NUMERICAL SIMULATION OF DRAINAGE BASINS FORMATION IN SMALL-SCALE POROUS MEDIA WITH AN UNALTERABLE REGION

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Abstract. *The fundamental matter of how tree-networks flow emerge in flow systems is addressed is this paper. A fully deterministic erosion model recently developed was used to explain the formation of tree flow networks in small scale basins. In this model, no optimization principle was used, nor any kind of performance criterium. The initial basin was a homogenous porous media . The goal of this work is to verify the influence of a barrier existing prior to the formation of such these tree networks. This barrier is a set of impermeable blocks, that neither are able to have their position changed, nor can be destroyed by the flow. The flow configuration must relief the higher pressure grandients while bypass these barriers. It was verified that, because of this additional necessity, the networks created in the presence of a barrier have a global flow resistance slightly higher than those created without any restraints, mainly if the barrier is close to the outlet. It was also verified the maitenance of a morphological stagnation region prior to the barrier, which have its total area related to the barrier to outlet distance.*

Keywords*: porous media flow, drainage networks formation, geomorphology, Constructal theory*

1. INTRODUCTION

Tree flow networks are geometrical structures broadly verified in nature and for that reason became an important subject of scientific research. A lot of efforts have been made with the goal of unveiling the characteristics and the nature of these structures. However, a consensus has not yet been built about the physical factors that drive a great fraction of natural systems to the creation of networks (e.g., Bejan, 2000 and Errera and Bejan, 2000).

Drainage networks have been widely studied in fluvial geomorphology. Important achievements have been made on this sense (a review is available at Rinaldo, 2006 and in Bejan and Lorente, 2008). A topological ordering scheme was developed that leaded to the Horton and Hack´s Laws and a river network formation model. The Optimal Channel Networks could be used to physically explain some natural parameters obtained, but the creation of the network *itself* has left yet some aspects unknown.

A fully deterministic erosion model (Errera and Bejan, 1998) was used to show that networks can be formed even in a low-scale homogeneous drainage basin, as a consequence of the "attenuation" of high level pressure gradients by the erosion process. In the present work, a flow and morph restraint was added to the erosion model previously mentioned, on the form of a set of impermeable blocks, that neither are able to have their position changed nor can be destroyed by the flow. The influence of this barrier was studied on the basis of the geometrical structures obtained and the global flow resistance reduction along the formational process.

2. THE EROSION MODEL OF A RECTANGULAR BASIN

A surface area of $A = HL$, and rigid shape H/L, is initially coated with a homogeneous porous layer of permeability K, with a very small thickness $W << (H,L)$, with the exception of a fraction φ that is formed by a impermeable material (Figure 1). The area receives a uniform mass flow rate of an incompressible Newtonian fluid (in the case of water, this is similar to rainfall) of value $\dot{m}''A$, that is collected and then discharged through a small opening of size D x W placed over the origin of the system. It thus create a pressure field P(x,y) (hydraulic potential) that drives the fluid to the outlet. If the flow through the K medium can be modeled as Darcy flow, it can be shown that this pressure field is obtained by a Poisson equation. Spaces with open channel flow are created because the porous media blocks can be dislodged. This happens when the resulting force created by the pressure field acting on the block (ΔPDW) is stronger than the cohesion force of the soil (τD^2), where τ is the maximum shear stress supported by the block). Using these facts, a dimensionless pressure field was obtained:

$$\frac{\partial^2 \tilde{P}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{P}}{\partial \tilde{y}^2} + M = 0 \qquad (1)$$

with:

$$(\tilde{x},\tilde{y}) = \frac{(x,y)}{D}\ ,\quad \tilde{P} = \frac{P}{\tau D / W}\ ,\ M = \dot{m}'' \frac{\nu D}{\tau K} \tag{2}$$

where v is the kinematic viscosity of the fluid.

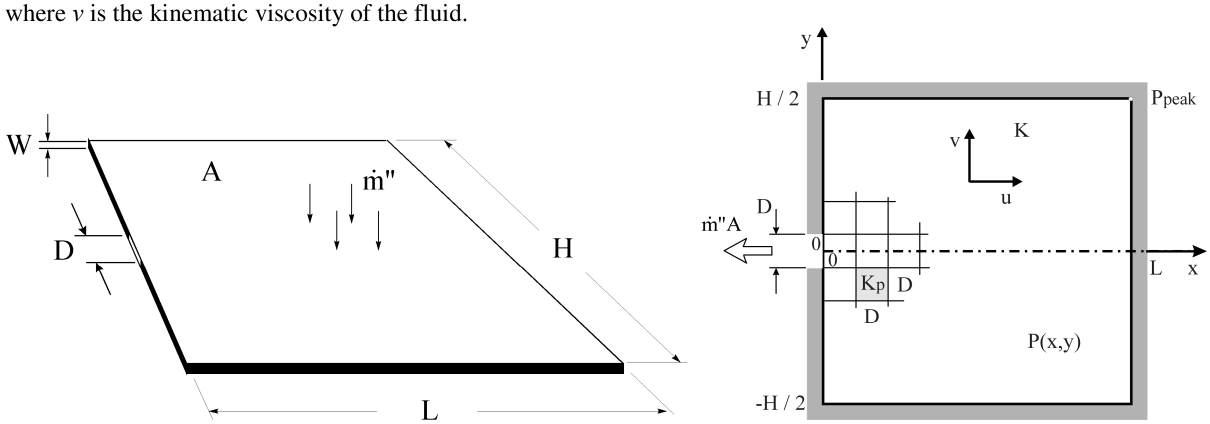

Figure 1.Two-dimensional model of "area-to-point flow" (Errera and Bejan, 1998) in saturated porous media with the solid porous matrix (e.g., soil) modeled as squares of side D that can be dislodged to generate a highly permeable channel

And the dislodging condition is expressed by Eq. (3) where s is the direction of the resulting force acting on the block. That is, whenever the equality is reached for a particular K-block, it becomes a K_p medium.

$$\left(\frac{\partial \tilde{P}}{\partial \tilde{s}}\right) < 1 \tag{3}$$

The basin is impermeable all round the perimeter except by the exit slot, where the dimensionless (scaled) pressure is null. This information (Dirichlet type boundary condition) and Darcy flow model provides the boundary conditions. At the interfaces between blocks of different porous media, continuity applies. Along the perimeter of the barriers we again apply the mathematical equivalent of impermeability (non-flow).

The set o equations were solved numerically by the finite element method for each step n of the evolution process. Once the pressure field was known, the stress condition of Eq. (3) was checked for each block. Time plays no role in the flow, time is represented by stages (n) when any change in the internal structure takes place. A computational code with the heuristics and the CFD simulation were developed. We made use of FIDAP® (FLUENT, 1993) the GPL application Octave and the programming language C++. Further details on the heuristics can be found in Errera and Bejan, 1998 and Marin, 2010.

3. SIMULATIONS WITH UNALTERABLE REGION (BARRIERS)

We introduced insertions that are unalterable and impermeable, that is, it plays no part in the flow and in the evolution process. The basin was held squared, $H = L = 1$, throughout the entire evolution process. The basin was formed by 51 x 51 (or 2601) blocks, and the unalterable region is also a square set of 15 x 15 blocks ($\varphi = 0.09$) lying symmetrically along the x axis, with the distance to the outlet changing.

The outer routine controls the evolution process by setting up the configuration (mesh and properties), calling the CFD routine to solve the pressure field, then evaluating the stress across each one of the eligible K-blocks to check whether condition of Eq. (3) is reached. The structure is then up-dated or the forcing term M is increased for new round.

The CFD routine to solve the pressure field is based on the Finite Element Method on a uniform mesh that was considerably finer than the D-size grid formed by the blocks (Fig. 2). There were four quadrilateral elements for each block. Local accuracy was further increased by using bi-quadratic interpolation with nine nodes each. The computer code was previously validated (Errera and Bejan, 1998) against finite volume method. The FEM method was chosen since it provided the pressure field along the perimeter of each block. The commercial CFD code was flexible in accommodating the changes that occur in the internal structure after each step. Simulations considered as many as 10,812 elements with 42,025 nodal points (for the half-domain of Fig. 1).

Further details on the numerical procedure and computer coding can be found in Errera and Bejan, 1998, Marin and Errera, 2009 and Marin, 2010.

Simulations were carried out for three different configurations in which the unalterable area (in gray) is positioned at the left most third, in the middle and at 2/3 of the horizontal width *L.* The forcing term, i.e., the flow load given by the precipitation is gradually increased as the flow resistance decreases with the "conversion" of low permeability area (K) to high permeability area K_p along the evolution process. For this paper, it was used a constant increment of 10^{-4} to the *M* values.

Each simulation of the dynamics of the network formation can take as many as 400 outer steps and consequently more then 800 solutions of the pressure field. Time wise, in linux based workstation in a Pentium IV, it took as long as four hours of computing time for each one of the simulations. Running time is definitely a major constraint for finer D-grids and FEM meshes, and forcing increments.

4. RESULTS AND DISCUSSION

For the sake of the lay-out of the paper, results of the simulations were included as appendix.

Figure 2 shows a reference case from Errera and Bejan, 1998 that consider the same conditions set in this work. In that case the whole K-medium is free to be dislodged.

The set of Figs. (3)-(5) represents the evolution of internal structure of three different basins. It is important to stress steps *n* do not mean instants in the sense of chronological time. It means history of events. Such events are noticed (or registered) by singular change in the internal structure of the basin. The flow filed is always considered steady. Therefore chronological time periods between steps (*n* to *n*+1) is of no concern.

In Fig. 3 one can see the structure obtained are clearly networks and that a morphological stagnation region prior to the barrier is sustained. The structures are symmetric because of the problem statement. Looking at the *M* dynamics, it can be seen that instability moments take place right before branching episodes. The variation of the overall flow resistances ($P_{*,max} = P_{max}/M$) is not monotonic as the curve of the forcing term *M*. The cause of such response is associated to the internal structure itself that in some steps passes through major change with minor increase in *M* and some times just consolidate (thickens and prolongs) one pattern.

It is worth mentioning the importance of the event is not measured by the intensity of the forcing term *M*, but by the changes it makes on the overall flow resistance and the structure.

Similar trends are seen in Fig. 4 . Two fingers appear with 45° with the *x*-direction near the step $n = 200$ when there is room to evolve a near straight line to the farthermost corners ($x = L$, $y = \pm H/2$). There is also a late branching near $n = 800$.

Following the same line of thought, Figure 5 shows a surprising change of pattern: sometime before the step $n = 200$, the early fingers curved to be parallel with the *x*-axis to embrace the fixed block (unalterable region).

The dynamics of Fig. 5 resembles the reference case basin of Fig. 2 since the fixed block is far enough not to significantly interfere the early stages of the drainage network.

The structures obtained are resilient to the corresponding *M* values, and they are influenced by the dynamics of this external forcing dynamics.

5. CONCLUSIONS

The simulations showed that internal structure such as flow network (fingering and branching) can occur with no à priori assumption of its existence. In addition there is no "optimization procedure" whatsoever involved in the model.

Results shed some light on how flow networks self-organize in order to overcome flow barriers in a basin. While this paper address the fundamental problem, it opens new possibilities to study how oil may flow in plane wells and to design drainage systems.

Studies can continue by varying parameters (initial condition, basin properties and forcing term) randomly while the principle itself remains deterministic.

There are also efforts in parallel computing of certain procedures and in the development of dedicated FEM code to speed up simulations and improve computer memory requirements to later use finer D-grids.

6. AKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.

APPENDIX

Figures 2 to 5 were placed as appendix for the sake of the lay-out of the paper.

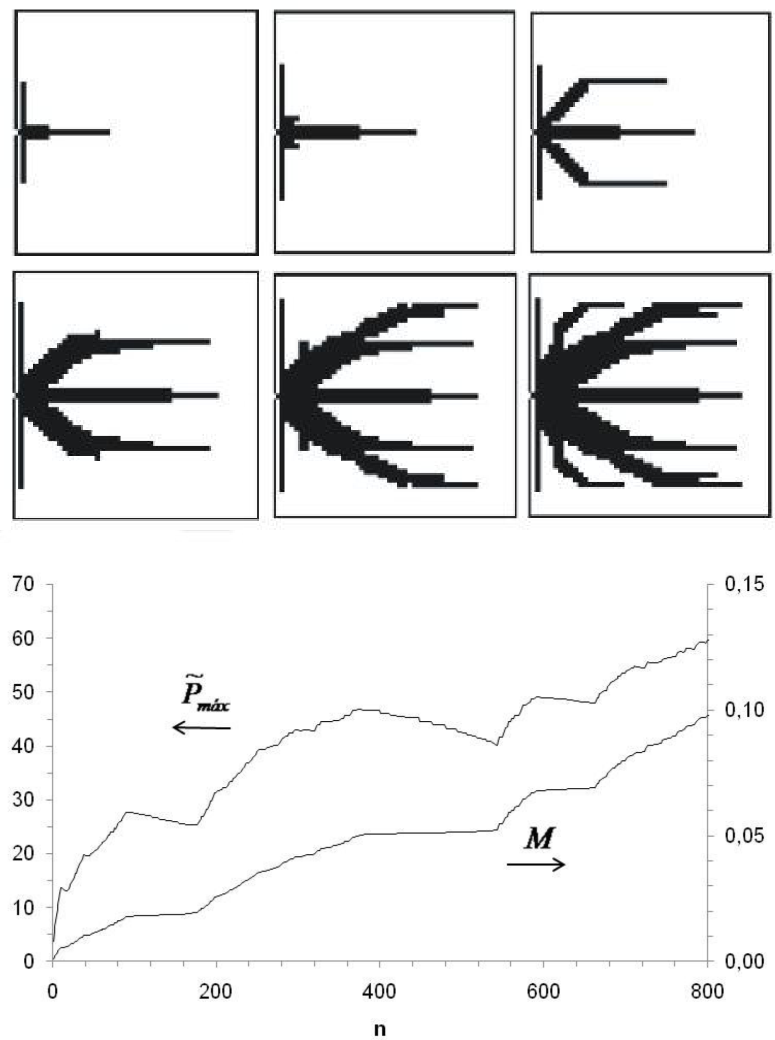

Figure 2. Dynamics of the network formation and of the pressure peak and M values, where n is the number of dislodged blocks, up to n = 800 (after Errera and Bejan, 1998).

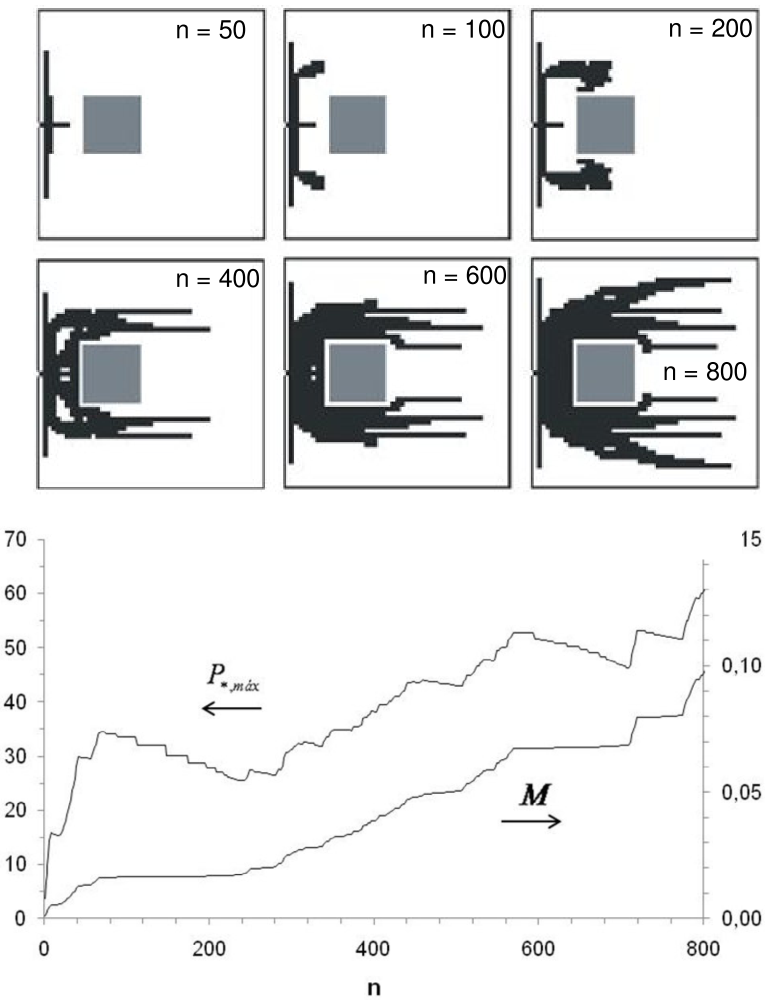

Figure 3. Dynamics of the network formation and of the pressure peak and M values, where n is the number of dislodged blocks, up to n = 800. (region at 1/3*L*)

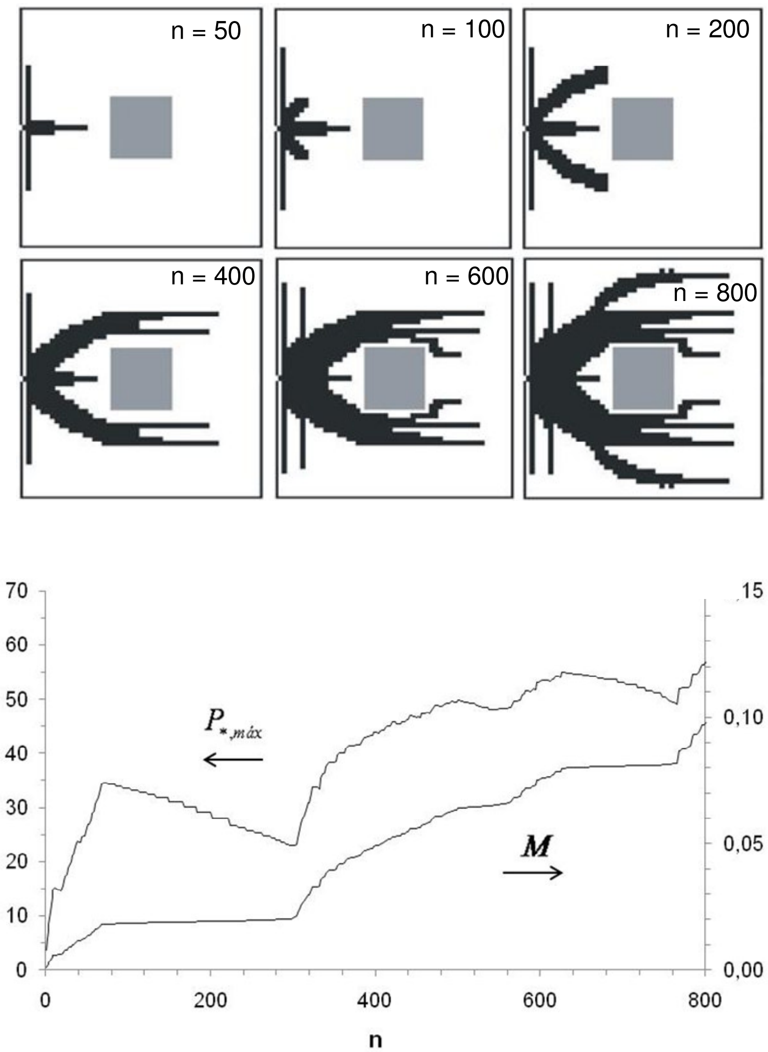

Figure 4. Dynamics of the network formation and of the pressure peak and M values, where n is the number of dislodged blocks, up to n = 800 (region at 1/2*L*)

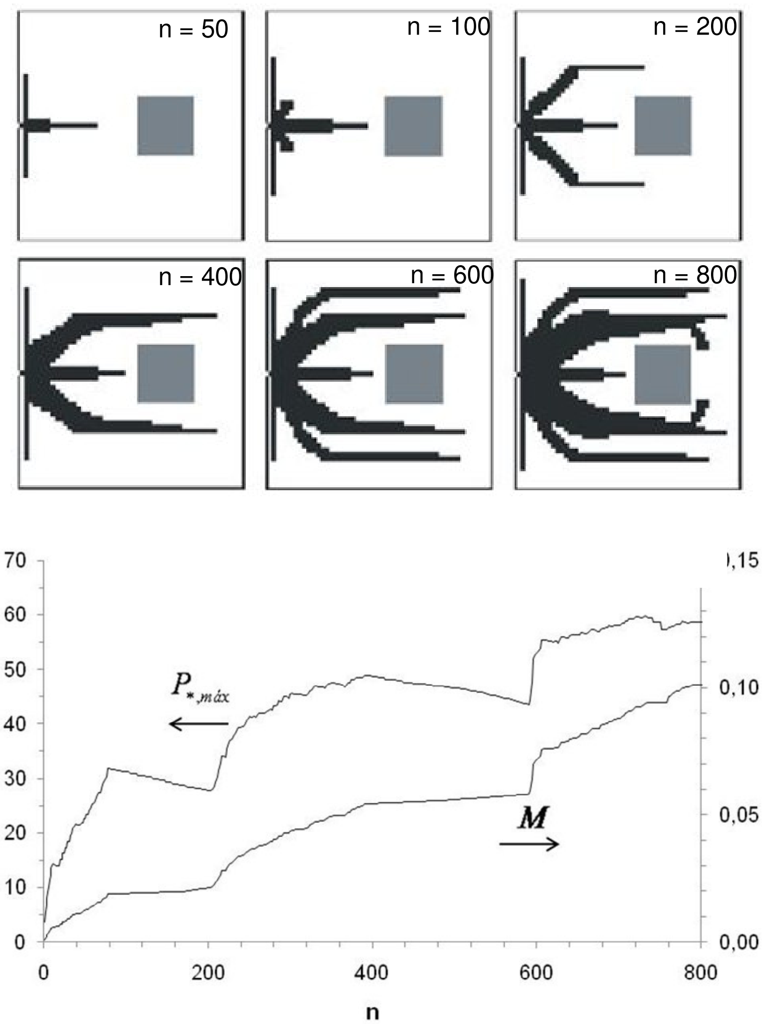

Figure 5. Dynamics of the network formation and of the pressure peak and M values, where n is the number of dislodged blocks, up to n = 800 (region at 2/3*L*)