COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

ESCAPE IN A NONIDEAL ELECTRO-MECHANICAL SYSTEM

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Abstract. *In this work it is investigated a particular system consisting of a pendulum whose point of support is vibrated along a horizontal guide by a two bar linkage driven from a DC motor, considered as a limited power source. This system is nonideal, since the speed of the motor is influenced by the response of the system, revealing a variety of nonlinear phenomena. The bifurcational behavior is complex mainly near a secondary resonance region* ($\theta' \approx 2.5$)*, where the escape phenomenon is observed. During passage through the critical speed, an important characteristic appears, where the system may pass through this region reaching a limited attractor or it may escape from the potential well to a remote attractor.*

Keywords: *Nonideal system, Pendulum, Escape.*

1. INTRODUCTION

A dynamic system is called nonideal, when we consider the influence of the oscillatory system on the driving force, i.e., there is a dynamic interaction between the motor and the pendulum and vice-versa. As the motor's action depends on the pendulum's motion, it can not be described by a determined function on the time, and its motion must be represented by a differential equation increasing the degrees of freedom of the system. In this case, the energy source is called of limited power (Nayfeh and Mook (1979)).

The escape from the potential well in the forced oscillator can be associated with homoclinic tangles and fractals basin, and consequently it is often followed by chaotic motions, Thompson *et al* (1987). Thompson (1989) studied the escape of the asymmetric potential and observed that the stability loss of the system occurs through a fold bifurcation. When an attractor set undergoes a discontinuous change, disappearing from the phase space, the bifurcation is catastrophic and it is known as blue-sky catastrophe or boundary crisis. A catastrophe in a dissipative dynamical system which causes an attractor to completely lose stability will result in a transient trajectory making a rapid jump in the phase space to some other attractor, Stewart and Ueda (1991). This event occurs due to a fractal basin boundary, and in the presence of even infinitesimal noise we cannot predict to which of the remote attractors the system will jump, Thompson and Soliman (1991). Stewart *et al* (1995) studied the optimal escape of periodically forced oscillations from a potential well. It is important to observe that the phenomena, above mentioned, are related with softening systems near the fundamental resonance region.

In this work, we will investigate through numerical simulation a particular nonideal system consisting of a pendulum whose support point is vibrated along a horizontal guide by a two bar linkage driven from a DC motor, which has limited power (See "Fig. 1"), in a secondary resonance region close to $\theta' \approx 2.5$. In this region, the system presents the escape phenomenon from the potential well, characterized by rotational solutions of the pendulum (Belato (1998)).

Other features of this system were analyzed in previous works, near the main resonance region ($\theta' \approx 1$), including: periodic, multi-periodic, quasiperiodic and chaotic motion (Belato (1998), Belato *et al* (1999a) and Belato *et al* (1999)), and near the secondary resonance region ($\theta' \approx 5$), where a saddle node bifurcation occurs, Belato (1998).

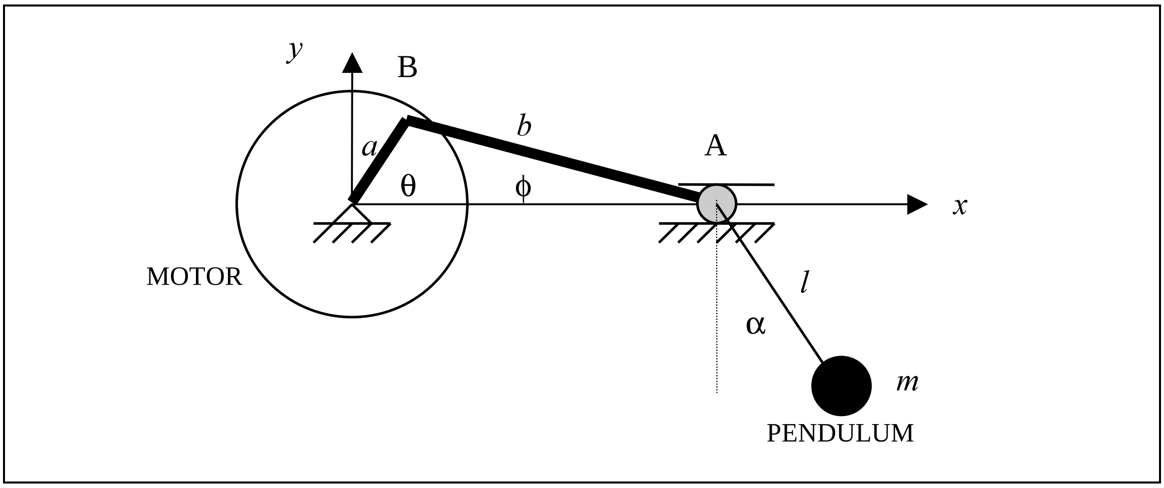

Figure 1 **-** Schematic of the system "electromotor-pendulum".

When the motor speed is close to the region determined by $\theta' \approx 2.5$, a global bifurcation known as blue-sky catastrophe occurs, which is characterized by the disappearance of a limited attractor of the phase space and the system jumps to a remote attractor that can be limited or not.

This paper is organized as follows. The Section 2 contains the differential equations that govern the motion of the electromotor-pendulum system. Section 3 gives the results of numerical simulation and its interpretation and Section 4 contains the conclusions.

2. EQUATIONS OF MOTION

The complete system of the dimensionless differential equations that determine the motion of the electro-motor pendulum (See details in Belato (1998)) are given by:

$$
\begin{aligned}
&(J + c_3 F^2 sin^2\alpha)\theta'' = -(c_1 + c_2 F^2)\theta' - c_3 sin^2\alpha F F'\theta' - c_4 F(\cos\alpha sin\alpha + \alpha'^2 sin\alpha) + \\
&\qquad\qquad + c_5 I - c_6 \\
&\alpha'' + sin\alpha = \in_2 (F\theta'' + F'\theta')\cos\alpha - c_7\alpha' \\
&I' = c_8 - c_9 I - c_{10}\theta'
\end{aligned}
\tag{1}
$$

where $F = \left[1 + \dfrac{\in_1 \cos\theta}{(1 - \in_1^2\ sin^2\theta)^{1/2}}\right] sin\theta$, $\in_1 = \dfrac{a}{b}$, $\in_2 = \dfrac{a}{l}$, $c_1 = \dfrac{\mathrm{c_m}}{\omega_0}$, $c_2 = \dfrac{\mathrm{c_A} a^2}{\omega_0}$, $c_3 = ma^2$, $c_4 = mal$, $c_5 = \dfrac{\mathrm{K_T}}{\omega_0^2}$, $c_6 = \dfrac{\mathrm{T_f}}{\omega_0^2}$, $c_7 = \dfrac{\mathrm{c_P}}{\omega_0 ml}$, $c_8 = \dfrac{\mathrm{V}}{\mathrm{L}\omega_0}$, $c_9 = \dfrac{\mathrm{R}}{\mathrm{L}\omega_0}$, $c_{10} = \dfrac{\mathrm{K_E}}{\mathrm{L}}$, and θ is the motor angular displacement, α is the pendulum angular displacement, J is the moment of the inertia of the rotor of the motor, m is the pendulum's mass, l is the length of the pendulum, $\mathrm{c_A}$ is the damping coefficient for the friction at the pin A, $\mathrm{c_P}$ is the damping coefficient for the friction on the pendulum, V is the motor voltage, L is the inductance, R is the electrical resistance, I is the current, $\mathrm{K_T}$ is the torque constant, $\mathrm{K_E}$ is the voltage constant, $\mathrm{c_m}$ is the constant for the internal loss coefficient in the motor, $\mathrm{T_f}$ is a constant friction torque in the motor, $\omega_0 = \sqrt{g/l}$ is the natural frequency of the pendulum. The primes denote derivatives with respect to $t^* = \omega_0 t$.

The chosen control parameter is the motor voltage, represented by the parameter c_8. In previous works, this parameter was considered as a constant and only the resulting steady state behavior of the system was analyzed. Here, we will adopt the control parameter as a function of the time, to analyze the transient behavior of the electromotor pendulum system, and it will be given by:

$$
c_8 = p\tanh(qt^*) \tag{2}
$$

where p and q are constants values.

Then, the emphasis here is on a qualitative description and identification of the pendulum behavior when the parameter q in the variable c_8 is varied.

3. NUMERICAL SIMULATIONS AND RESULTS

We carried out a large number of numerical simulations with different initial conditions and numerical values of the physical parameters of the problem. Simulink™ Toolbox of the MATLAB™, is used to simulate the system of differential equations and the numerical integrator used is the Runge Kutta fifth order (RK45) with variable steplenght. The parameters in this paper used in the simulation of the system of differential equation (1) are: $\in_1 = \in_2 = 0.233$, $c_1 = c_2 = c_6 = 0$, $c_3 = 0.00098$, $c_4 = 0.0042$, $c_5 = 0.01055$, $c_7 = 0.01$, $c_9 = 33.83$, $c_{10} = 78.40$ and $c_8 = p\tanh(qt^*)$ is the parameter used to control the diversity of nonlinear behavior of this system, with p = 290 and q is varied. The initial conditions for the following results are: $\theta'(0) = \theta(0) = 0$, $\alpha'(0) = \alpha(0) = 0$ and $I(0)$ = 0.

When the control parameter is constant, results for different motor voltages will not consider the transient motion, and it is possible to detect different pendulum behaviors for increasing motor speed. This system presents three main regions: the fundamental resonance, where a chaotic attractor appears, and two secondary resonance regions. Near the first one, $\theta' \approx 2.5$, it is verified that the escape from the potential well leads to divergence to infinity and in this case the pendulum starts to have rotational motion, i.e., the pendulum escapes from the potential well, its motion is captured by the motor motion and both enter in a synchronized state, Belato (1998). This system behavior is detected inside of the potential well determined by the point of minimum $(\alpha', \alpha) = (0,0)$, "Fig. 2b".

In "Figure 2a" it is shown the behavior of the pendulum motion as the motor speed increases, when the control parameter c_8 is constant. We can note the existence of the separatrix (heteroclinic orbits), obtained by the undamped unforced pendulum equation, that separates bounded and unbounded solutions, "Fig. 2b". When we introduce the motor in the mechanism, some bifurcational characteristics appear on the system. One of them is observed in the region close to $\theta' \approx 2.5$, where a global bifurcation known as blue-sky catastrophe occurs. In this case, a periodic or quasiperiodic attractor loses stability disappearing from the phase space and the pendulum jumps to one of two remote unlimited attractors, i.e., it enters in a rotational motion. An explanation for this phenomenon is that there occurs the appearance of a homoclinic connection in the system. Due to this connection, the point $(\alpha', \alpha) = (0,0)$ becomes unstable when the motor speed travels through the domain $2.5 < \theta' < 5.0$, and for the values $\theta' > 5.0$ the pendulum motion converges to bounded periodic solutions oscillating around the point $\alpha \approx \pm(\pi/2)$, Belato (1998). This graphic only represents a schematic of the phenomenon of the electromotor pendulum, and results with more precision will be presented in future works.

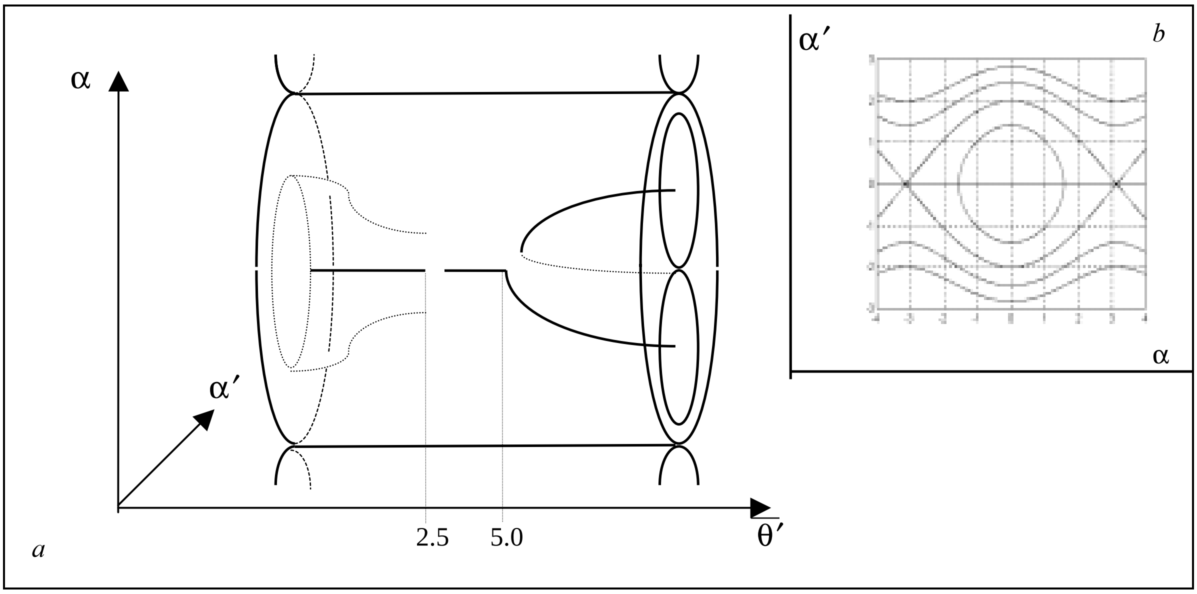

Figure 2 - Schematic representation of the phase portrait of the pendulum. (a) Characteristics of the phase portrait as the motor speed is increased. For $\overline{\theta'} \approx 2.5$ the solid line is discontinuous because in this region occurs a global bifurcation (called blue-sky catastrophe), where the attractor makes a rapid jump to a remote attractor. $\overline{\theta'}$ represents the mean of the motor's speed θ'. (b) Projection of the phase portrait in the plane $\alpha' \times \alpha$, as $\overline{\theta'} < 5.0$.

Now, we study the pendulum behavior considering the control parameter as defined in “Eq. 2”. The possibility of passage through resonance, where the motor speed may be captured by resonant speed, inciting the pendulum’s escape from the potential well will be investigated. Adopting the value p = 290 we obtain the numerical results presented in “Fig. 3” and “Fig. 4”.

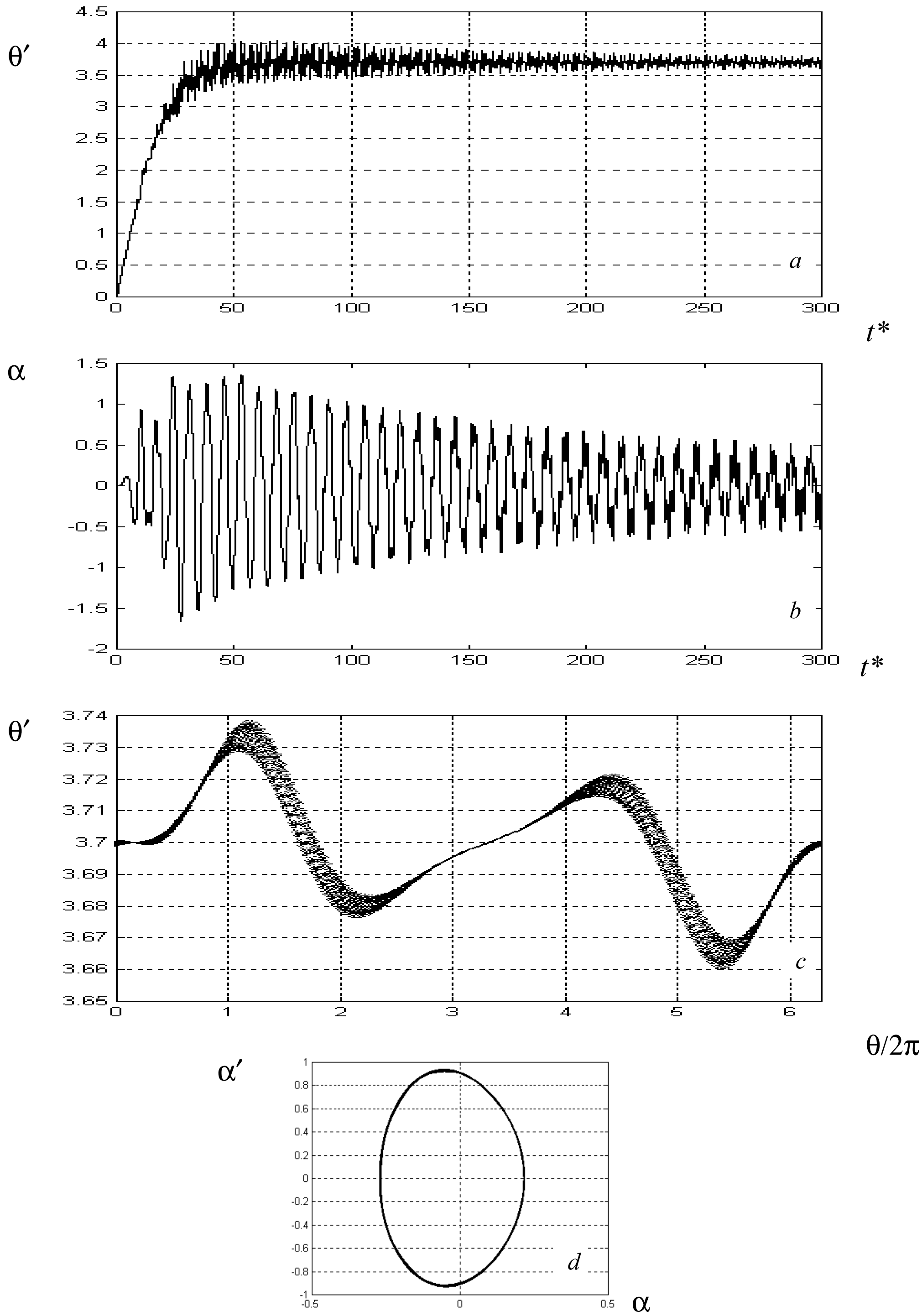

Figure 3 **-** System behavior for p = 290 and q = 0.05. (a) Motor speed transient. (b) Transient motion of the pendulum, asymptotically converging to the periodic attractor, presented in (d) with small amplitude. (c.) Phase space of the motor when the periodic attractor is reached. (d) The periodic attractor is not symmetric indicating that the point $(\alpha',\alpha)=(0,0)$ becomes unstable.

For q = 0.05, we noticed that the pendulum motion is limited and the solution converges to a periodic attractor represented in “Fig. 3d”. This solution is not symmetric, indicating the presence of instability in the point $(\alpha',\alpha)=(0,0)$, when the mean motor’s speed is $\overline{\theta}' \approx 3.7$. “Fig. 3c” shows the steady state of the motor behavior, when the periodic attractor is reached.

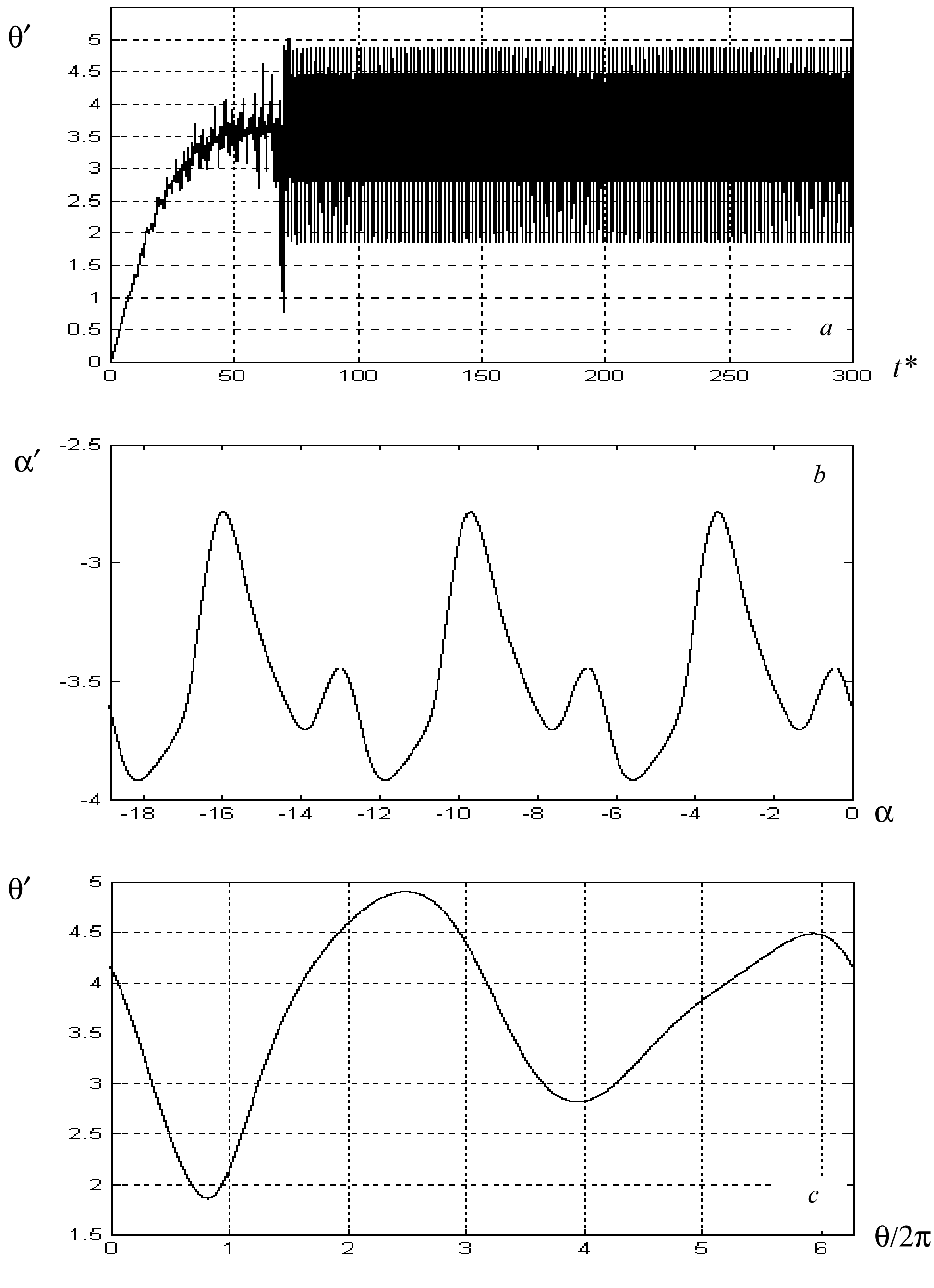

Figure 4 **-** System behavior for p = 290 and q = 0.04. (a) Temporal representation of the motor speed when the system enters in rotational motion. (b) Phase space of the pendulum steady state. (c.) Phase space of the motor steady state. Note the existence of a synchronized state among the motor motion and the pendulum motion.

Taking q = 0.04, we simulate a slower passage through the critical speed of this system. For this case, when the system pass through the resonant motor speed, $\theta' \approx 2.5$, the pendulum escapes from the potential well to an unlimited remote attractor, i.e., it can enter in rotational motion in clockwise (or anticlockwise) direction, "Fig. 4". In "Fig. 4a ", we can observe an increase in the amplitude of the motor motion, but its behavior looks regular and synchronized with the pendulum motion, "Fig. 4b and 4c". Although, the pendulum escape occurs due to the existence of a homoclinic trajectory, i.e., an oscillation of infinite period, but no irregular or chaotic motion in the system is detected when the motor velocity $\theta' \approx 2.5$ is reached. However, the proximity of the control parameter c_8 from the bifurcation point reveals a lack of precision to predict the final state of this system, because there occurs a decrease of the basin of attraction in this domain, leading to a basin catastrophe (this topic will be analyzed in a future work).

In "Fig. 5" is represented a initial variation of the control parameter c_8, when q = 0.04 and q = 0.05.

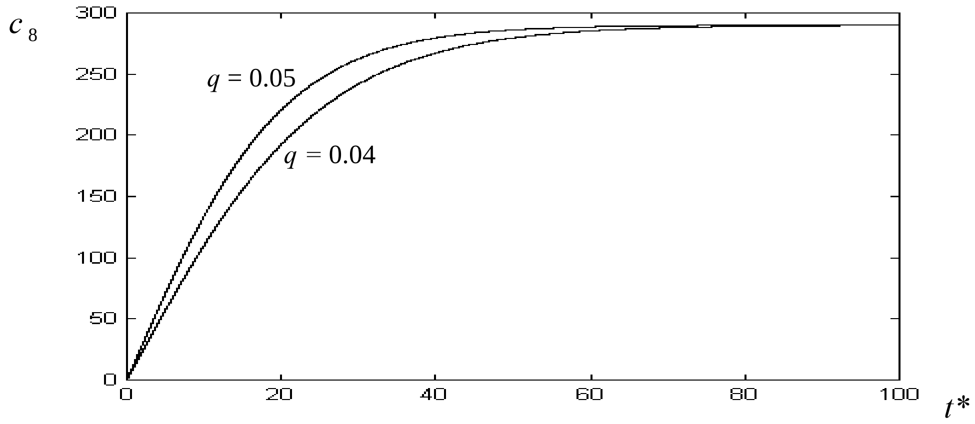

Figure 5 **-** Temporal representation of the function $c_8 = p\tanh(qt^*)$,
for q = 0.04 (rotational solution), and q = 0.05 (periodic attractor), when p = 290.

4. CONCLUSIONS

In this work, a particular nonideal nonlinear electro-mechanical system was analyzed. In the secondary resonance region, close to $\theta' \approx 2.5$, the system undergoes a global bifurcation (blue-sky catastrophe), which turns the solutions in this domain of the control parameter unbounded. When we consider a small variation of the transient behavior of the control parameter, we cannot predict when the system converges to the limited attractor or when it will be captured by the resonant motor speed and it will escape to a remote attractor. For the adopted parameters, the final solution becomes unpredictable but it does not reveal any irregular or chaotic behavior.

Acknowledgement

The authors thank the financial support of the FAPESP and CNPq.

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