ANALYSIS OF REGULAR AND IRREGULAR DYNAMICS OF A NON-IDEAL GEAR RATTLING PROBLEM

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Abstract. *This paper presents a study on the dynamics of the rattling problem in gear boxes under non-ideal excitation. The subject has been analyzed by a number of authors, such as Karagiannis and Pfeiffer (1991), for the ideal excitation case. An interesting model of this same problem by Moon (1992) has been recently used by Souza and Caldas (1999) to detect chaotic behavior. We consider two spur gears with different diameters and gaps between the teeth. Suppose the motion of one gear to be given while the motion of the other is governed by its dynamics. In the ideal case, the driving wheel is supposed to undergo a sinusoidal motion with given constant amplitude and frequency. In this paper, we consider this motion to be a function of the system response and a limited energy source is adopted. Thus, an extra degree of freedom is introduced in the problem. The equations of motion are obtained via a Lagrangian approach with some assumed characteristic torque curves. Next, extensive numerical integration is used to detect some interesting geometrical aspects of regular and irregular motions of the system response.*

Key-words: *Gear rattling, non linear dynamics, non-ideal systems.*

1. INTRODUCTION

Rattling in change-over gears of automobiles in an unwanted comfort problem. It is excited by the torsional vibrations of the drive train system at the entrance of the gear box, where these torsional vibrations themselves are generated by imbalances of the engine. All gear-wheels not under load rattle due to backlashes in the meshes of the gears.

In recent years, general models have been developed to analyze these rattling phenomena, mainly with the goal to find some means to reduce them by parameter variation. They are founded on an procedure based on impact theory. The rattling vibrations pos-

sess typical non linear behavior leading to periodic and chaotic regimes. The subject has been analyzed by a number of authors, such as Karagiannis and Pfeiffer (1991), for the ideal excitation case. An interesting model of this same problem by Moon (1992) has been recently used by Souza and Caldas (1999) to detect chaotic behavior.

We confine our considerations to single stage rattling. We consider two spur gears with different diameters and gaps between the teeth. Suppose the motion of one gear to be given while the motion of the other is governed by its dynamics. In the ideal case, the driving wheel is supposed to undergo a sinusoidal motion with given constant amplitude and frequency. In this paper, we consider this motion to be a function of the system response and a limited energy source is adopted. Thus, an extra degree of freedom is introduced in the problem. The equations of motion are obtained via a Lagrangian approach with some assumed characteristic torque curves. Next, extensive numerical integration is used to detect some interesting geometrical aspects of regular and irregular motions of the system response.

2. MATHEMATICAL MODEL

Our model is presented in Fig. 1. We consider a cart of mass M connected to a inertial reference frame by a spring k (whose stiffness has a linear part k_1 and a non linear part k_2) and a linear viscous damper c. Its displacement is denoted by x. In the upper part of the cart, a d wide gap is carved. Within the boundaries of this gap a point mass m_2 (whose displacement is denoted by S) is free to move and eventually impact against them. The motion of the cart is induced by an in-board non-ideal motor driving an unbalanced rotor, whose angular displacement is φ and whose moment of inertia is J. This situation is modeled by a small point mass m_1 at a r eccentricity.

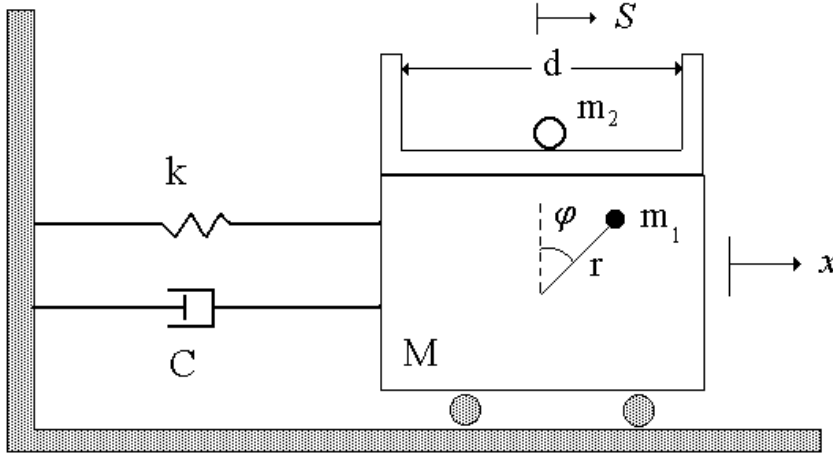Figure 1: The mathematical model

We now derive the equations of motion of the cart via Lagrange's equations:

$$\frac{d}{dt}\left[\frac{\partial Y}{\partial \dot{x}}\right] - \frac{\partial Y}{\partial x} = -c\dot{x}$$

$$\frac{d}{dt}\left[\frac{\partial Y}{\partial\dot\varphi}\right] - \frac{\partial Y}{\partial\varphi} = L(\dot\varphi) + D(\dot\varphi) \tag{1}$$

where Y is the Lagrangian function comprising the total potential energy and the kinetic energy. The active and resisting torques are $L(\dot\varphi)$ and $D(\dot\varphi)$, respectively, functions of the angular speed of the motor.

The total potential energy is:

$$V = \frac{1}{2}k_1 x^2 + \frac{1}{4}k_2 x^4 \tag{2}$$

The kinetic energy is:

$$T = \frac{1}{2}M\dot x^2 + \frac{1}{2}J\dot\varphi^2 + \frac{1}{2}m_1[\dot x - r\dot\varphi\cos(\varphi)]^2 + \frac{1}{2}m_1[r\dot\varphi\sin(\varphi)]^2 \tag{3}$$

Thus, we obtain the following equations of motion for the cart:

$$\frac{dx_1}{d\tau} = x_2$$
$$\frac{dx_2}{d\tau} = -x_1 - \epsilon[2\mu x_2 + \alpha x_1^3 - \ddot x_4\cos(x_3) + x_4^2\sin(x_3)]$$
$$\frac{dx_3}{d\tau} = x_4$$
$$\frac{dx_4}{d\tau} = \epsilon[G + I\ddot x_2\cos(x_3)] \tag{4}$$

where:

$$x_1 = x, \quad x_2 = \dot x, \quad x_3 = \varphi, \quad x_4 = \dot\varphi$$

$$2\epsilon\mu = \frac{c}{\sqrt{k_1(M+m_1)}}, \quad \tau = \sqrt{\frac{k_1}{M+m_1}}t$$

$$\epsilon\alpha = \frac{k_2}{k_1}r^2, \quad \epsilon = \frac{m_1}{M+m_1}, \quad L = E_1\exp(E_2 x_4)$$

$$\epsilon G = \frac{L+D}{J+m_1 r^2}\left[\frac{M+m_1}{k_1}\right], \quad D = x_4$$

where E_1 and E_2 are constants of the motor.

These equations will render movable boundaries to the motion of the point mass m_2. The motion of this point mass is simply given by the solution of the homogeneous equation

$$\ddot S = 0 \tag{5}$$

with initial conditions given by each impact with the movable boundaries.

In the next section, motions of both cart and free point mass are obtained via numerical integration.

3. NUMERICAL SIMULATIONS

First, a parameter study of the effect of the motor constant E_1 upon the motion of the cart is performed. Results displayed in Figures 2 and 3 show that this parameter will effect both the frequency and the amplitude of that motion.
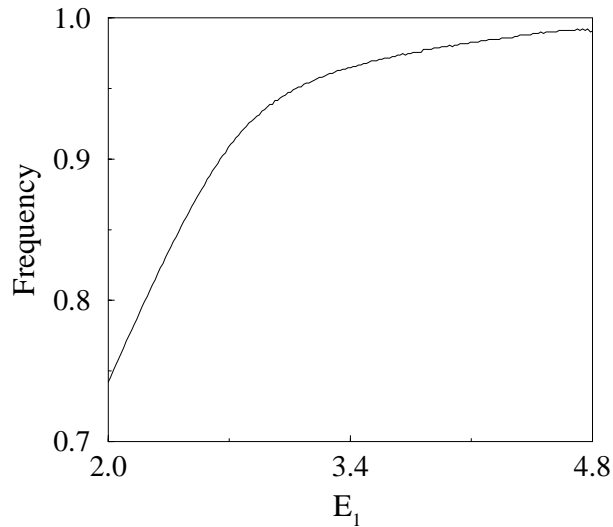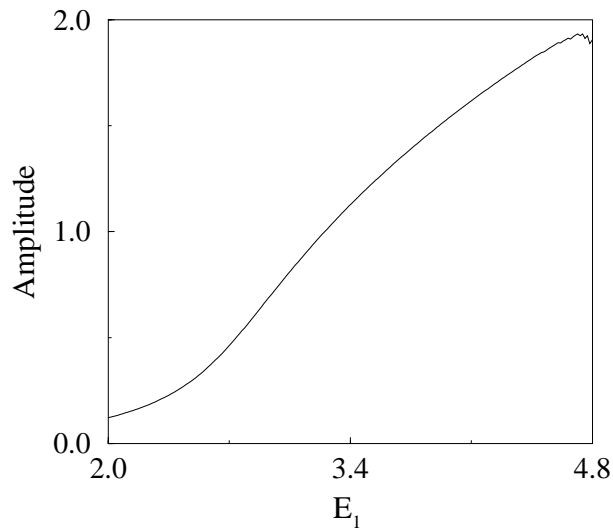
Figure 2: Frequency versus the motor parameter E_1.

Figure 3: Amplitude versus the motor parameter E_1.

The motion of the cart imply movable boundaries to the displacement of the point mass that is free to move inside the gap. It is interesting to see that in Figures 4 and 5

where heavy lines represent the position of the boundaries of the gap at each time τ and the thin lines the position of the free point mass. Figure 4 prompts a condition of regular motions while Fig. 5 depicts one of chaotic motions.
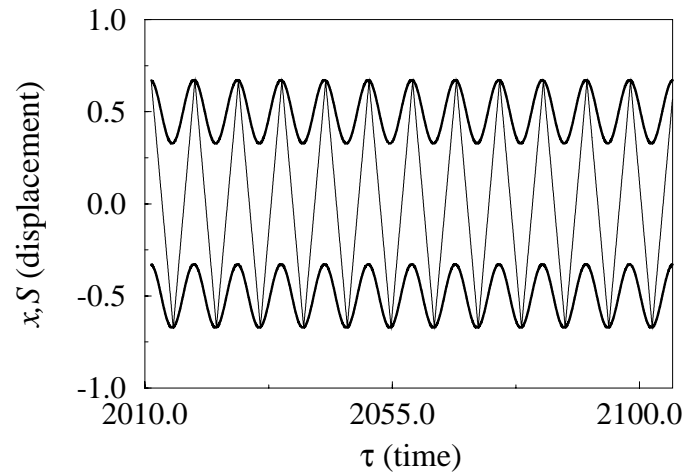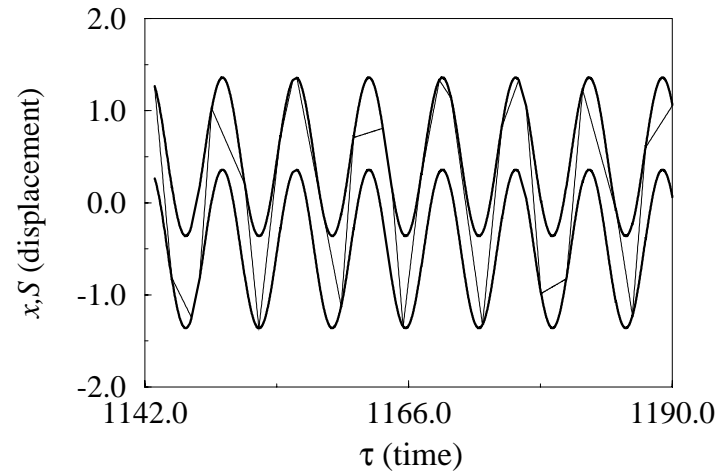
Figure 4: Periodic impact motion.

Figure 5: Chaotic impact motion.

For a better understanding of the phenomenon, a bifurcation diagram is prompted in Fig. 6, showing the speed $\dot{S}$ of the point mass in the moment just before the impact against the movable boundaries as function of the motor parameter E_1.
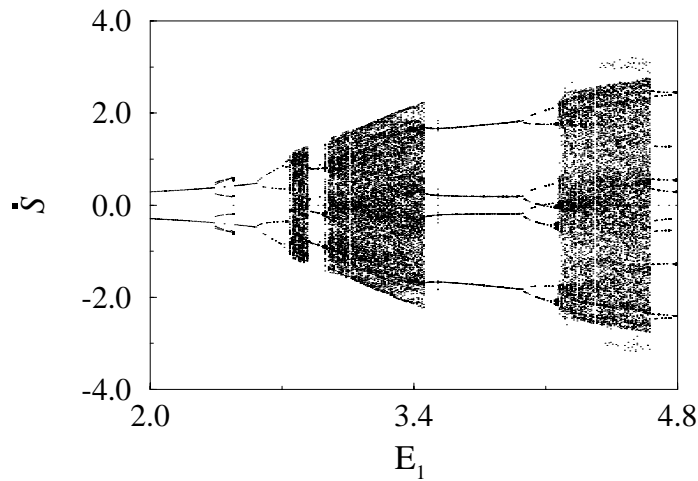
Figure 6: Bifurcation diagram of the velocity $\dot{S}$ versus the motor parameter E_1.

4. COMMENTS ON THE RESULTS

A simple mathematical model of the gear rattling phenomenon is proposed allowing for consideration of limited energy sources such as those liable to occur in practice.

Our investigations confirm that a rich bifurcation structure and chaotic behavior is present in the adopted model. This could be of interest as a possible explanation of the undesirable rattling observed in practical gear boxes.

Acknowledgements

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