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BAYESIAN ESTIMATION OF NANOFLUIDS THERMAL PROPERTIES WITH THE TRANSIENT LINE HEAT SOURCE PROBE

Lamien Bernard, lamienbernard@hotmail.com
Helcio Rangel Barreto Orlande, helcio@mecanica.coppe.ufrj.br
PEM/COPPE/UFRJ, Caixa Postal: 68503, Cidade Universitária, Rio de Janeiro, 21941-972

Abstract. *The transient line heat source probe technique is widely used for the measurement of the thermal conductivity of nanofluids. In this work, we present a mathematical model that takes into account the probe and the surrounding nanofluids;and thus permits the simultaneous estimation of the nanofluid and the probe thermal properties. Since several parameters appearing in the formulation are not deterministicaly known, a technique within the Bayesian framework is used for the solution of the inverse problem. To ensure minimum variance in the estimated parameters, the D-optimal approach is used, together with the analysis of the sensitivity coefficients for the design of the experiment. Simulated temperature measurements are used in the inverse analysis.*

Keywords: Nanofluids, Thermophysical Properties, Line heat source probe, Bayesian statistics, MCMC method.

1. INTRODUCTION

The thermal loads are increasing in a wide variety of applications like microelectronics, transportation, etc. Micromechanical systems (MEMS) technology and nanotechnology are also rapidly emerging as a new revolution in miniaturization. Hence the management of high thermal loads in these systems offers challenges and the thermal conductivity of heat transfer fluids have become vital. Traditional heat transfer fluids such as water, engine oil, and ethylene glycol (EG) usually have low thermal conductivity (Chandrasekar and Suresh, 2009). Metals in the solid form have thermal conductivities larger by three orders-of-magnitude than usual heat transfer fluids. For example, the thermal conductivity of water at room temperature is 0.6 W/m K, while for copper is 386 W/m K. These three orders of magnitude between the thermal conductivity of fluids and metals makes one consider since one hundred years ago, the possible enhancement of thermal conductivity of liquids by suspending metal particles in them (Patel et al, 2003). However, early investigations have concerned millimeter or micrometer-sized particles, and the resulting solutions suffered from severe sedimentation (Murshed et al., 2008).

Following this idea, Choi (1995) imagined the application of the emerging nanotechnology to thermal engineering to create a new generation of fluids with enhanced thermal properties. This new class of fluids, named as nanofluids, is engineered by dispersing metal-oxides or metals nanoparticles or nanotubes in traditional heat transfer fluids. In contrast to micrometer-sized particles, nanoparticles, due to their high surface to volume ratio, can remain in suspension and thereby reduce erosion and channels obstructions. Furthermore, due to their small size, nanoparticles are more appropriate for use in micro-systems (Murshed et al., 2008). Most of the published works on nanofluids shows that they present enhanced thermal properties than the pure base fluids. As thermal conductivity is one of the most important parameter responsible for enhancing heat transfer, numerous experiments in nanofluids were concerned with its determination. The literature survey shows that the obtained results are dispersed, and many factors such as particles size and shape, particles volume fraction, temperature, base-fluid, dispersion, etc. are supposed to have an influence on the nanofluid's thermal properties. On the other hand, several mechanisms and models of enhanced conductivity have been proposed, but none has gained universal support (Choi, 2009).

Most of the nanofluid thermal conductivity measurements reported in the literature have been conducted by using the transient hot wire method, or the line heat source probe. However, in the classical use of this technique, some ideal hypotheses like the non-participation of the probe in heat transfer and the infinite size of the sample, can lead to errors in the resulting estimates. Recently, inverse parameter estimation which involves accurate modeling has been successfully applied to the line heat source probe, allowing the simultaneous estimation of more than one parameter, in addition to the thermal conductivity (André et al., 2003; Thomson and Orlande, 2006; Banaszkiewicz et al., 1997; Carvalho and Neto, 1999). A common feature of these works is that the solution of the inverse problem was retrieved deterministically. Therefore, they provide no means to quantify the uncertainties associated with the supposedly "known" parameters. On the other hand, Bayesian inference approaches to inverse problems offer a way to cope with the uncertainties of all parameters appearing in the formulation, by using a complete probabilistic description via prior modeling. This approach provides a natural framework for uncertainties quantification.

In this work, we present a mathematical model for a line heat source probe that takes into account the probe and the surrounding material. Since all the parameters appearing in the model are not accurately known, a technique within the Bayesian framework, known as the Markov Chain Monte Carlo based on Metropolis-Hastings algorithm (MCMC-MH) is used for the solution of the inverse problem, as described below.

2. LINE HEAT SOURCE PROBE

The classical Hot-Wire technique (Blackwell, 1954) consists in a constant heat power generation by Joule effect through a thin cylindrical wire placed inside the material that is assumed to be a semi-infinite medium (no heat losses). The temperature rise of the wire is then measured. At longer times, the Hot-Wire temperature evolution is shown to be a linear function of the logarithm of time. Knowing the heat power dissipation, the thermal conductivity of the material is computed according to the following formulae, where m is the slope of the curve (Blackwell, 1954):

$$k = \frac{Q}{4\pi m} \tag{1}$$

In this work we use a commercial line heat source probe, the Hukseflux TP-02 which is shown in figure 1. It consists of a needle of stainless steel with 150 mm length and 1.5 mm external diameter, connected to a stainless steel base with 50 mm length and 10 mm external diameter. Inside the needle there's a heating wire, as well as two K type thermocouples connected in a way to provide the temperature difference between the probe and the medium. Inside the base there's a PT-1000 temperature sensor for the measurement of the temperature of the cold joints of the thermocouples**.**

Figure 1: TP02 line heat source probe from Hukseflux

3. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem under analysis consists of a long and thin cylinder (the probe) of radius r_s and high thermal conductivity k_s, assumed as a lumped system. The probe is inserted into the fluid with unknown properties, which is contained in a cylindrical cell. The fluid and the cell are both considered to be hollow cylinders with internal radius r_s, r_{int} and external radius r_{int}, r_{ext}, respectively. We assume heat conduction in homogeneous and isotropic media, with constant thermal properties. Furthermore, it is assumed a perfect thermal contact at both probe/liquid, and liquid/cell interfaces, while for t > 0, the surface at $r = r_{ext}$ exchange heat with a surrounding liquid in a thermostatic bath. The system is assumed to be initially in thermal equilibrium, at the temperature T_0. For the time scale of interest, t > 0, the probe is uniformly heated with a time dependent heat source g(t). By neglecting end-effects, the physical problem under picture can be formulated as one dimensional. The proposed mathematical formulation is divided into two parts. The first one corresponds to the heating period and the second one to the non-heating period. For the sake of brevity, it is presented below only the mathematical formulation for the heating period.

$$C_s^* * \frac{d\,\Theta_s(\tau)}{d\tau} = 1 + 2 * K_f^* * \left.\frac{\partial \Theta_f(R,\tau)}{\partial R}\right|_{R=1} \quad , \ for \quad \tau > 0 \tag{2}$$

$$\frac{1}{\alpha_f^*} * \frac{\partial \Theta_f(R,\tau)}{\partial \tau} = \frac{1}{R} * \frac{\partial}{\partial R}\left(R * \frac{\partial \Theta_f(R,\tau)}{\partial R} \right) \ , in \ 1 < R < \mathrm{R_{int}} \ , for \ \tau > 0 \tag{3}$$

$$\frac{1}{\alpha_m^*} * \frac{\partial \Theta_m(R,\tau)}{\partial \tau} = \frac{1}{R} * \frac{\partial}{\partial R}\left(R * \frac{\partial \Theta_m(R,\tau)}{\partial R} \right) \ , in \ \mathrm{R_{int}} < R < \mathrm{R}_{ext} \ , for \ \tau > 0 \tag{4}$$

$$\Theta_s(\tau) = \Theta_f(R,\tau) \quad at\ R=1,\ \ \tau>0 \tag{5}$$

$$\Theta_f(R,\tau) = \Theta_m(R,\tau)\ \ at\ R = R_{\text{int}}\ ,\ \tau>0 \tag{6}$$

$$K_f^* * \frac{\partial \Theta_f(R,\tau)}{\partial R} = K_m^* * \frac{\partial \Theta_m(R,\tau)}{\partial R}\quad at\ R = R_{\text{int}}\ ,\ \tau>0 \tag{7}$$

$$K_m^* \frac{\partial \Theta_m(R,\tau)}{\partial R} + Bi * \Theta_m(R,\tau) = 0\ \ at\ \ R = \mathrm{R}_{ext}\ ,\ \tau>0 \tag{8}$$

$$\Theta_s(\tau) = \Theta_f(R,\tau) = \Theta_m(R,\tau) = 0 \quad in\ 1 \le R \le R_{ext},\ at\ \tau = 0 \tag{9}$$

where the subscripts *f*, *m* and *s* denote the fluid, the cell and the probe, respectively. The following dimensionless parameters were introduced:

$$\Theta(R,\tau) = \frac{T(r,t)-T_0}{\frac{g(t)*r_s^2}{k_{ref}}};\qquad \tau = \frac{k_{ref} * t}{C_{ref} * r_s^2};\qquad \alpha_f^* = \frac{\alpha_f}{\alpha_{ref}};\quad R = \frac{r}{r_s} \tag{10.a-d}$$

$$K_f^* = \frac{k_f}{k_{ref}};\qquad K_m^* = \frac{k_m}{k_{ref}};\qquad \alpha_m^* = \frac{\alpha_m}{\alpha_{ref}};\qquad C_s^* = \frac{C_s}{C_{ref}};\qquad Bi = \frac{h*r_s}{k_{ref}} \tag{10.e-i}$$

where the subscript *ref* denotes a reference value.

4. DIRECT PROBLEM AND INVERSE PROBLEM

The *direct problem*, associated with the formulation given above for the physical problem, consists in determining the temperature distribution from the knowledge of the heating and final times, the geometry and the thermal properties of the probe and of the surrounding materials. The solution of the *direct problem* was obtained from an implicit finite volume discretization, which yields a tridiagonal matrix system of equations (Patankar, 1980). Grid parameters were obtained, by solving analytically an analogous heat conduction problem in a single medium with a time-dependent heat source (probe) of strength g(t) along its axis (ASME V&V, 2009). In order to match the lumped body assumption for the probe, it was considered as a highly conductive material. This problem was solved with the *Classical Integral Transform Technique* (Ozisik, 1993). The numerical solution graphically matched the analytical one for 140 and 20 volumes, respectively, for the fluid and the cell.

The *inverse problem* of concern aims at the simultaneous estimation of the probe's volumetric heat capacity (C^*_s), the liquid's thermal properties (K_f^* and α_f^*), and the cell's thermal properties (K_m^*, α_m^*), as well as the Biot number (*Bi)* at the surface of the cell, which are regarded as unknown. In addition, temperature measurements of the probe temperature are supposed available. Such measurements may contain random errors, which are assumed to be additive, uncorrelated, normally distributed with zero mean and with known and constant standard deviation.

For the solution of the *inverse problem*, a technique within the Bayesian framework is used. Such technique is described next.

5. MARKOV CHAIN MONTE CARLO (MCMC) METHODS

In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision-making problem. As new information is obtained, it is combined with any previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes' theorem (Lee, 2004). Therefore, the term Bayesian is often used to describe the so-called statistical inversion approach, which is based on the following principles (Kaipio and Somersalo, 2004):

1. All variables included in the model are modeled as random variables.
2. The randomness describes the degree of information concerning their realizations.

3. The degree of information concerning these values is coded in probability distributions.
4. The solution of the inverse problem is the posterior probability distribution.

Consider, for the sake of generality, the vector of parameters appearing in the physical model formulation as

$$\boldsymbol{P}^T = [P_1, P_2, \dots, P_N] \tag{11}$$

and the vector of the available measurements as

$$\boldsymbol{Y}^T = [Y_1, Y_2, \dots, Y_I] \tag{12}$$

where N is the number of parameters and I is the number of measurements. Bayes' theorem can then be stated as (Kaipio and Somersalo, 2004):

$$\pi_{posterior}(\boldsymbol{P}) = \pi(\boldsymbol{P}|\boldsymbol{Y}) = \frac{\pi_{prior}(\boldsymbol{P})\pi(\boldsymbol{Y}|\boldsymbol{P})}{\pi(\boldsymbol{Y})} \tag{13}$$

where $\pi_{posterior}(\boldsymbol{P})$ is the posterior probability density, that is, the conditional probability of the parameters $\boldsymbol{P}$ given the measurements $\boldsymbol{Y}$; $\pi_{prior}(\boldsymbol{P})$ is the prior density, that is, the coded information about the parameters prior to the measurements; $\pi(\boldsymbol{Y}|\boldsymbol{P})$ is the likelihood function, which expresses the likelihood of different measurement outcomes $\boldsymbol{Y}$ with $\boldsymbol{P}$ given; and $\pi(\boldsymbol{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant.

In practice such normalizing constant is difficult to compute and numerical techniques like Markov Chain Monte Carlo are required in order to obtain samples that accurately represent the posterior probability density. In order to implement the Markov Chain, a density $q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})$ is required, which gives the probability of moving from the current state in the chain $\boldsymbol{P}^{(t-1)}$ to a new state $\boldsymbol{P}^*$.

The Metropolis-Hastings algorithm (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004) was used in this work to implement the MCMC method. It can be summarized in the following steps:

1. Sample a *Candidate Point* $\boldsymbol{P}^*$ from a jumping distribution $q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})$

2. Calculate:

$$\alpha = \min\left[1, \frac{\pi(\boldsymbol{P}|\boldsymbol{Y})\, q(\boldsymbol{P}^{(t-1)}, \boldsymbol{P}^*)}{\pi(\boldsymbol{P}^{(t-1)}|\boldsymbol{Y})\, q(\boldsymbol{P}^*, \boldsymbol{P}^{(t-1)})}\right] \tag{14}$$

3. Generate a random value U which is uniformly distributed on (0,1).

4. If $U \le \alpha$, define $\boldsymbol{P}^{(t)} = \boldsymbol{P}^*$; otherwise, define $\boldsymbol{P}^{(t)} = \boldsymbol{P}^{(t-1)}$

5. Return to step 1 in order to generate the sequence $\{\boldsymbol{P}^{(1)} \boldsymbol{P}^{(2)}, \dots, \boldsymbol{P}^{(n)}\}$.

In this way, we get a sequence that represents the posterior distribution and inference on this distribution is obtained from inference on the samples $\{\boldsymbol{P}^{(1)}, \boldsymbol{P}^{(2)}, \dots, \boldsymbol{P}^{(n)}\}$. We note that values of $\boldsymbol{P}^{(i)}$ must be ignored until the chain has not converged to equilibrium. For more details on theoretical aspects of the Metropolis-Hastings algorithm and MCMC methods, the reader should consult references (Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004).

We assume in this work that the errors in the measured variables are additive, uncorrelated, normally distributed, with zero mean and known constant standard-deviation. Hence, the likelihood function is given by (Beck and Arnold, 1977, Kaipio and Somersalo, 2004, Tan et al, 2006, Lee, 2004):

$$\pi(\boldsymbol{Y}|\boldsymbol{P}) = (2\pi)^{-M/2}\, |\boldsymbol{W}|^{-1/2}\, \exp\left\{-\frac{1}{2}[\boldsymbol{Y} - \boldsymbol{T}(\boldsymbol{P})]^T \boldsymbol{W}^{-1}[\boldsymbol{Y} - \boldsymbol{T}(\boldsymbol{P})]\right\} \tag{15}$$

where $\boldsymbol{T}$ is the vector of estimated temperatures obtained from the solution of the direct problem with an estimate for the parameters $\boldsymbol{P}$, and $\boldsymbol{W}$ is the covariance matrix of the measurements

Generally, before the solution of an inverse problem is investigated, a sensitivity analysis together with a D-optimal design of the experiment needs to be performed. Such examinations give an indication of the best sensor location and measurements times to be used in the inverse analysis which correspond to linearly independent sensitivity coefficients with large absolute values and large magnitudes of the information matrix determinant (Ozisik and Orlande, 2000).

6. SENSITIVY ANALYSIS AND D-OPTIMAL DESIGN OF THE EXPERIMENT

The sensitivity coefficient J_{ij} is defined as the first derivative of the temperature at time t_i, with respect to the unknown parameter P_j, that is,

$$J_{ij} = \frac{\partial T_i}{\partial P_j} \tag{16}$$

Together with the information matrix determinant $(|\boldsymbol{J}^T\boldsymbol{J}|)$, the analysis of the sensitivity coefficients plays an important role in inverse parameter estimation. As a whole, it is desirable to obtain linearly independent sensitivities coefficients with large magnitudes. Such conditions enable the solution of the inverse problem to be less sensitive to measurements errors. The sensitivity matrix is defined as (Ozisik and Orlande, 2000):

$$\boldsymbol{J} = \left[\frac{\partial \boldsymbol{T}^T}{\partial \boldsymbol{P}}\right]^T = \begin{bmatrix} \frac{\partial T_1}{\partial P_1} & \frac{\partial T_1}{\partial P_2} & \frac{\partial T_1}{\partial P_3} & \cdots & \frac{\partial T_1}{\partial P_N} \\ \frac{\partial T_2}{\partial P_1} & \frac{\partial T_2}{\partial P_2} & \frac{\partial T_2}{\partial P_3} & \cdots & \frac{\partial T_2}{\partial P_N} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial T_I}{\partial P_1} & \frac{\partial T_I}{\partial P_2} & \frac{\partial T_I}{\partial P_3} & \cdots & \frac{\partial T_I}{\partial P_N} \end{bmatrix} \tag{17}$$

Optimum experiments can be designed by maximizing the determinant of the information matrix ($max|\boldsymbol{J}^T\boldsymbol{J}|$). Such an analysis is performed, to ensure minimum variance for the estimates, and experimental variables such as the heating time, the duration of the experiment, location and number of sensors, the measurements methods can be chosen based on this analysis (Ozisik and Orlande, 2000).

For a case involving a single sensor, each element of the sensitivity matrix $F_{m,n}$, m,n=1,…,N, of the matrix $\boldsymbol{F}=\boldsymbol{J}^T*\boldsymbol{J}$ is given by (Ozisik and Orlande, 2000):

$$F_{m,n} = \left[\boldsymbol{J}^T * \boldsymbol{J}\right]_{m,n} = \sum_{i=1}^{I}\left(\frac{\partial T_n}{\partial P_m}\right)*\left(\frac{\partial T_n}{\partial P_m}\right) \quad m,n = 1,...,N \tag{18}$$

where I is the number of measurements and N is the number of unknown parameters.

7. RESULTS AND DISCUSSIONS

In order to carry out the simulations, the following material properties were selected: steel for the probe and the cell container (k_m= 43.2 W/m°C; α_m=11.8x10^{-6}; C_s=3661300 J/m^3 °C;), and water as the fluid whose thermal properties are to be determined (k_f=0.6 W/m°C, α_f=1.4x10^{-7} m^2/s). We assume a heat transfer coefficient h = 20 W/m^2°C from the cell to the external medium. The chosen reference material properties are those of the base-fluid; such a choice enables to get directly, in the case of nanofluids, a relative enhancement in thermophysical properties if such is the case. For the case involving the estimation of the thermal properties of water, we have the following dimensionless parameters: K_m^*= 72; α_m^*=82.2067; Bi =0.02; K_f^*=1; α_f^*=1 and C^*_s =0.8759. The diameter of the probe is taken as 1.5 mm, and the sample dimension (40 mm) was chosen, as specified in the probe user's manual.

We make use of reduced sensitivity coefficients in the analysis presented below. The reduced sensitivity coefficients are defined as the sensitivity coefficients multiplied by their corresponding parameters, and, therefore, can have the temperature as a basis of comparison. In this work, the sensitivities coefficients were computed with forward finite differences.

Figure 2 presents the transient behavior of the reduced sensitivity coefficients with respect to the parameters appearing in the formulation, for a temperature sensor within the probe. The variation of the dimensionless temperature of the probe was included in this figure, as well. This figure shows that the reduced sensitivity coefficients of the parameters of the fluid under study and of the probe have the same order of magnitude of the dimensionless temperature and are linearly independent. However, after the heating is ceased, we can notice a rapid decrease of the sensitivity coefficients with respect to the thermal conductivity and thermal diffusivity of the fluid, as well as of the volumetric heat capacity of the probe. Furthermore, we can notice that the sensitivity coefficient with respect to the parameters of the material of the cell and the Biot number are quite small. Therefore, such parameters cannot be estimated with measurements taken at such sensor position.

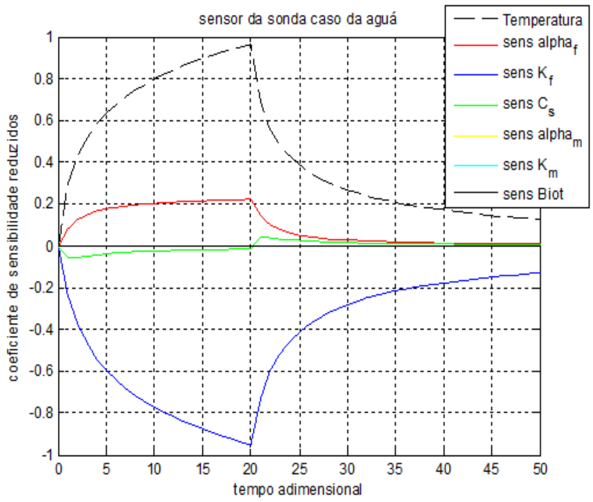


Figure 2: Sensitivity coefficients analysis

Figure 3 shows the results of the D-optimal design, by considering that the temperature measurements are collected with variable frequencies, for a dimensionless heating time of 20. The maximum temperature in the region was taken into account in such analysis (Beck and Arnold, 1977, Ozisik and Orlande, 2000). The maximum value of the determinant is reached for a short time, when the sensitivity coefficients are varying from their null initial values.

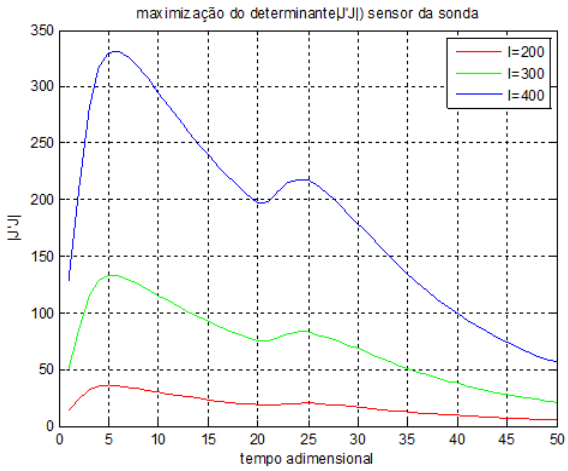


Figure 3: Determinant of the Information matrix

Figure 4 presents the results of the D-optimal design, now by considering that the temperature measurements are collected with a fixed frequency of one measurement every 0.5 second. It can be noticed in figure 4 that, when the heating time is taken equal to the duration of the experiment, the maximization of the determinant yields a relatively lower value as compared to the cases in which the heating is ceased before the end of the experiment. Thus, more informative experiments can be obtained by choosing a heating time smaller than the duration of the experiment. As shown in fig.5, by increasing the final time, the determinant increases, but less significantly after 275 s. Hence, the duration of the experiment can be chosen equal to 275 s and the heating time to 200 s, as this value yields the maximum determinant of the information matrix.

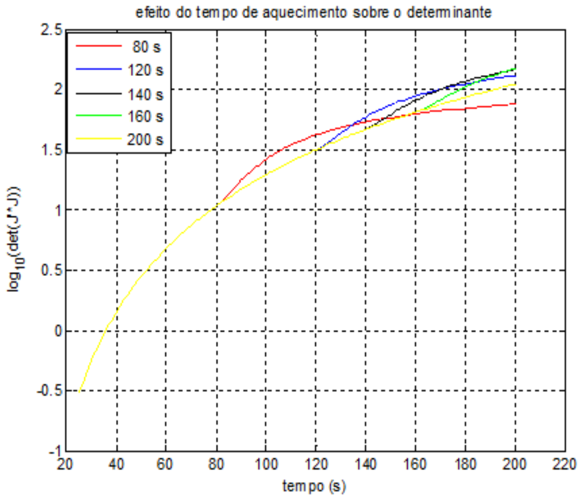


Figure 4: Effect of heating-time on the determinant

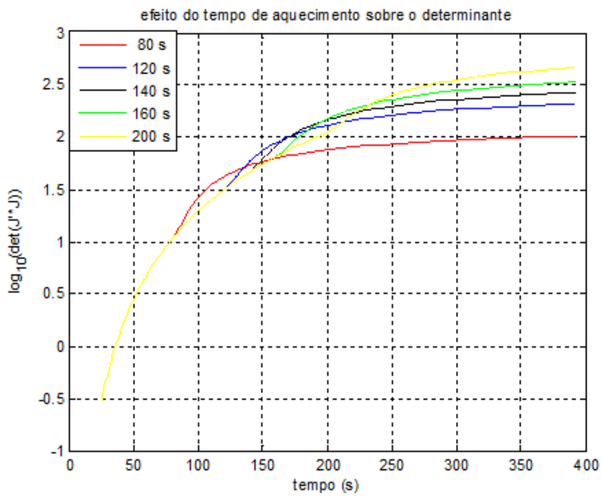


Figure 5: effect of heating-time on the determinant

Based on the results obtained from the D-optimal design of the experiment presented above, as well as of the previously defined thermal and geometrical parameters, simulated measurements were generated for a sensor within the probe by solving the *direct problem*. As the measurements may contain errors, simulated measurements errors are added, which are supposed to be Gaussian random values, with a standard deviation of 1% of the maximum estimated temperature.

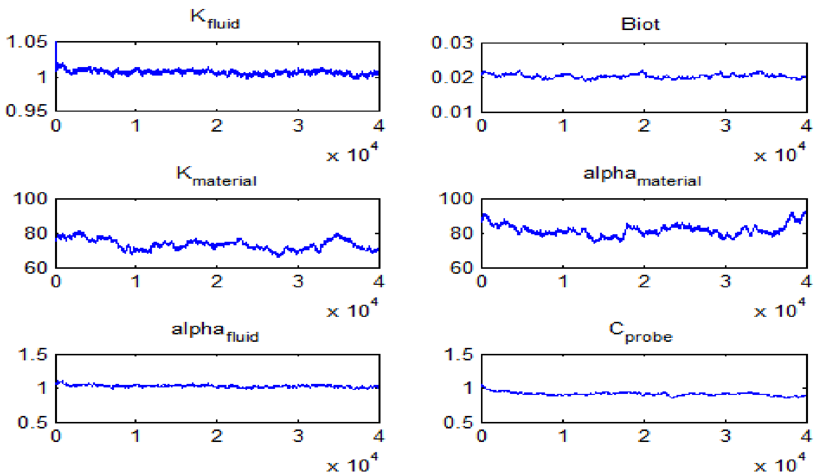
The solution of the inverse problem was considered in two steps, as follows. Firstly, we consider a fluid with known thermal properties where the objective of the inverse problem is to get an estimate of the volumetric heat capacity of the probe. As mentioned above, accurate estimates of the cell container cannot be obtained in the time interval used in the analysis, thus it is assumed that they are known from another experiment, like the flash method. In order to solve the inverse problem for this first step, the prior distributions presented in Table 1 were assumed for each parameter.

Table 1: Prior Distributions for the first step of the inverse problem solution

Dimensionless Parameter	Distribution	
C_s^* Volumetric Heat Capacity of the Probe	Uniform	Lower Bound: C_s^*-20%* C_s^* Upper Bound: C_s^*+20%* C_s
K_f^* Fluid Thermal Conductivity	Normal	Mean: K_f^* Standard-deviation: 5%* K_f^*
α_f^* Fluid Thermal Diffusivity	Normal	Mean: α_f^* Standard-deviation: 5%* α_f^*
K_m^* Cell Container Material Thermal Conductivity	Normal	Mean: K_m^* Standard-deviation: 5%* K_m^*
α_m^* Cell Container Material Thermal Diffusivity	Normal	Mean: α_m^* Standard-deviation: 5%* α_m^*
Bi Biot Number	Normal	Mean: Bi Standard-deviation: 5%* Bi

Figures 6 show the states of the Markov Chain for each parameter, and also the burn-in period of roughly of 10.000 samples, which are required for the Markov Chain to reach equilibrium. From the posterior distribution, we compute the mean value of the parameters. Table 2 presents the obtained results with their associated standard deviation and 99% confidence interval.

As shown in this table, the thermal properties of the fluid and the volumetric heat capacity of the probe are more accurately estimated than the thermal properties of the cell and the Biot number. Such a result was expected, from the sensitivity analysis.



Figures 6: States of the Markov Chain

Table 2: Estimated parameters with their related statistics – first step

Parameters	K_f	α_f	Biot	K_m	α_m	C_s
Exact	1	1	0.02	72	82.2067	0.8759
Estimates	1.0061	1.0257	0.0202	72.3559	81.1371	0.9081
Standard Deviation	0.0029	0.0153	$5x10^{-4}$	2.5515	3.3175	0.0203
Confidence Interval (99 %)	0.9986 1.0135	0.9863 1.0651	0.0188 0.0215	65.7833 78.9285	72.5913 89.6836	0.8558 0.9604

The estimation of the probe volumetric heat capacity obtained from the previous step in the inverse problem solution was then used in a second step as an informative normal prior. The second step was concerned with the estimation of the thermal properties of the fluid and, thus, non-informative uniform priors were assumed for these parameters. Table 3 presents the specified prior distributions used for the second step. The prior distributions for the cell container thermal properties and for the Biot number remain the same as previously.

Figures 7 show the states of the Markov Chain for each parameter. The burning period was roughly of 3000 samples. The parameters were computed from the mean value of the posterior distribution, by discarding the samples of the burning-period. Table 4 shows the obtained results and their related statistics. As shown in this table, accurate estimates for the fluid thermal properties and for the probe volumetric heat capacity can be obtained from the proposed methodology. The stability of the inverse solution with respect to the initial state of the Markov Chain was also investigated to the results obtained were not affected by such initial state of the Markov Chain.

Table 3: Prior Distributions for the second step of the inverse problem solution

Dimensionless Parameter	**Distribution**	
C_s^* Volumetric Heat Capacity of the Probe	Normal	Mean & Standard-deviation: Resulting of the previous inverse solution
K_f^* Fluid Thermal Conductivity	Uniform	Lower Bound: K_f^* -20%* K_f^* Upper Bound: K_f^*+20%* K_f^*
α_f^* Fluid Thermal Diffusivity	Uniform	Lower Bound: α_f^*- 20%* α_f^* Upper Bound: α_f^*+20%* α_f^*
K_m^* Cell Container Material Thermal Conductivity	Normal	Mean: K_m^* Standard-deviation: 5%* K_m^*
α_m^* Cell Container Material Thermal Conductivity	Normal	Mean: α_m^* Standard-deviation: 5%* α_m^*
Bi Biot Number	Normal	Mean: Bi Standard-deviation: 5%* Bi

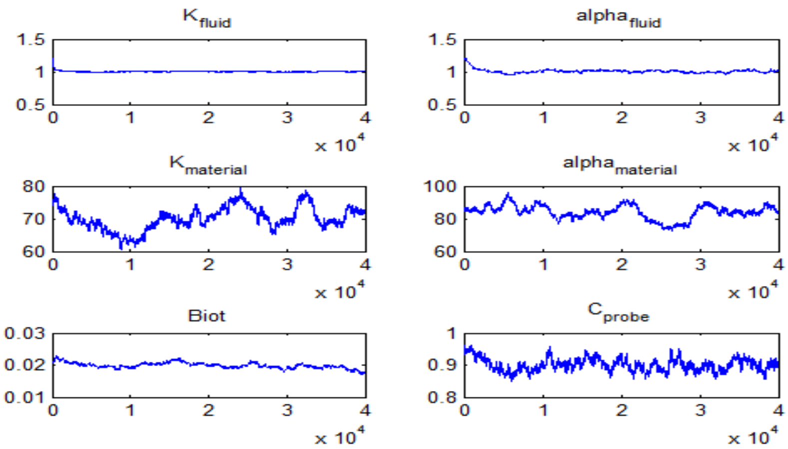


Figure 7: States of the Markov Chain

Table 4: Estimated parameters with their related statistics – second step

Parameters	K_f	α_f	Biot	K_m	α_m	C_s
Exact	1	1	0.02	72	82.2067	0.8759
Estimates	1.0014	1.0034	0.0198	69.8635	83.9171	0.8978
Standard Deviation	0.0034	0.0175	$8x10^{-4}$	3.5774	4.4473	0.0180
Confidence Interval (99 %)	0.9925 1.0102	0.9582 1.0486	0.0177 0.0219	60.6481 79.0789	72.4609 95.3733	0.8515 0.9442

8. CONCLUSIONS

In this paper a Bayesian inverse parameter estimation technique was used for the thermal characterization of fluids, by using the transient line heat source probe. The Markov Chain Monte Carlo method, coded in the form of the

Metropolis-Hastings algorithm, was used to obtain the posterior probability density for the parameters. Simulated temperature measurements were used in the inverse analysis. The proposed methodology enables to get accurate and simultaneous estimates of the thermal conductivity and thermal diffusivity of nanofluids, and the volumetric heat capacity of the probe.

9. ACKNOWLEDGEMENTS

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10. REFERENCES

André, S., Remy, B., Pereira, F.R., Cella, N., Neto, A.J.S., 2002, ''Hot Wire Method for the Thermal Characterization of Materials: Inverse Problem Application'', Engenharia Térmica, No 4, 2003 p.55-64.

ASME V&V, 20-2009, ''Standard for Verification and Validation in Computational Fluids Dynamics and Heat Transfer'', The American Society of Mechanical Engineers, Three Park Avenue, New York, NY.

Banaszkiewicz, M., Seiferlin, K., Spohn, T., Kargl, G., Komle, N., 1997, ''A New Method for the Determination of Thermal Conductivity and Thermal Diffusivity from linear Heat Source Measurements'', Rev. Sci. Instrum., vol. 68, No 11, pp. 4185-4190, American Institutes of Physics.

Beck, J.V. and Arnold, K.J.,1977, ''Parameter Estimation in Engineering and Science, Wiley Interscience, New York

Blackwell, J.V. and Arnold, K.J.,1954, ''A Transient-Flow Method for the Determination of Thermal Constants of Insulating Materials in Bulks'', Journal of Applied Physics, 25, pp.137-144

Carvalho, G., Neto, A.J.S., 1999, ''An Inverse Analysis for Polymers Thermal Properties Estimation'',Proceedings of the 3rd Int. Conf. Inv. Problems in Engineering: Theory and Practice, pp. 495-500

Chandrasekar, M., Suresh S., 2009, ''A Review on the Mechanisms of Heat Transport in Nanofluids'', Heat Transfer Engineering, 30:14, 1136-1150

Choi S.U.S, 1995, ''Enhancing Thermal Conductivity of Fluids with Nanoparticles'', Proceedings of the 1995 ASME International Mechanical Engineering Congress and Exposition, San Francisco, CA, USA

Choi, S.U.S., 2009, "Nanofluids: from vision to reality through reasearch", J. Heat Transfer 131 033106-1-033106-9

Jin B., '' Stochastique Numerical Methods for Inverse Problems, < http://www6.cityu.edu.hk/ma/events/conference/icam2008/abstract/jin,bangti.pdf>

Kaipio, J. and Somersalo, E., 2004,''Statistical and Computational Inverse Problems'' Applied Mathematical Sciences 160, Springer-Verlag,

Lee, P., 2004, ''Bayesian Statistics'', Oxford University Press, London.

Murshed, S.M.S, Leong, K.C., Yang, C., 2008, ''Thermo Physical properties of Nanofluids- A Critical review'', Applied Thermal Engineering, vol.28 pp. 2109-2125

Ozisik, M.N., 1993, ''Heat Conduction'', 2nd Edition, Wiley, New York.

Ozisik, M.N., Orlande H.R.B., 2000,''Inverse Heat Transfer: Fundamentals and Applications'', Taylor and Francis, New York

Patankar, S.V., 1980,''Numerical Heat Transfer and Fluid Flow'', Minkowycz, W.J. and Sparrows, E.M. Editors

Patel, H.E., Das, S.K., Sundarajan, T., Nair, S.A., George, B., Pradeep, T., 2003, ''Thermal Conductivities of Naked and Monolayer Protected Metal Nanoparticle based Nanofluids: Manifestation of Anomalous Enhancement and Chemical Effects'', Applied Physics Letters vol.83, No 14, pp. 2931-2933

Tan, S., Fox, C., Nicholls, G., 2006, '' Inverse Problems'', Course Notes for Physics 707, University of Auckland.

Thomson, N.H., Orlande, H.R.B, 2006, ''Computation of Sensitivity Coefficients and Estimation of Thermophysical Properties with the Line Heat Source Method'', Proceedings of the 3rd European Conference on Computational Mechanics Solids, Structures and Coupled Problems in Engineering, Lisbon, Portugal

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