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A HYBRID METHOD FOR AEROELASTIC ANALYSIS IN TRANSONIC FLOW

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Abstract. *Aeroelastic analyses of a fighter aircraft and of the standard aeroelastic wing AGARD 445.6 in transonic flow are presented. The analyses were performed using a finite element model and a modal model respectively. The doublet-lattice based module in NASTRAN™ was used with aerodynamic influence coefficients modified for transonic flow analysis. The aerodynamic modification was based on results from a finite-difference Navier-Stokes simulation of flow around the wings in transonic flow.*

Keywords: Transonic flow, aeroelasticity, unsteady aerodynamics, computational fluid dynamics, finite element analysis.

1. INTRODUCTION

It has been known for quite some time (Landahl,1951) that transonic flow conditions are critical for flutter, with the flutter dynamic pressure being substantially reduced for Mach numbers near unity, in a phenomenon usually termed as “transonic dip” (Whitlow,1987). The severity of flutter at transonic speeds is linked to the presence of moving shock waves over the wing surface (Ashley,1980). From these considerations, it is clear that accurate flutter predictions depend on the ability of the aerodynamic model to predict correct shock strength and location, in a time accurate fashion.

Most flutter computations use commercial finite-element codes with aeroelastic modeling capability such as NASTRAN™. These codes, however, are usually based on linear aerodynamic methods and thus limited to subsonic or supersonic analyses. Transonic flutter clearance relies on experience combined with costly and time-consuming wind tunnel and/or flight tests. More recently, computational aeroelasticity has allowed coupled aerodynamic /structural dynamic computations in the transonic regime . However, the computational resources needed for this coupled analysis are quite significant so its industrial application is still limited (Baker et al,1998).

As an alternative there are some methods for approximate modeling of non-linear transonic aerodynamics based on corrections of the linear aerodynamic influence coefficient matrix (AIC). The Transonic Equivalent Strip method (TES) (Liu et al, 1988) is one approach that shows good results on the capturing of the transonic dip phenomenon. This method is based on the application of two consecutive correction steps: one chordwise (mean flow) and other spanwise (phase correction) to a given steady mean pressure input from measured or computed data. Another approach is based on the local equivalence concept (Baker et al, 1998) which is based in an optimization procedure using computed or wind tunnel results.

The present work is based on a method proposed by Pitt and Goodman, who developed modifications of doublet-lattice influence coefficients using results from a Transonic Small Disturbance (TSD) code. That method was capable of simulating the transonic dip phenomenon with small differences with respect to wind tunnel data. These discrepancies were attributed by the authors to viscous effects. Indeed, viscous effects alter the strength and location of shock waves over the wing surface which in turn may have a significant effect on the flutter computations. In order to take viscous effects into account, the present method uses results from viscous simulations to modify the doublet lattice influence coefficients.

2. STRUCTURAL DYNAMIC MODEL

The structural properties used in these models have been basically extracted from manufacturer reports (Kolar & Lile, 1971) for the fighter aircraft and from technical reports for the AGARD wing (Yates,1988).

A beam model has been used for the F5E aircraft consisting of a finite element model of the aircraft structure represented by straight beams. It is an approximation of the major substructures of the airframe, namely the wings, the fuselage, and the tail cone. The beams are joined together with the adequate constraints applied. The resulting structure is, therefore, an approximation of the whole airframe. The criteria used here to match the experimental dynamic properties have been based on a manual adjustment of some parameters like the beam elements moments of area, the material elastic moduli, or the constraints. The airframe structural mass has also been represented as lumped masses located on the resulting nodes of the beam discretization. Since the mass properties are better known than the stiffness properties, changes in mass parameters during the adjustment process have been avoided. As a result of this adjustment process, the updated dynamic model has become coherently scaled with the dynamic properties of the actual aircraft. Damping effects were neglected in this model. The missiles and other external stores have dynamic characteristics such that, when compared with those of the airframe itself, create the possibility of considering the missiles as rigid bodies (Silva,1996).

For the AGARD wing the mode shapes, generalized mass and stiffness have been used to construct directly a modal model of this wing. With the modal vectors, it is possible to relate the structural degrees of freedom (nodes) via constraint relations, closing the mathematical model as a set of algebraic equations. In this case it is not necessary to perform any adjustment .

The dynamic analysis has been performed via finite element method for the assembly of the mass and stiffness matrices. The equations of motion , in the absence of aerodynamic forces, can be written as :

$$[M]\{\ddot{x}\}+[K]\{x\}=0 \tag{1}$$

The eigensolution of the resulting system has generated the natural frequencies and associated mode shapes. Remembering again structural damping has been neglected in the analytical modeling.

3. AERODYNAMIC MODEL

In the transonic regime, nonlinear aerodynamic effects are known to become very important. Of particular interest in this case is the sudden reduction in aeroelastic stability shown by some systems, in a phenomenon known as “transonic dip”. Then, it is necessary to consider these non-linearities with care by the use of an appropriate formulation.

The methodology presented here uses a mixed formulation based in the determination of the steady transonic pressure coefficient from a non-linear code in order to correct the nominal steady pressure coefficient distribution of the Doublet Lattice Method (DLM) (Albano & Rodden, 1969).

The correction has been introduced in the DLM code as a weighting factor that multiplies the generalized aerodynamic forces vector, and then the analysis has been made in the same manner as in the subsonic case.

A brief discussion of the aerodynamic theory will be presented next for a better understanding of the correction method proposed.

The inclusion of the aerodynamic loading which depends on the state variables of the structure in the equations of motion (1) yields:

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F(x,\dot{x})\} \tag{2}$$

The aerodynamic model for the right hand side of equation (2) has been based on a standard version of the doublet lattice method (DLM) (Rodden & Johnson, 1994). All the lifting surfaces of the aircraft have been discretized in terms of interfering panels which contain singular solutions of the unsteady acceleration potential equation for a given value of reduced frequency. These solutions are based on the Küssner relation between the acceleration potential (pressure) and the normalwash on two distinct points. The individual solution of each panel, as well as the interference of one panel onto others, can be represented by an algebraic form as:

$$\left\{\frac{P}{q_d}\right\}=[AIC]\left\{\frac{w}{U_\infty}\right\} \tag{3}$$

where $[AIC]$ is the influence coefficient matrix. It relates the panel to the downwash induced on all surface panels.

For the determination of the pressure vector in (3) it is necessary to know the induced downwash. From the boundary conditions for small perturbations the relationship between the normalwash and the solid boundary displacement is given by:

$$\left\{\frac{w}{U_\infty}\right\}=\frac{1}{U_\infty}\left(\frac{\partial\{z\}}{\partial t}+U_\infty\frac{\partial\{z\}}{\partial x}\right) \tag{4}$$

In aeroelastic analysis it is usually more convenient to employ a modal representation of the aeroelastic model, thus :

$$[m]\{\ddot{\eta}\}+[k]\{\eta\}=\{\overline{F}(\eta,\dot{\eta})\} \tag{5}$$

and equation (4) can be written as:

$$\left\{\frac{w}{U_\infty}\right\} = \frac{1}{U_\infty}\left([\Phi]\frac{\partial\{\eta\}}{\partial t} + U_\infty[\Phi_x]\{\eta\}\right) \quad (6)$$

remembering that $\{z\} = [\Phi]\{\eta\}$, and $\{\overline{F}\} = [\Phi]^T\{F\}$.

In the right hand side of equation (4) one may note that a substantial derivative is applied to the displacement vector {z}. The aerodynamic loading vector can be expressed by the use of equation (4) with the multiplication of the pressures by an integration matrix, which is constructed from the panel elements geometry. The normalwash vector can be substituted by relation (6), closing the right hand side as a function of the generalized coordinates of the system.

The next step is the correction of the AIC in order to account for non-linear effects present in transonic flow. The procedure is based in the correction of the AIC matrix using steady state pressure coefficients obtained by a Navier-Stokes code. The modification of the aerodynamic properties will be done only over the wing, where non-linear aeroelastic effects are more relevant, since the presence of shock waves over the wing generates strongly non-linear behavior.

The viscous computations used here were performed using an implementation of Roe's Flux Difference Splitting (FDS) method (Roe,1981),(Vinokur,1988), which is capable of good shock resolution. The computational grid used for the F-5 wing is illustrated in Fig. 1.

The Navier-Stokes computational fluid dynamics (CFD) procedure yields pressure coefficient C_p distributions over the wing surface. The difference in C_p between lower and upper surfaces of the wing is $\Delta C_p = C_p^l - C_p^u$. In the DLM, the lifting forces at each panel are concentrated on the ¼ element chord, at element midspan. Therefore, the CFD computed ΔC_p's are linearly interpolated to these locations on the wing surface.

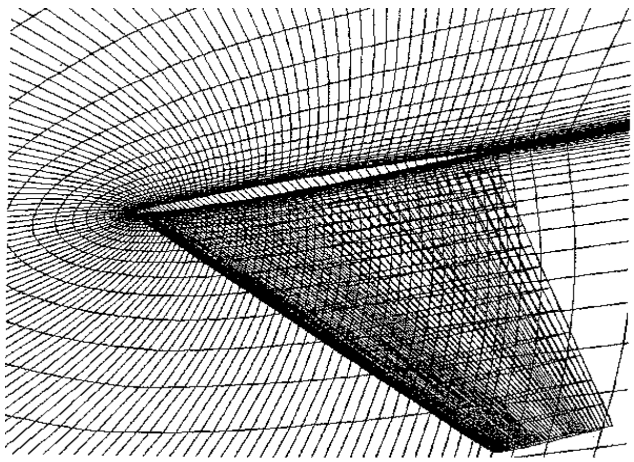


Figure 1- Finite difference mesh that discretizes the domain surrounding the wing.

The AIC matrix form the DLM corresponds to pressure per unit angular displacement at the panel. In the present formulation, CFD-computed panel pressures per unit angular displacement are calculated by performing numerical simulation at two angles of attack. The difference between the two results is then divided by the angle-of-attack perturbation. For the results presented in this paper the angles of attack were 0^o and 1^o. Future computation may be performed at higher angles in order to study flutter speed non-linearity with angle-of-attack.

In order to illustrate the above statement that the DLM AIC matrix corresponds to pressures per unit angular displacement, let us express equation (6) in the frequency domain:

$$\{w\}(ik) = \left\{[\Phi]ik + \left[\Phi_x\right]\right\}\{\eta\} \tag{7}$$

for $k = \omega b / U_\infty$ known as the reduced frequency and here $b = 1$ (semi-chord length). When the reduced frequency is set to zero, relation (7) becomes:

$$\{w\}(ik)_{k=0} = \left[\Phi_x\right]\{\eta\} \tag{8}$$

that is, the lifting surface motion reduces to a rotation in turn of the *y* axis, which is aligned with the wing span. The steady AIC matrix is obtained in NASTRAN^{TM} by setting a zero reduced frequency. This yields purely real elements in the AIC matrix. In this matrix, the columns associated to plunge movements are null, but columns associated with pitch will be non-null, as the panel plunge will appear as a translational component of the panel pitch.

The steady load vector, computed by the doublet lattice method, is formed by a real AIC matrix pre-multiplied by the integration matrix and post-multiplied by the physical displacements expressed in the form of equation (8). Then the final relation is:

$$\left\{F^D\right\} = [Q]\left\{\alpha_d\right\} \tag{9}$$

where $[Q] = [S][AIC]$ is the generalized AIC matrix that relates the loads for a given physical displacement of the panels, $[S]$ is the integration (area) matrix, and $\left\{\alpha_d\right\}$ is the assumed disturbance input resulting from the steady state boundary conditions.

The AIC matrix relates pressure coefficients to non-dimensional normalwash (equation 6), which is directly related to displacements for zero reduced frequency (8).

Consequently, a modified AIC matrix can be constructed by equating the left hand side of relation (9) with the computed load vector form the non-linear analysis:

$$\left\{F^C\right\} = [Q][W]\left\{\alpha_d\right\} \tag{10}$$

A weighting matrix $[W]$ is necessary to satisfy the system of equations (10) in order to match the desired loads. This matrix has a diagonal form and can be obtained solving the system:

$$\left\{F^C\right\} = [Q]\{C\} \tag{11}$$

Where $\{C\}$ is a vector of correction factors which can be expressed as:

$$\{C\} = [W]\left\{\alpha_d\right\} \tag{12}$$

recalling that $\left\{\alpha_d\right\}$ is a unit displacement vector and the diagonal elements of $[W]$ are the column elements of $\{C\}$.

These corrections can be included in NASTRAN^{TM} through the left-multiplication of matrix $[Q]$ by $[W]$ (Rodden & Johnson, 1994). This assumption was made in order to use the default feature which provides for a weighting matrix with correction factors. Of course, left-multiplying $[Q]$ by $[W]$ is not the same operation as right-multiplying $[Q]$ by $[W]$. Considering:

$$\{F^C\} = [Q][W]\{\alpha_d\} \qquad \text{and}$$

$$\{F^C\} = [W'][Q]\{\alpha_d\} \tag{13}$$

thus

$$[W'] = [Q][W][Q]^{-1} \tag{14}$$

The $[W']$ matrix is a full matrix. However, the off-diagonal elements are typically more than twenty times smaller than the diagonal elements. Therefore an approximate matrix $[W']$ with diagonal elements only is used instead of the full matrix. This approximation greatly simplifies implementation in NASTRAN^{TM}, since the built-in diagonal weighting matrix may be directly used.

4. TEST CASES

F-5E aircraft: The F-5E fighter aircraft, including external stores, has been modeled by sets of panels that discretize the lifting surfaces and some parts of the aircraft fuselage, such as the junctures between the wings and the main body (Fig. 2). A total of 546 panels are used, of which 112 are on the wing.

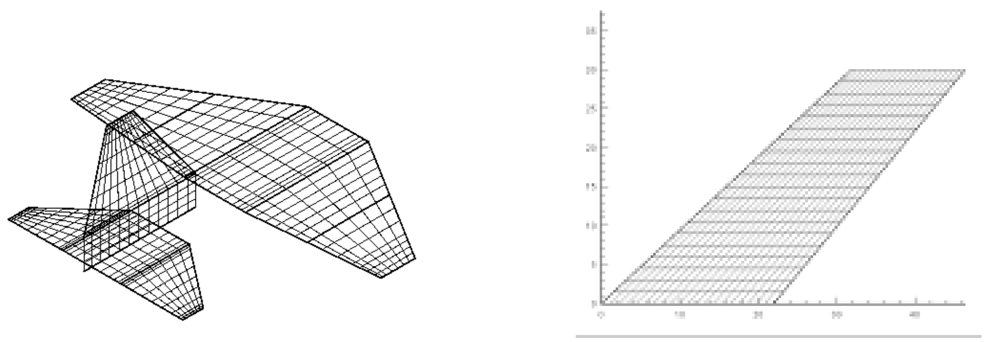


Figure 2- DLM paneling of the F-5E and the AGARD 445.6 wing

The missile main body and flippers have also been modeled as the projection of their spans into the main lifting surface plane (the aircraft wing). Like the aircraft lifting surfaces, they have also been subdivided into panels.

The transonic correction procedure is applied only to the main lifting surface (wing) panels. The analysis has been carried out for different Mach numbers, at sea level altitude in order to evaluate the behavior of the flutter velocity in these flow conditions.

Table 1- Flow conditions for F-5E aircraft aeroelastic analysis.

Mach Number	Density (kg/m^3)
0.60	1.225 (S.L.)
0.70	1.225 (S.L.)
0.80	1.225 (S.L.)
0.95	1.225 (S.L.)

AGARD 445.6 Wing: Known as a standard aeroelastic configuration, the AGARD 445.6 wing was discretized by the doublet lattice method as an isolated wing in different flow

conditions. The aerodynamic model is comprised of 400 panels (Fig. 2). The test conditions are the same as in (Yates,1988) and (Lee-Rausch and Batina, 1993), which present experimental and numerical results, respectively. The model under study is the weakened model (no. 3) which is described in (Yates,1988). The Mach numbers and air densities for the cases considered here are presented in Table 2.

Table 2- Flow conditions for AGARD wing 445.6 aeroelastic analysis

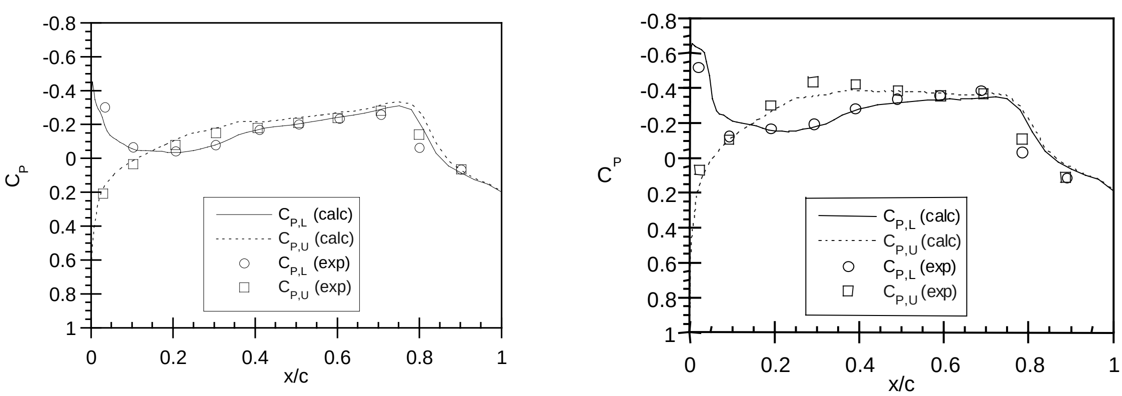
Mach Number	Density (slugs/ft^3)
0.678	0.000404
0.901	0.000193
0.960	0.000123

5. RESULTS

F-5E aircraft: The computational results of the Navier-Stokes simulation are compared with experimental data(Tijdeman et al,1979) for the case of the F-5E wing at a Mach number of 0.95, in order to verify the accuracy of the FDS solution in transonic flow. Results at two spanwise stations are presented in Fig. 3. Good agreement may be observed over all wing stations, although a few discrepancies are noted for the station near the wing tip. These discrepancies are associated with difficulties in obtaining a smooth grid around the wing tip within reasonably economical grid sizes.

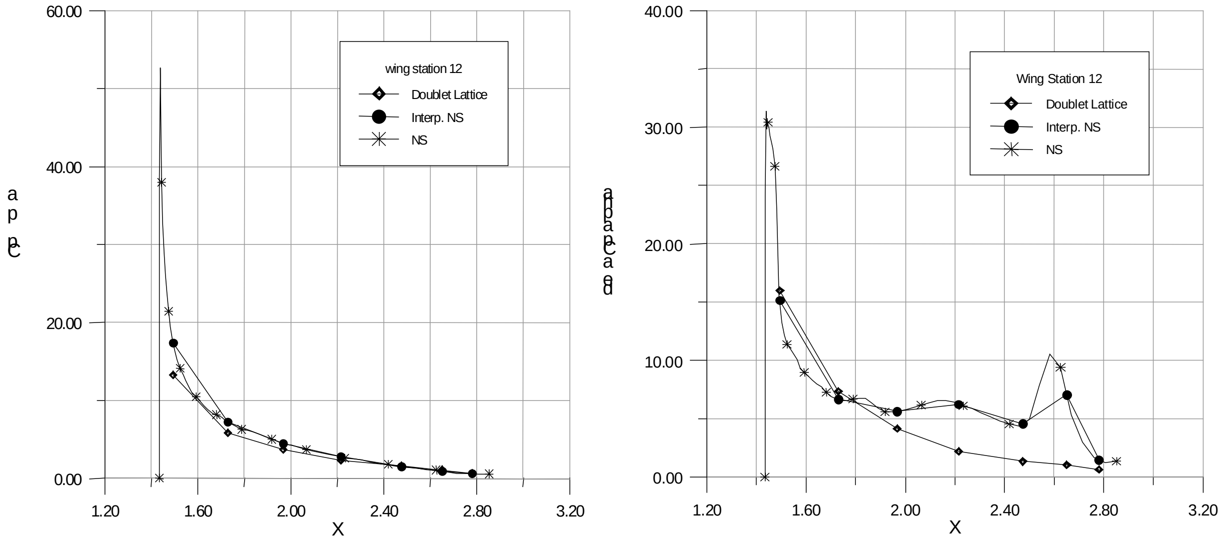
It should be noted that the corrections applied to the doublet lattice method take in account not only the non-linear effects of the transonic flow, but also thickness and viscosity effects. In a Doublet Lattice panel model the lifting surface is approximated by elementary flat plates (panels), that is, wing thickness is not represented. Fig. 4 presents a comparison between the pressure coefficient difference ΔC_p distributions obtained from doublet lattice and viscous methods, for subsonic flow (Mach number 0.6) . A similar comparison is presented also in Fig. 4 for a transonic condition (Mach number 0.95).

From Fig. 4, it may be observed that, for the subsonic case, the Prandtl-Glauert corrected result obtained from the DLM has a pressure difference close to the CFD-computed results. For the transonic case (Fig. 4) the computed pressures for the DLM and the CFD case are quite different, especially around the shock location, as expected.



(M=0.95, 18.1% span). (M=0.95, 97.7% span).

Figure 3- Pressure coefficient distribution over F-5E wing.



(Mach number = 0.6) (Mach number = 0.95)

Figure 4- Pressure per unit displacement in pitch.

The aeroelastic analysis results obtained with NASTRAN™ using the standard Prandtl-Glauert correction and the weighted non-linear correction method are summarized in Fig. 5. The configuration under study includes external stores which were ballasted in order to reduce their stability margin. The present results are for the aircraft with two air-to-air missiles and a centerline bomb. Four different mach numbers were investigated. The maximum dynamic pressure (sea level) has been considered.

From Fig. 5, it is seen that the flutter velocities obtained with the non-linear correction are smaller than the velocities obtained via Prandtl-Glauert correction.

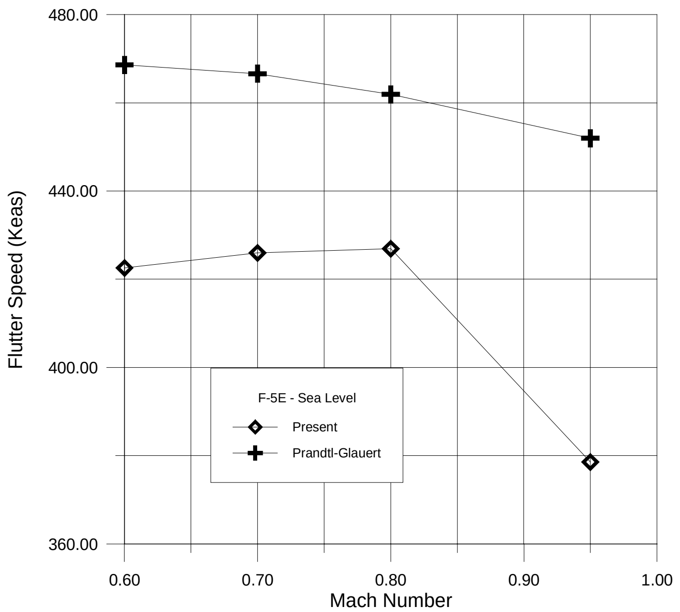


Figure 5- Variation of flutter speed with Mach no. (F-5E, sea level, armed configuration).

AGARD wing 445.6 weakened (no. 3): For this case, results obtained with the present non-linear correction method are compared to those obtained with the standard Prandtl-

Glauert correction, to results from the ZTAIC code (Chen et al, 1997) and to wind-tunnel results. These comparisons are illustrated in Fig. 6, for flutter speed index and frequency ratio, respectively. From these figures, it may be seen that the present method gives more conservative results than the standard linear method and shows better agreement with experiment in flutter speed index. However, flutter frequency ratios are not so well predicted. The authors believe that unsteady non-linear corrections using CFD-computed unsteady pressure coefficients may provide better agreement in flutter frequency. Nevertheless, considering that in a typical external store integration program one is concerned with safe flight envelopes, the capability of the present method to provide improved flutter speed predictions with respect to the linear method is an encouraging result.

It should be noted that a contributing factor to the present results may have been the fact that the case under analysis has a flutter speed very close to the transonic Mach number used for the non-linear correction. Therefore, additional validations have to be carried out before the method's capabilities may be established.

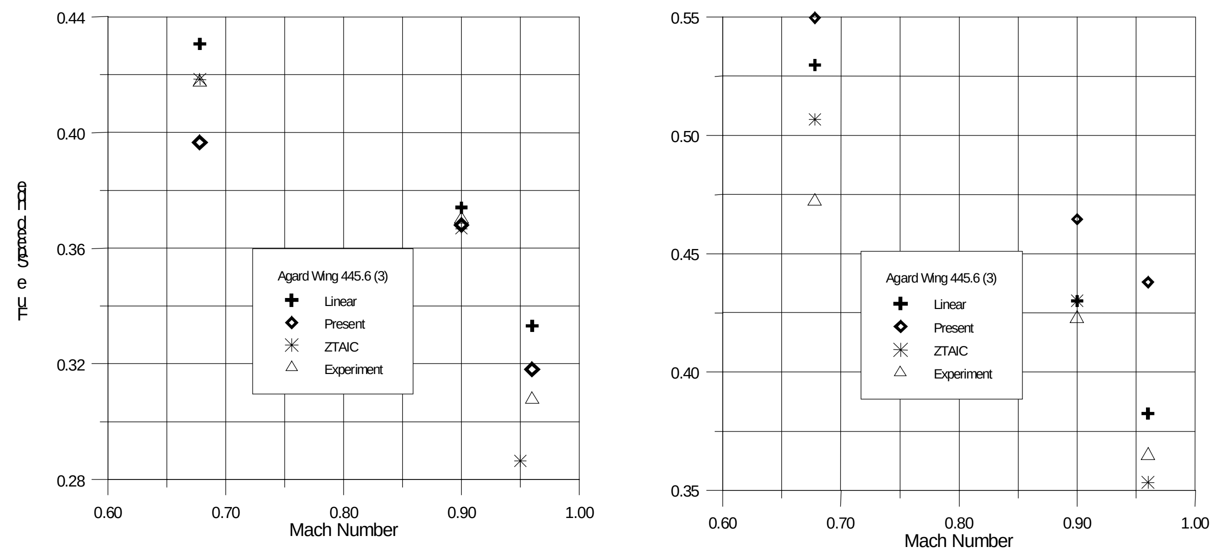


Figure 6: Variation of flutter speed index and frequency ratio with Mach No. (wing 445.6 weakened model no. 3).

6. CONCLUSIONS

An approximate non-linear correction method has been developed for transonic aeroelastic analysis. This correction modifies considerably the steady loading on the wing due to thickness and viscous effects and appropriate non-linear representation of transonic flow.

For the test cases under study, the proposed approximate non-linear correction yields essentially the same flutter frequency as the traditional Doublet Lattice based method. The flutter speed was better predicted by the present method, which is an encouraging result. However, it should be noted that a contributing factor may have been the fact that the case under analysis has a flutter speed very close to the transonic Mach number used for the non-linear correction. In future work, more cases will be studied in order to completely validate the procedure.

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