LIQUID-LIQUID DISPLACEMENT FLOWS IN A HELE-SHAW CELL INCLUDING NON-NEWTONIAN EFFECTS

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Abstract. *Viscous fingering in non-Newtonian fluids in a rectangular Hele-Shaw cell is investigated. This cell is filled with an aqueous solution of Xanthan gum, and then a Newtonian mineral oil is injected into the cell, thus displacing the non-Newtonian liquid. A digital camera is used to capture images of the interface between the fluids during the flow. Applications include displacement of heavy crude oil in reservoirs. The main parameters that govern this flow are the viscosity ratio, the rheological capillary number, and the (dimensionless) flow rate. The interface shape is given for different values of flow rate and viscosity ratio.*

Keywords: *Second recovery stage, Viscous fingering, Hele-Shaw cell*

1. INTRODUCTION

Oil recovery operations traditionally have been subdivided into three stages: primary, secondary and tertiary. In surging oil wells, the secondary recovery stage consists of injection of water or gas to displace the oil from the porous rock. The study of such displacement flows is important because the production of oil in the primary stage, obtained by the natural pressure of the reservoir, is rather limited. The displacement efficiency can be evaluated by the shape of the interface between the liquids. Injected fluids tend to flow towards the more permeable layers or zones, bypassing a large amount of oil in the unswept region [10] as can be seen in Fig. 1. This will create an early breakthrough of the injected fluid, therefore implying low oil recovering rates and eventually, an uneconomical process.

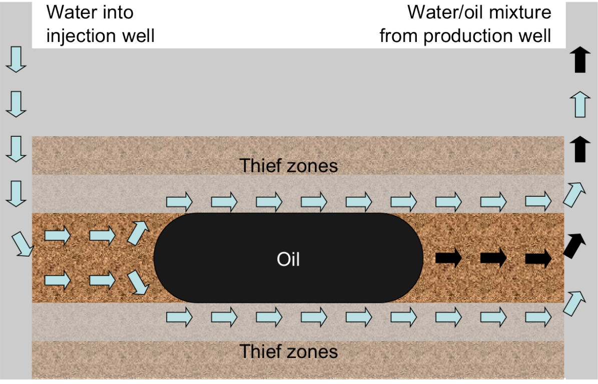

Figure 1. Poorly swept efficiency in reservoir

The Saffman-Taylor or viscous fingering instability occurs when, in a porous medium or low-gap rectangular channel (Hele-Shaw cell), one fluid pushes a more viscous one. The interface between the fluids may become unstable, leading to the formation of fingerlike patterns [11, 4] according to the left part of Fig. 2. Otherwise, there is a formation of a flat interface (right part of Fig. 2).

Figure 2. Viscous fingering and plug displacement, respectively

The Saffman-Taylor instability has been extensively studied for Newtonian fluids [6]. For gas-displaced non-Newtonian liquids (zero viscosity ratio), a large number of articles is also found in the literature [1, 3, 7, 9, 8, 13, 12]. Yamamoto [12] studied the growth phenomenon of viscous fingering which repeat three patterns: spreading, splitting and shielding. Chevalier et al. [3] investigated the effects of inertia on the width of the viscous fingers. Amar and Poiré [1] simplified the Hele-Shaw cell into a two-dimensional problem. Displacement flows involving two liquids of comparable viscosity, however, have received very little attention.

2. EXPERIMENTS

Visualization experiments were performed to investigate the phenomenon of viscous fingering during the displacement of Xanthan Gum by a less viscous Newtonian oil flowing through a Hele-Shaw cell. The interface shape is recorded as it proceeds along the cell. Different flow rate values are investigated to determine the conditions under which fingering occurs.

Figure 3 illustrates a schematic diagram of the displacement flow in a Hele-Shaw cell and gives the cell dimensions in millimeters. Figures 4 and 5 depict the flow visualization apparatus. Two pumps are used to fill the channel, one for each liquid. The Reynolds number is kept low for all cases investigated, to ensure negligible inertia [3]. The rheological capillary number is kept low too.

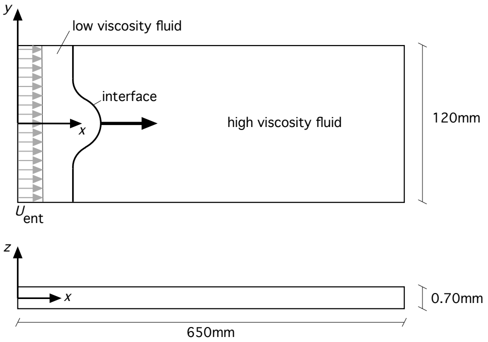

Figure 3. Schematic diagram of a rectangular Hele-Shaw cell

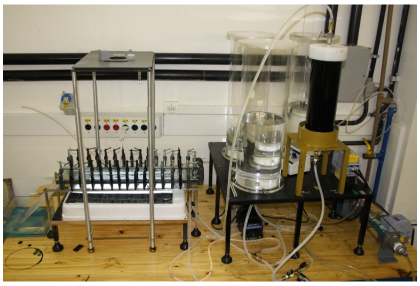

Figure 4. Experimental setup

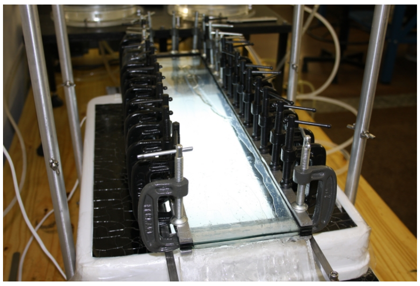

Figure 5. The Hele-Shaw cell apparatus

The problem is non-dimensionalized in accordance with [5] (Fig. 6). The main governing parameters that arise are the dimensionless flow rate, $\bar{u}^*$, the dimensionless viscosity ratio, η^*, and the rheological capillary number, Ca.

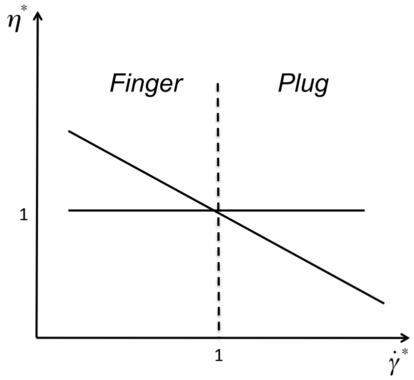

Figure 6. Dimensionless viscosity function

The wall shear rate, $\dot{\gamma}$, is given by Eq. 1 for this flow [2]. The characteristic shear rate, $\dot{\gamma}_c$, is defined as the shear rate at which the viscosities of the two fluids are equal (Eq. 2). The characteristic viscosity, η_c, is the viscosity evaluated at the characteristic shear rate (Eq. 3).

$$\dot{\gamma} = \frac{6\bar{u}}{b}\left(\frac{1+2n}{3n}\right) \tag{1}$$

$$\dot{\gamma}_c = \left(\frac{\mu}{K}\right)^{(1/(n-1))} \tag{2}$$

$$\eta_c \equiv \eta(\dot{\gamma}_c) = \mu \tag{3}$$

The dimensionless volume flow rate is chosen to be equal to a dimensionless shear rate, $\bar{u}^* = \dot{\gamma}^*$. It is defined as the ratio of the wall shear rate, $\dot{\gamma}$ to the characteristic shear rate, $\dot{\gamma}_c$ (Eq. 4). In this manner, the dimensionless viscosity function of the non-Newtonian fluid, $\eta^*(\dot{\gamma}^*)$ is just the viscosity ratio at the given flow rate (Eq. 5, Fig. 6). Finally, the rheological capillary number is given in Eq. 6.

$$\bar{u}^* \equiv \frac{\dot{\gamma}}{\dot{\gamma}_c} \equiv \dot{\gamma}^* = \left\{\frac{6}{b}\left(\frac{K}{\mu}\right)^{1/n-1}\frac{2n+1}{3n}\right\}\bar{u} \tag{4}$$

$$\eta^* \equiv \frac{\eta\left(\dot{\gamma}\right)}{\eta_c} = \dot{\gamma}^{*n-1} \tag{5}$$

$$Ca \equiv \tau_o b/\sigma \tag{6}$$

The parameters that appear in the above equations are: the Newtonian viscosity μ, the consistency index K, the behavior index n, the gap b, the mean flow rate $\bar{u}$ and the interfacial tension σ.

Our proposal is to identify the threshold value from which the flow patterns changes from fingers to plugs, as can be seen in Fig. 6. If the dimensionless viscosity ratio is greater than 1 and the dimensionless volume flow rate is less then 1, fingering occur. Otherwise, there is a plug displacement.

3. RESULTS

3.1 Rheology

The flow curves were determined with the aid of a rotational rheometer (Fig. 7). For the Xanthan gum, a shear thinning behavior was observed and the power-law viscosity function was employed to fit the data.

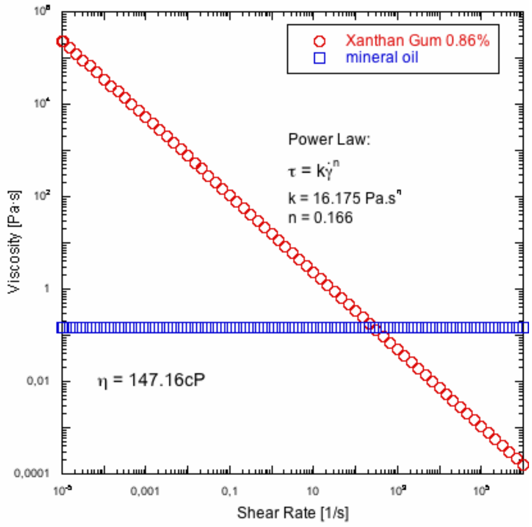

Figure 7. Flow curves

3.2 The displacement experiments

Figures 8, 9, 10, 11 and 12 represents the cases in which fingers occurs ($\eta^* > 1$ and $\bar{u}^* < 1$). It can be observed that the higher the dimensionless flow rate, the higher the branching.

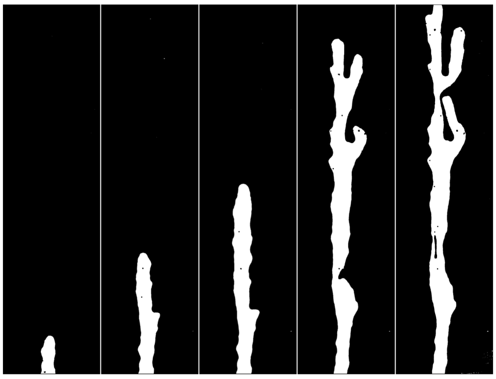

Figure 8. Flow pattern for $\eta^* = 4.48$, $\bar{u}^* = 0.17$

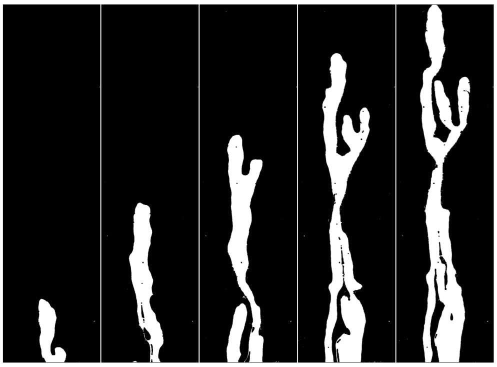

Figure 9. Flow pattern for $\eta^* = 2.56$, $\bar{u}^* = 0.32$

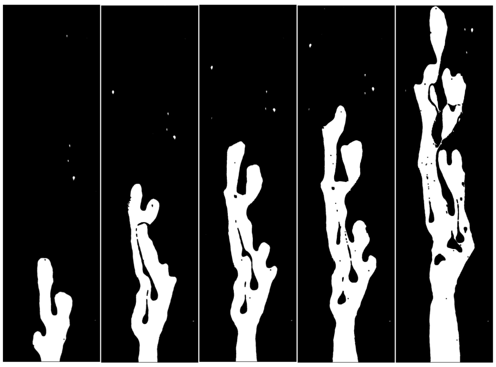

Figure 10. Flow pattern for $\eta^* = 1.92$, $\bar{u}^* = 0.46$

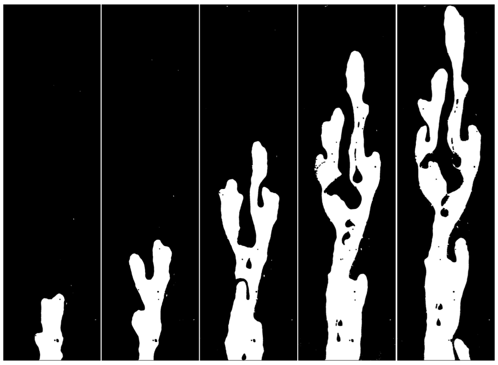

Figure 11. Flow pattern for $\eta^* = 1.43$, $\bar{u}^* = 0.65$

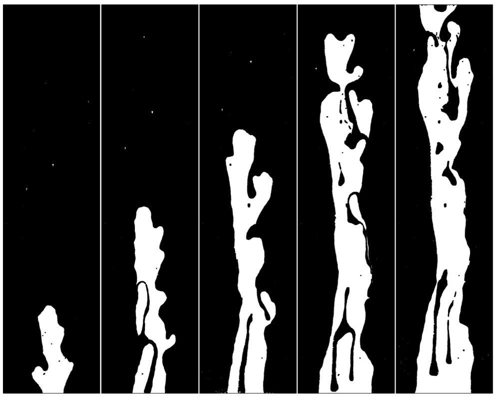

Figure 12. Flow pattern for $\eta^* = 1.32$, $\bar{u}^* = 0.71$

Figures 13 and 14 represents the cases where the dimensionless viscosity ratio is less than 1, so a plug displacement is observed.

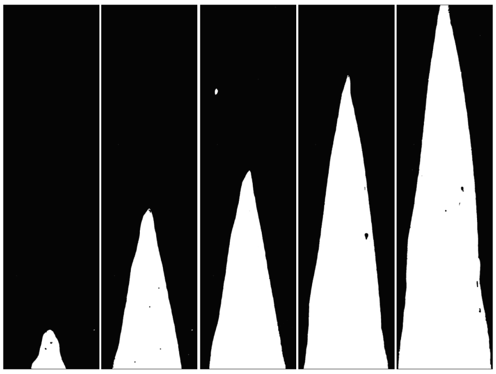

Figure 13. Flow pattern for $\eta^* = 0.33$, $\bar{u}^* = 3.82$

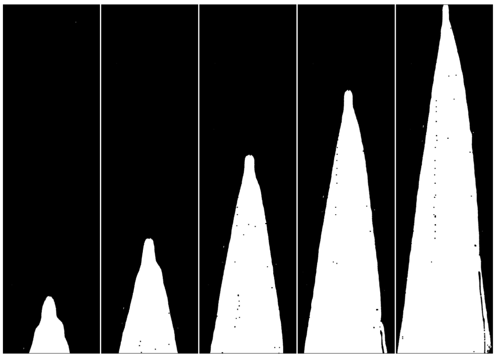

Figure 14. Flow pattern for $\eta^* = 0.28$, $\bar{u}^* = 4.54$

According to Fig. 15, the displacement efficiency increases with the flow rate, while decreases with the viscosity ratio. The experimental results predict that when dimensionless viscosity ratio is greater than 1 fingering occurs, otherwise a plug displacement occurs which is consistent with our model.

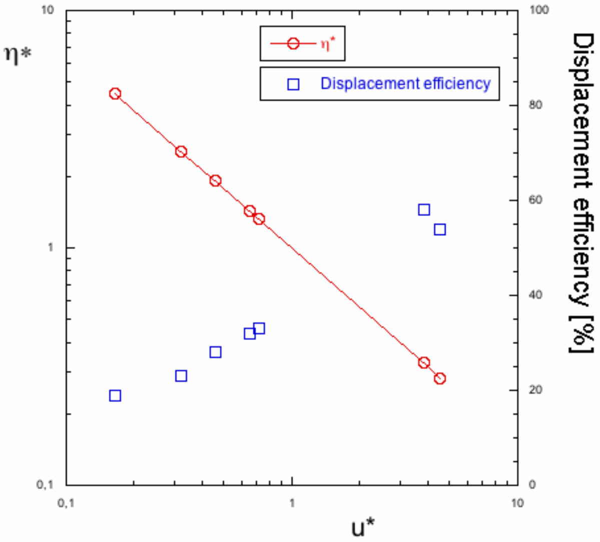

Figure 15.

4. FINAL REMARKS

A systematic study of the influence of the parameters that govern this flow is under way, and more results will be presented at the conference. In general, it was already observed that the displacement efficiency increases with the viscosity ratio, while decreasing with the flow rate. The experimental results predict that when $\eta^* > 1$ fingering occurs, and when $\eta^* < 1$ a plug displacement occurs. In the cases where fingering occurs, the higher the flow rate (lower the viscosity ratio) the higher the branching.

5. ACKNOWLEDGEMENTS

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7. Responsibility notice