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A Constitutive Model for Anisotropic Soft Biological Tissue Subject to Mechanical Damage

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ABSTRACT

Many soft biological tissues have the important task of transmitting necessary forces to control movements, hold organs in place or ensure a relatively continuous flow of fluids. Examples of these tissues are ligaments, tendons, skin and blood vessels. Biological tissues can be seen as composite materials whose constituents are assembled into a hierarchical structure. At a macroscopic structural level, the main structural elements of such tissues are the bundles of fibers that provide mechanical resistance and viscoelastic anisotropic behavior. This paper intends to extend a previously proposed anisotropic viscoelastic model in order to include mechanical damage that may be experimentally observed on biological tissues subject to large strains. The material model is based on a variational thermodynamically consistent framework, in which a local minimization provides the internal variables updates for each load increment. The main advantage of this model is its capability to represent the mechanical behavior of different materials depending on the choice or construction of potential functions. Numerical examples are presented in order to illustrate the features of the proposed model.

Keywords: Viscoelasticity, Anisotropy, Mechanical Damage, Finite strains.

1 INTRODUCTION

In this work is proposed a variational constitutive model appropriate to simulate the mechanical behavior of connective soft tissues (e.g. ligaments and tendons) subject to large strains, different loading velocities and damage. These soft biological tissues are mainly formed by arrangements of collagen fibers embedded in a cellular matrix. This internal structure provides an anisotropic mechanical response dependent on the fiber directions as well as a viscosity due to interstitial fluid and interaction among fibers. Many models have been proposed to model these kinds of materials relating different phenomena (large strains, hyperelasticity, anisotropy, viscosity, damage, etc.). Among them, we can mention [1], [2], [3], [4], [5] and [6]. In [7] a variational framework for viscoelastic anisotropic materials submitted to finite strain regime is proposed. The aim of the present work is to extend that model to include damage behaviors.

The theoretical framework in which this paper is based and the specific variational model that includes viscoelasticity, anisotropy and damage are described in Section 2 and Section 3 respectively. Section 4 shows some preliminary results using different damage functions proposed in literature. Final remarks are enclosed in Section 6.

2 VARIATIONAL CONSTITUTIVE MODEL

Hyperelastic models are based on the existence of a free energy function W which is dependent only on the total strain. Then, the first Piola-Kirchhoff stress tensor **P** is defined as

$$\mathbf{P} = \frac{\partial \mathrm{W}(\mathbf{F})}{\partial \mathbf{F}} = 2\mathbf{F}\frac{\partial \mathrm{W}(\mathbf{C})}{\partial \mathbf{C}} \tag{1}$$

where $\mathbf{F}$ is the gradient of deformation and $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$ the Cauchy-Green tensor.

The stress state of dissipative materials is dependent not only on the total strain but also on the strain history. In order to overcome this difficulty an approach based on variational concepts was proposed in [8] in which the constitutive problem can be stated analogously to Eq.(1) in an incremental way. Variational constitutive models are based on pseudo-hyperelastic potentials where the constitutive problem can be stated analogously to a hyperelastic model. In this approach a pseudo-potential energy, also called the Incremental Potential, is defined at each load step, providing the first Piola-Kirchhoff stress as follows:

$$\mathbf{P}_{n+1} = \frac{\partial \psi(\mathbf{F}_{n+1};\, \xi_n)}{\partial(\mathbf{F}_{n+1})} = 2\mathbf{F}_{n+1}\frac{\partial \psi(\mathbf{C}_{n+1};\, \xi_n)}{\partial(\mathbf{C}_{n+1})} \tag{2}$$

In this expression, $\xi = \{\mathbf{F}, \mathbf{F}^i, \mathbf{Q}\}$ is the set of external and internal state variables. The elastic and inelastic gradients of deformation $\mathbf{F}^e$ and $\mathbf{F}^i$ are obtained from the multiplicative decomposition of $\mathbf{F}$. The symbol $\mathbf{Q}$includes all remaining internal variables related to the dissipative phenomena. In [8] it is shown that the Incremental Potential may have the expression:

$$\Psi(\mathbf{F}_{n+1};\, \xi_n) = \min_{\mathbf{F}^i_{n+1}, \mathbf{Q}_{n+1}} \left\{W(\xi_{n+1}) - W(\xi_n) + \Delta t \psi\left(\dot{\mathbf{F}}^i, \dot{\mathbf{Q}}; \xi_n\right)\right\} \tag{3}$$

where the free energy is conveniently decomposed additively into elastic (φ,φ^e) and inelastic contributions (φ^i):

$$W(\xi) = \varphi(\mathbf{F}) + \varphi^e\left(\mathbf{F}{\mathbf{F}^i}^{-1}\right) + \varphi^i(\mathbf{F}^i, \mathbf{Q}) \tag{4}$$

The strain energy is decomposed additively into elastic and inelastic contributions φ, $\varphi^e$and φ^i, depending on the total value of $\mathbf{F}$ on the elastic part $\mathbf{F}^e$ and on the inelastic part $\mathbf{F}^i$ and internal variables $\mathbf{Q}$respectively. $\psi$is the (pseudo) potential that provides the dependence of the stress on the rate (incremental approximation of rate) variables $\dot{\mathbf{F}}^i$ and $\dot{\mathbf{Q}}$.

The minimization problem (3) identifies the optimal values of $\mathbf{F}^i{}_{n+1}$and $\mathbf{Q}_{n+1}$, which define the internal variables associated with the new state $\mathbf{F}_{n+1}$. Once this minimization problem is solved, stresses may be computed by Eq.(2) as in hyperelastic models. Different material models may be constructed in this general framework depending on the particular choices and arrangements of potentials φ, φ^e, φ^i and ψ.

3 ANISOTROPIC VISCOELASTIC MODEL SUBJECT TO DAMAGE

The anisotropic viscoelastic damage model proposed here is an extension of the work [7] in order to introduce mechanical damage on the fiber-reinforcement. The objective of this inclusion is twofold: Firstly, to account for the loss of resistance due to fibers damage that becomes significant on finite strains. Secondly, the proposed damage model allows for the representation of the so called Mullins effect; according to experimental observations during a load cycle, there exist a loss of stiffness at strain levels below the maximum previously attained in a previous cycle. The consideration of this phenomenon has an important impact in the capability of the model to represent experimental data.

The inclusion of fibers is performed following concepts shown in [9] by an additive decomposition of energies:

$$\Psi = \Psi_{\text{iso}} + \Psi_{\text{f}}^{\text{d}} \tag{5}$$

where, Ψ_{iso} was proposed in [10] and accounts for the isotropic response of the material, while $\Psi_{\text{f}}^{\text{d}}$ is related to the fiber contribution subject to mechanical damage.

The fiber-reinforcement contribution proposed in [7] is modified to take into account the mechanical damage. Figure 1 shows a rheological representation of the addition (5) in which both the isotropic and the fiber contributions are connected in parallel, reacting independently of each other for the same total strain. The additive decomposition in Eq.(5) states that the incremental potential of the isotropic matrix and that of the fibers are uncoupled. The same happens for the elastic and Maxwell branches of the fiber contribution. Each branch depends on the given strain increment over Δt and the constitutive response of the composite comes only from the additive constitutive response of each component, which is clearly illustrated in Figure 1. Moreover, this model considers that fibers are continuously distributed in the isotropic ground substance (matrix) [9] and therefore, no distinction is made on the size or length of them.

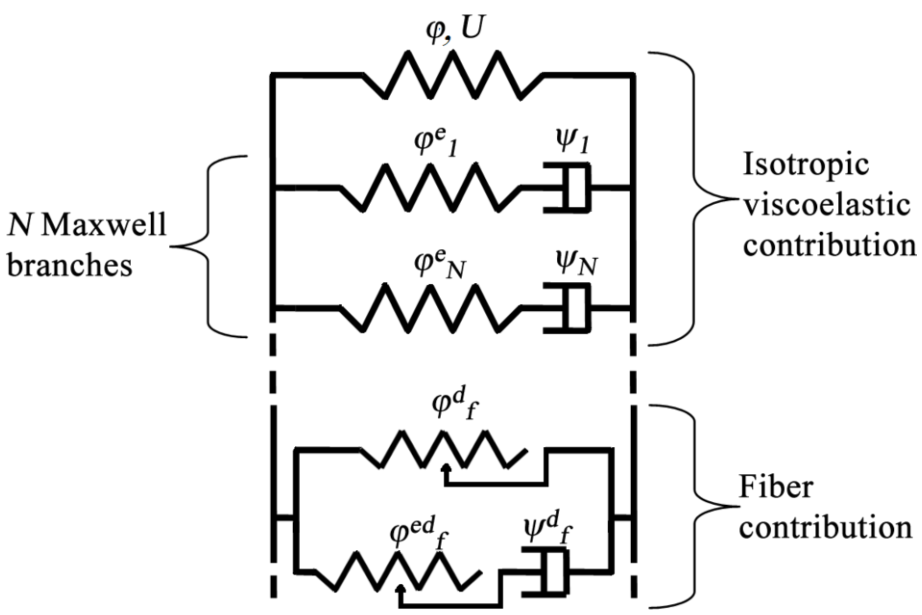

Figure 1: Rheological model

3.1 Isotropic Incremental Potential

As mentioned above, the incremental potential Ψ_{iso}used in this work is exactly that proposed in [10] and correspond to the rheological representation of Figure 1. The gradient of deformation is decoupled into volumetric and isochoric parts $\mathbf{F} = J\hat{\mathbf{F}}$with $J = \det(\mathbf{F})$. The isochoric part allows also for a multiplicative separation into elastic and viscous contributions: $\hat{\mathbf{F}} = \hat{\mathbf{F}}^e\mathbf{F}^v$. With these hypotheses, the free energy W is defined as

$$\Psi(\mathbf{C}) = U(J) + \varphi(\hat{\mathbf{C}}) + \varphi^e(\hat{\mathbf{C}}^e); \quad \hat{\mathbf{C}} = \hat{\mathbf{F}}^T\hat{\mathbf{F}}; \quad \hat{\mathbf{C}}^e = \hat{\mathbf{F}}^{e\,T}\hat{\mathbf{F}}^e \tag{6}$$

Potential φ and U, associated to the first branch of Figure 1, account for the elastic strain energy accumulated due to total isochoric and volumetric quantities $\hat{\mathbf{C}}$ and J, respectively. The potentials φ, $\varphi^e$and ψ are assumed to be isochoric functions of the Cauchy tensors $\hat{\mathbf{C}}$, $\hat{\mathbf{C}}^e$ and of the viscous stretching $\mathbf{D}^v$ by means of their respective eigenvalues c_i , c_i^e and d_i^v , i.e., $\varphi(\hat{\mathbf{C}}) = \varphi(c_1, c_2, c_3), \varphi^e(\hat{\mathbf{C}}^e) = \varphi^e(c_1^e, c_2^e, c_3^e)$ and$\psi(\mathbf{D}^v) = \psi(d_1^v, d_2^v, d_3^v)$.The viscous stretching $\mathbf{D}^v$ is defined by

$$\mathbf{D}^v = \mathrm{Sym}(\mathbf{L}^v) = \mathbf{L}^v = \dot{\mathbf{F}}^v\mathbf{F}^{v-1} \tag{7}$$

The viscous flow is assumed to be isochoric by means of the following constrains on the spectral components of $\mathbf{D}^v$:

$$\mathbf{D}^v = \sum_{j=1}^{3} \frac{\Delta q_j{}^v}{\Delta t}\mathbf{M}^v{}_j \tag{8}$$

$$\Delta q_j{}^v \in K_Q = \{p_j \in \mathbb{R}: p_1 + p_2 + p_3 = 0\}$$

$$\mathbf{M}^v{}_j \in K_M = \{\mathbf{N}_j \in sym: \mathbf{N}_j.\mathbf{N}_j = 1, \mathbf{N}_i.\mathbf{N}_j = 0, i \neq j\}$$

From these definitions, it is shown in [10] that at each time increment Δt, the isotropic Incremental Potential in Eq.(3) takes the form

$$\Psi_{iso} = \Delta U(J_{n+1}) + \Delta\varphi(\hat{\mathbf{C}}_{n+1}) + \min_{\mathbf{M}^v{}_j, \Delta q_j{}^v}\{\Delta\varphi^e(\hat{\mathbf{C}}^e{}_{n+1}) + \Delta t\psi(\mathbf{D}^v{}_{n+1})\} \tag{9}$$

subject to the set of constrains (Eq.8) that keeps the viscous stretching isochoric where:

$$\Delta[\cdot]((\cdot)_{n+1}) = [\cdot]((\cdot)_{n+1}) - [\cdot]((\cdot)_n) \tag{10}$$

The minimization of (9) with respect to $\mathbf{M}^v{}_j$is analytically performed, while the minimization with respect to $\Delta q_j{}^v$leads to a system of four nonlinear optimality conditions. If Newton method is chosen to solve it, the solution provides the stress tensor calculated by the classic "hyperelastic-like" expression and the symmetric analytical tangent modulus needed for the global equilibrium problem. Detailed of the model and all mathematical operations are found in[10].

3.2 Anisotropic Viscoelastic Damage Incremental Potential

The corresponding Incremental Potential for anisotropic viscoelastic damage behavior Ψ_f^d follows the same structure proposed in [7] and can be schematically represented in the second part of the Figure 1. Due to anisotropic contribution of fibers, the response depends not only on the Cauchy tensor $\mathbf{C}$, but also on the structural tensor $\mathbf{A}_f = \mathbf{a}_f \otimes \mathbf{a}_f$, where $\mathbf{a}_f$ is the unit vector defining the fiber direction. This dependence in the present case is related to the invariant I_f [1]:

$$I_f = \hat{\mathbf{C}} : \mathbf{A}_f = \mathbf{a}_f . \hat{\mathbf{C}} . \mathbf{a}_f = \lambda_f^2 \tag{11}$$

and has the particular physical meaning of the quadratic stretch λ_f^2 in the direction of the fiber. Other possible invariants were here avoided to keep the model as simple as possible and also due to the difficulties associated to material parameter identification. The second branch of the fiber contribution in Figure 1 is associated to the decomposition of the total elongation λ_f into elastic and viscous contributions $\lambda_f = \lambda_f^e \lambda_f^v$. The logarithmic strains related to the elongations and viscous stretching are defined as usual:

$$\epsilon_f = \ln \lambda_f \, ; \quad \epsilon_f^e = \ln \lambda_f^e \, ; \quad \epsilon_f^v = \ln \lambda_f^v \, ; \quad {d^v}_f = {\dot{\lambda}_f^v} \big/ {\lambda_f^v} \tag{12}$$

The incremental evolution of the viscous stretch is obtained using the exponential mapping proposed in [10] that allows us to write

$$\lambda^v_{f_{n+1}} = \exp\left(\Delta t {d^v}_f\right) \lambda^v_{f_n} \, ; \quad {d^v}_f = \frac{1}{\Delta t} \ln\left(\frac{\lambda^v_{f_{n+1}}}{\lambda^v_{f_n}}\right) \tag{13}$$

The novelty of this work is the inclusion of two new internal variables η and η^e related to damage, modifying the free energy accumulated by the reinforcement. The free energy of the fibers involves two terms of the form

$$W = \varphi_f^d(\lambda_f, \eta) + \varphi_f^{ed}(\lambda_f^e, \eta^e) = (1-\eta)\varphi_f(\lambda_f) + (1-\eta^e)\varphi_f^e(\lambda_f^e) \tag{14}$$

Finally, the dissipative function takes the form:

$$\psi = \psi_f\left(d_f^v\right) + \chi_f(\dot{\eta}, \eta) + \chi_f^e(\dot{\eta}^e, \eta^e) \tag{15}$$

and completes the model providing with the necessary information to define the evolution of internal variables λ_f^v, η and η^e.

Substituting all former potentials in (3) and rearranging terms, the anisotropic incremental potential associated to fibers takes the expression:

$$\Psi_f^d = \Psi_f^E + \Psi_f^M \tag{16}$$

where

$$\Psi_f^{\mathrm{E}} = \min_{\eta_{n+1}} \left\{ \Delta\varphi_f^d \left(\lambda_{f_{n+1}}, \eta_{n+1} \right) + \Delta t \chi_f(\dot{\eta}, \eta_{n+\alpha}) \right\} \tag{17}$$

$$\Psi_f^{\mathrm{M}} = \min_{\lambda^v_{f_{n+1}}, \eta^e_{n+1}} \left\{ \Delta\varphi_f^{ed} \left(\lambda^e_{f_{n+1}}, \eta^e_{n+1} \right) + \Delta t \psi_f \left(d^v_{f_{n+1}} \left(\lambda^v_{f_{n+1}} \right) \right) + \Delta t \chi^e_f(\dot{\eta}^e, \eta^e_{n+\alpha}) \right\} \tag{18}$$

such that

$$\Delta\varphi_f = (1 - \eta_{n+1}) \varphi_f \left(\lambda_{f_{n+1}} \right) - (1 - \eta_n) \varphi_f \left(\lambda_{f_n} \right) \tag{19}$$

$$\Delta\varphi_f = \left(1 - \eta^e_{n+1}\right) \varphi^e_f \left(\lambda^e_{f_{n+1}} \right) - \left(1 - \eta^e_n\right) \varphi^e_f \left(\lambda^e_{f_n} \right) \tag{20}$$

$$\dot{\eta} = \frac{\Delta\eta}{\Delta t} = \frac{\eta_{n+1} - \eta_n}{\Delta t}; \ \dot{\eta}^e = \frac{\Delta\eta^e}{\Delta t} = \frac{\eta^e_{n+1} - \eta^e_n}{\Delta t} \tag{21}$$

As mentioned, fibers are assumed to contribute in strain energy only for positive stretching. To this aim, it is assumed that

$$\varphi_f = \begin{cases} \bar{\varphi}_f & \text{se } \lambda_f > 0 \\ 0 & \text{se } \lambda_f \le 0 \end{cases}; \quad \varphi^e{}_f = \begin{cases} \bar{\varphi}^e_f & \text{se } \lambda^e_f > 0 \\ 0 & \text{se } \lambda^e_f \le 0 \end{cases}; \quad \psi_f = \begin{cases} \bar{\psi}_f & \text{se } d^v_f > 0 \\ 0 & \text{se } d^v_f \le 0 \end{cases} \tag{22}$$

Finally, χ_f and χ^e_f are proposed to be homogeneous functions of degree one of $\dot{\eta}$ and $\dot{\eta}^e$ respectively. This characteristic turns the response of the model (stress) independent of rates in damage. Rate dependence on this variable may however be included later without any theoretical difficulty, if needed. Finally, an exact penalization for negative damage rates (increments), is included as follows:

$$\chi_f(\dot{\eta}, \eta) = \begin{cases} Y(\eta)\dot{\eta} & \text{se } \dot{\eta} > 0 \\ +\infty & \text{se } \dot{\eta} \le 0 \end{cases}; \quad \chi^e_f(\dot{\eta}^e, \eta^e) = \begin{cases} Y^e(\eta^e)\dot{\eta}^e & \text{se } \dot{\eta}^e > 0 \\ +\infty & \text{se } \dot{\eta}^e \le 0 \end{cases} \tag{23}$$

From this definition, it is possible to see that functions Y and Y^e play the role of thermodynamic forces conjugated to η and η^e, since

$$Y(\eta) = \frac{\partial \chi_f(\dot{\eta}, \eta)}{\partial \dot{\eta}}; \quad Y^e(\eta^e) = \frac{\partial \chi^e_f(\dot{\eta}^e, \eta^e)}{\partial \dot{\eta}^e} \tag{24}$$

Expressions (17) and (18) need the evaluation of the dissipative functions (and their derivatives) at an intermediate time $n + \alpha$, where α is an algorithmic parameter that influences the precision and convergence of model, such that $\alpha \in (0, 1]$. Then, the classical midpoint-rule is used:

$$Y_\alpha = (1 - \alpha) Y(\eta_n) + \alpha Y(\eta_{n+1}) \tag{25}$$

$$Y^e_\alpha = (1 - \alpha^e) Y^e(\eta^e_n) + \alpha^e Y^e(\eta^e_{n+1}) \tag{26}$$

During computations, however, specific values for $\alpha$and α^e are needed. It is possible to show that convenient optimal values for the algorithmic parameters are obtained by the following expressions:

$$\alpha = \frac{Y(\eta_{n+1}) - Y(\eta_n)}{Y(\eta_{n+1}) - Y(\eta_n) + \frac{\partial Y(\eta_{n+1})}{\partial \eta_{n+1}} \Delta\eta} \tag{27}$$

$$\alpha^e = \frac{Y^e(\eta^e_{n+1}) - Y^e(\eta^e_n)}{Y^e(\eta^e_{n+1}) - Y^e(\eta^e_n) + \frac{\partial Y^e(\eta^e_{n+1})}{\partial \eta^e_{n+1}} \Delta\eta^e} \tag{28}$$

3.3 Material Models

Once the general constitutive framework is defined by expressions (9) and (17, 18), different particular models can be obtained depending on the specific expressions for the potentials.

For the isotropic contribution (9), different classical hyperelastic laws, like the Neo-Hookean, Mooney-Rivlin, Ogden and Hencky models, can be used for potentials φ, $\varphi^e$and ψ. The Hencky expressions are of the type

$$\varphi = \mu \sum_{j=1}^{3} (\varepsilon_j)^2 ; \quad \varphi^e = \mu^e \sum_{j=1}^{3} (\varepsilon^e{}_j)^2 ; \quad \psi = \eta^v \sum_{j=1}^{3} (d^v{}_f)^2 \tag{29}$$

while Ogden expressions are written as

$$\varphi = \sum_{j=1}^{3} \sum_{p=1}^{N} \frac{\mu_p}{\alpha_p} \left((\exp \varepsilon_j)^{\alpha_p} - 1 \right); \varphi^e = \sum_{j=1}^{3} \sum_{p=1}^{N} \frac{\mu^e_p}{\alpha^e_p} \left((\exp \varepsilon_j)^{\alpha^e_p} - 1 \right); \tag{30}$$

$$\psi = \sum_{j=1}^{3} \sum_{p=1}^{N} \frac{\eta^v_p}{\alpha^v_p} \left((\exp d^v_j)^{\alpha^v_p} - 1 \right) \tag{31}$$

Symbols μ,μ^e,η^v,μ_p, α_p, μ^e_p, α^e_p, $\eta^v_p$and $\alpha^v_p$are material parameters to be identified. More details on this issue are found in [10].

For the reinforcement contribution given by (17, 18), different functions of hyperelastic fiber-materials are found in the literature. In this paper we use that shown in [1]:

$$\bar{\varphi}_f = {k_1}/{2k_2} \left\{ \exp\left[k_2 (I_f - 1)^2 \right] - 1 \right\} \tag{32}$$

Identical potential is used for the elastic term $\bar{\varphi}^e_f$. For the dissipative potential $\bar{\psi}_f$ the quadratic Henky-type law (29) showed to be adequate.

Finally, a key issue in the present context is the appropriate expression for the damage laws $Y(\eta)$ and $Y^e(\eta^e)$present in (23). In the present case, the function $Y(\eta)$is related to the energy released by a variation in damage, which is the classical interpretation in damage models. Within this approach, a simple but effective choice is:

$$Y(\eta) = \xi \eta^{p} \tag{33}$$

where ξ and p are material parameters.

4 RESULTS

In order to verify the ability of the proposed model to represent the viscoelastic-damage behavior, two uniaxial strain controlled tests are presented. Since the anisotropic behavior was extensively investigated in [7], here only the damage effect on the fiber direction is presented. The formulation proposed in this work was implemented in the commercial software MatLab (Mathworks, Natick, MA). The material parameters presented in Table 1 do not have any correlation with real materials and have been chosen to test the capabilities of the model.

Two uniaxial strain-controlled tests were performed. The first of them use Eq.(17) only, which represents a single anisotropic elastic-damage branch of the rheological model (see Figure 1). The other was performed using Eq.(18) only, which account for the behavior of the viscous elastic-damage branch of Figure 1. The history of the applied strain and the resulting stress-strain curves are presented in Figs. 2-3. In Figure 2 it is possible to see the stress softening in subsequent cycles due to damage (simulation of the Mullins effect). As expected, each new loading follows the same path of the previously unloading until reaching the highest strain of the previous cycle. In addition, for conveniently large strain values, a clear damage is observed causing a pronounced drop of stress values.

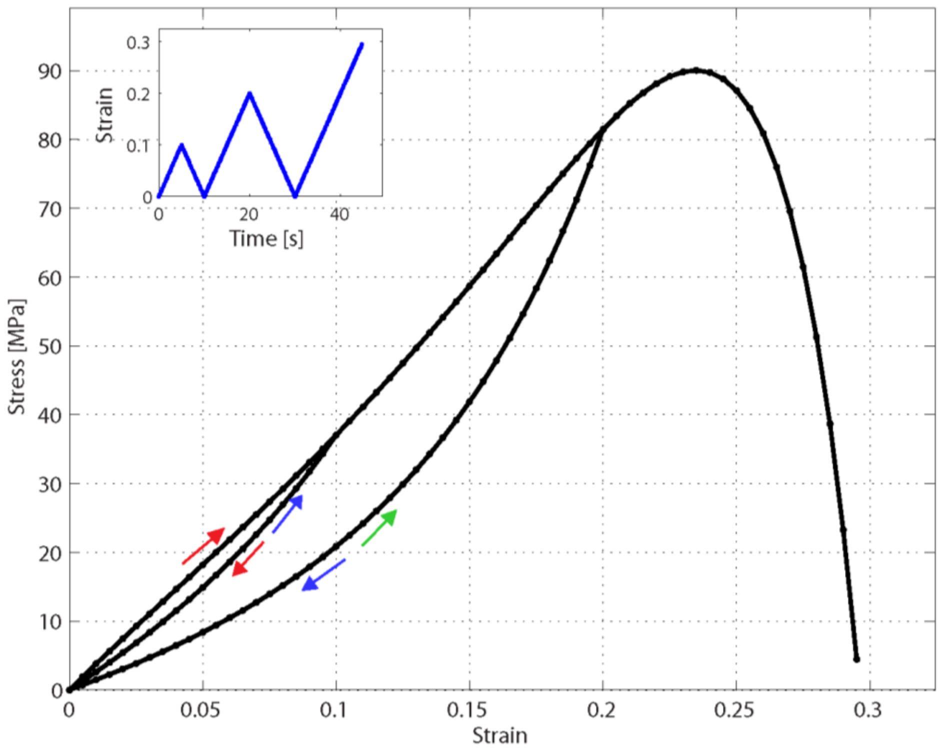

Figure 2: Stress response of the elasto-damage fiber contribution

Furthermore, it is clearly seen in Figure 3 the characteristic rate dependency of the viscous elastic-damage branch. In this case, besides the reproduction of the Mullins-like phenomenon, it is visible a hysteretic behavior within each cycle.

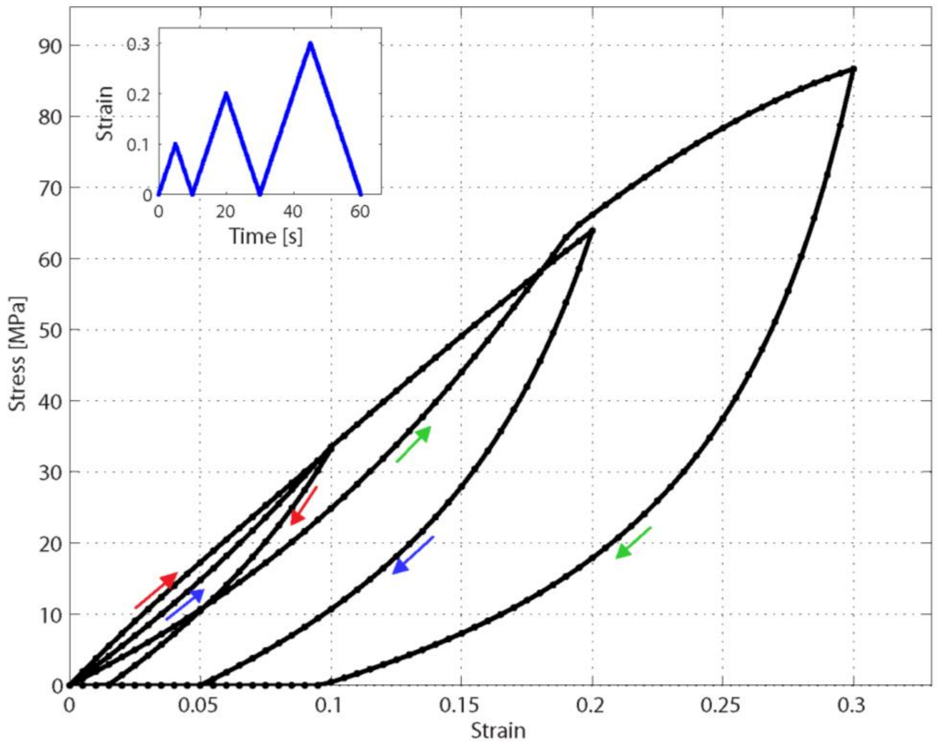

Figure 3: Stress response of the viscoelastic damage fiber contribution

Table 1: Material parameters

Potential	Model	Parameter
φ^e	Holzapfel	$k_1 = 100$MPa; $k_2 = 3$
Y	Eq. (33)	ξ =50 MPa; $p = 3$
φ_f^e	Holzapfel	$k_1 = 100$MPa; $k_2 = 3$
ψ_f	Hencky	$\eta_f = 5000$MPa/s^{-1}
Y^e	Eq. (33)	ξ^e =50 MPa; $p^e = 3$

5 CONCLUSIONS

The main objective of this work that consists on extending a previous viscoelastic model for fiber-reinforced viscoelastic materials to include damage was achieved. Preliminary numerical results show that the addition of two new damage-like internal variables turns the model capable of representing the well-known Mullins effect as well as the stress-drop at high strains. Moreover, these phenomena are consistently coupled with the viscoelastic behavior typically found in soft biological tissues. Again, it is worth repeating that present examples have the goal of verifying the ability of the proposed approach to follow expected qualitative behaviors. The use of experimental tests in order to perform the identification of material parameters is the subject of ongoing work.

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REFERENCES

[1] Holzapfel G, Gasser TC, (2001), "A viscoelastic model for fiber-reinforced composites at finite strains: Continuum basis, computational aspects and applications", Computer Methods in Applied Mechanics and Engineering,190 (34):4379-4403.

[2] Limbert G, Taylor M, Middleton J., (2004), "Three-dimensional finite element modeling of the human acl: simulation of passive knee flexion with a stressed and stress-free acl.",Journal of Biomechanics, 37(11):1723-1731.

[3] Calvo B, Peña E, Martinez MA, Doblaré M., (2007),"An uncoupled directional damage model for fibred biological soft tissues. Formulation and computational aspects.", International Journal for Numerical Methods in Engineering, 69(10):2036-2057.

[4] Peña E, Calvo B, Martinez MA, Doblaré M., (2008), "Infinite-strain damage of viscoelastic-fibred materials. Application to soft biological tissues", International Journal for Numerical Methods in Engineering, 74(7):1198-1218.

[5] Rodriguez JF, Alustre V, Doblaré M., (2008),"Finite element implementation of a stochastic three dimensional finite-strain damage model for fibrous soft tissue. Application to soft biological tissues", Computer Methods in Applied Mechanics and Engineering, 197(9-120):946-958.

[6] Peña E., (2008), "Damage functions of the internal variables for soft biological fibred tissues", Mechanics Research Communications, 38(8):610-615.

[7] Vassoler JM, Reips L, Fancello EA., (2012), A variational framework for fiber-reinforced viscoelastic soft tissues", International Journal for Numerical Methods in Engineering, 89(13):1691-1706.

[8] Ortiz M, Stainier L., (1999),"The variational formulation of viscoplastic constitutive updates", Computer Methods in Applied Mechanics and Engineering, 171(3-4):419-444.

[9] Holzapfel GA., (2000), "Nonlinear solid mechanics: a continuum approach for engineering", Wiley, 2000.

[10] Fancello EA, Ponthot JP, Stainier L., (2006), "A variational formulation of constitutive models and updates in nonlinear finite viscoelasticity", International Journal for Numerical Methods in Engineering", 65(11):1831-1864.

[11] Anand L, Weber G., (1990), "Finite deformations constitutive equations and a time integration procedure for isotropic hyperelastic-viscoplastic solids", Computer Methods in Applied Mechanics and Engineering, 79(2):173-202.

[12] Ogden RW, Roxburgh DG., (1999),"A pseudo-elastic model for the Mullins effect in filled rubber", Proceedings: Mathematical, Physical and Engineering Sciences, 455(1988):2861-2877.