	PK	K	PK	
1	487.1	478.5	434.5	451.2	495
2	493.4	485.5	441.2	459.5	517
3	549.4	522.5	493.8	463.2	550
4	798.4	775.7	688.2	714.2	836
5	299.5	303.2	342.1	338.6	302.0

TABLE 7 – FLUTTER FREQUENCY RESULTS

MODEL	FLUTTER FREQUENCY, Hz				
	DOUBLET-LATTICE METHOD		STRIP THEORY		TESTED
	K	PK	K	PK	
1	117.4	122.0	106.9	110.9	120
2	119.7	121.6	107.6	121.6	120
3	118.5	140.0	124.0	97.8	120
4	158.5	162.5	90.6	174.0	110
5	10.7	10.6	8.4	8.4	10.2

TABLE 8 – FLUTTER REDUCED FREQUENCY RESULTS

MODEL	FLUTTER REDUCED FREQUENCY				
	DOUBLET-LATTICE METHOD		STRIP THEORY		TESTED
	K	PK	K	PK	
1	0.1307	0.1383	0.1334	0.1332	0.1314
2	0.1467	0.1515	0.1474	0.1600	0.1403
3	0.1597	0.1984	0.1859	0.1564	0.1616
4	0.2079	0.2194	0.1379	0.2552	0.1378
5	0.1121	0.1097	0.07714	0.07794	0.1061

CONCLUSIONS

A study of the aeroelastic stability characteristics of cantilever wings has been presented. The structural representation is made using the finite element method of analysis which is the state-of-art representation of complex structural dynamic configurations. The aerodynamic representation is made using the strip theory and the doublet-lattice method. The aeroelastic solution is obtained in the modal base representation using the *k* and the *p-k* methods of solution. The results obtained using the various methods have been compared with previous analytical solutions and wind tunnel experimental findings. It can be concluded that the results obtained agree favorably with previous analytical and experimental findings in terms of accuracy of the final results. As expected the *p-k* method is more excessive in terms of computational requirements as compared to the *k* methods of the aeroelastic solution.

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