# SIMULATION OF TWO DIMENSIONAL TEMPERATURE RECONSTRUCTION BY INVERSE RADIATION ANALYSIS WITH DIFFERENT NUMBER OF IR DETECTORS

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***Abstract.*** *This work presents an inverse analysis for temperature field estimation in a two-dimensional gray media with boundary surfaces without external incident radiation; it uses a multidimensional spatial scheme and high order angular quadrature in the discrete ordinates method. Different angular quadratures for DOM and a different number of sensors are used in this numerical study to examine the effects of these on the accuracy of the estimation. The participating media system contains absorbing, emitting, isotropic scattering of gray medium. The radiative intensities exiting in some points of boundary surfaces, which simulate data of sensing devises, are known. In this work, the Discrete Ordinates Scheme with Infinitely Small Weight (DOS + ISW) is used to calculate data values. The radiative properties such as the single scattering albedo and absorption coefficient of the medium are assumed to be uniform everywhere. The boundaries are considered to be transparent. The conjugate gradient method is used to solve the inverse radiation problem for determining the temperature field. The inverse problem is formulated as an optimization problem that minimizes the error between the calculated and the simulated measurement of radiation intensity leaving the media that is sensing at one, two or four points at the boundary of the cavity. Different angular quadratures are tested; from comparison between these results it is found that the low order angular quadratures do not give accurate estimations. The numerical results are obtained by considering simulated data with and without noise. Different arrangements of the position of the sensors at the cavity boundary were analyzed. The results using one sensor at the cavity boundary give poor estimations for the temperature field. The temperature field has been estimated with accuracy by using LC11 and Tn6 angular quadratures of DOM when four or two sensors were used and the results for four and two sensors are very close between them. Also, the effects on the estimations of non-uniform distribution of the absorption coefficient had values with random errors of 10% when they were analyzed.*

***Keywords***: *radiative transfer, inverse radiation analysis, conjugate gradient method,2-D temperature field, discrete ordinates method.*

## 1. INTRODUCTION

Radiative heat transfer is the predominant mode of heat transfer in combustion chambers and furnaces. It is a very complicated phenomenon due to the gases that absorb, emit and scatter radiation. Since radiation affects the temperature field, accurate knowledge of the gas heat rate is required. In practice, two ways to predict the gas heat rate distribution are commonly used. The first one is to solve the theoretical combustion model or empirical formula that is not able to get an accurate result. The second one is to solve the inverse radiation problem.

Many researchers have reported inverse problems that deal with the prediction of the temperature distribution in a medium from either simulated or experimental radiation measurements. Yi et al. (1992), Li and Ozisik (1992), Siewert (1993, 1994), Liu et al. (1998, 2000), Li (1994, 1997a), have reconstructed the temperature profiles or source terms in a one-dimensional plane-parallel, spherical and cylindrical media by inverse analysis from the simulated data of the radiation intensities exiting at the boundaries. Most of the first works have considered that one-dimensional systems assumed that the bounding surface is transparent or the emissivity of the boundary surface is known. On many occasions, the bounding surface is opaque and its emissivity is unknown; for example, the boundary emissivity of the combustion chamber is changed with the operating condition. Under this condition the unknown temperature profile needs to be estimated simultaneously with the unknown emissivity of the boundary surfaces. Li and Ozisik (1993), Liu et al. (1999), have reconstructed simultaneously temperature profiles and wall emissivity in parallel plane media. The optical thickness, the albedo and the scattering phase function were simultaneously estimated by Neto and Ozisik (1995) in a parallel plane media. Li (1997b) has studied the inverse problem of an unknown source term in a two dimensional rectangular medium with transparent boundaries. Zhou et al. (2000) used an optimization procedure for estimating simultaneously the temperature and scattering albedo profiles.

In most of the above works, the discrete ordinates method (DOM), the discrete transfer method (DTM), the zonal method or the Monte Carlo method were employed to solve the direct and the sensitivity problems.

For a system governed by radiation, the inverse problem is represented by a set of Fredholm equations of the first kind, which are known to be ill-posed. If the resulting system is solved by simple techniques, like the Gauss elimination or Gauss-Siedel iteration, the ill-posed character of the governing equations leads to non-physical solutions that are highly affected by small perturbations in the input. Therefore, in order to produce physically reasonable, yet accurate solutions, the system must be regularized. Regularization leads to a set of solutions that ignores some information that is the source of the ill-posed character. Consequently, the solutions are subject to different levels of errors that result from ignored information, and an optimal solution must be sought that satisfies the physical requirements of the problem with acceptable accuracy.

There exist a number of regularized solution techniques that have been used for solving similar problems, including truncated singular value decomposition (TSVD), the conjugate gradient method (CGM), the bi-conjugate gradient method (BiCGM) that is a method based on the CGM and improves the method to solve any arbitrary MxN system. Also, the Tikhonov regularization method that was first proposed by Tikhonov (1975) for solving inverse conduction problems. Reviews of regularization methods were provided by Daun and Howell (2005).

The inverse radiation problem considered in this paper is concerned with the estimation of the source term distribution or temperature profile in 2-D participating media systems containing absorbing, emitting and scattering gray medium from knowing the radiative intensities exiting in some points of boundary surfaces that simulate dates received by sensing devises. In practical experimental measurements of radiation intensity at a point (in fact, a pinhole) of the boundary of some domain, it is possible to obtain a set of directional intensities in 180, 240 or more directions for 2-D calculations in a cross section depending on the resolution of the sensor in the experimental device. Then it is possible to select different angular quadratures that can be adjusted to the experimental data. Based on this, different angular quadratures for DOM and a different number of sensors are used in this numerical study to examine the effects of these on the accuracy of the estimation. The inverse problem is formulated as an optimization problem and the conjugate gradient method is used for its solution. In this work, the discrete ordinates method is used to solve the direct and the sensitivity problems. A multidimensional high order spatial scheme and different angular quadratures for the discrete ordinates method are used to examine the accuracy of the estimation. Analysis is made of the direct problem, the gradient equation and the sensitivity problem. The procedure for each of these steps is described and then an algorithm for the solution of the inverse radiation problem is presented. Finally, several inverse problems of the source term in two-dimension with different arrangements of the position of the sensors at the cavity boundary were analyzed.

## 2. ANALYSIS

The analysis consists of the direct problem, the gradient equation, and the sensitivity problem.

### 2.1 Direct Problem

It considers the radiative transfer process in an absorbing, emitting, scattering, gray and two dimensional rectangular medium. The boundary surfaces are considered to be transparent. There is no external incident radiation. The radiative properties, such as single scattering albedo $\omega$ absorption coefficient of the medium, are assumed to be uniform everywhere. The direct problem of concern here is to find the radiative intensities exiting at the boundaries for the known source term distribution and radiative properties.

The radiative transport equation for an absorbing, emitting gray gas with isotropic scattering can be written as Siegel and Howell (1992) did,

$$(\Omega.\nabla)\boldsymbol{I}(\boldsymbol{r},\Omega) = -(\kappa+\sigma)\boldsymbol{I}(\boldsymbol{r},\Omega) + S(\boldsymbol{r}) + \frac{\sigma}{4\pi}\int_{4\pi}\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \tag{1}$$

with

$$S(\boldsymbol{r}) = \frac{(1-\omega)\bar{n}^2\bar{\sigma}\boldsymbol{T}^4(\boldsymbol{r})}{\pi} \tag{2}$$

where $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity in $\boldsymbol{r}$, and in the direction $\Omega$; $\boldsymbol{I_b}(\boldsymbol{r})$, is the radiation intensity of the blackbody in position $\boldsymbol{r}$ and at the temperature of the medium; $\kappa$ is the gray medium absorption coefficient; $\sigma$ is the gray medium scatter coefficient; and the integration is in the incident direction $\Omega'$. The source term $S(\boldsymbol{r})$ is related to the temperature $T(\boldsymbol{r})$ of the medium by. $\bar{n}$ is the refractive index and $\bar{\sigma}$ is the Stefan Boltzmann constant.

For diffusely reflecting surfaces the radiative boundary condition for Eq. (1) is

$$\boldsymbol{I}(\boldsymbol{r},\Omega) = \varepsilon\boldsymbol{I_b}(\boldsymbol{r}) + \frac{\rho}{\pi}\int_{\boldsymbol{n}.\Omega'<0}|\,\boldsymbol{n}.\Omega'|\,\boldsymbol{I}(\boldsymbol{r},\Omega')\boldsymbol{d}\Omega' \tag{3}$$

where $\boldsymbol{r}$ lies on the boundary surface Γ, and Eq. (3) is valid for $\boldsymbol{n}.\Omega>0$. $\boldsymbol{I}(\boldsymbol{r},\Omega)$ is the radiation intensity leaving the surface at the boundary condition, $\varepsilon$ is the surface emissivity, $\rho$ is the surface reflectivity and $\boldsymbol{n}$ is the unit vector normal to the boundary surface.

In the method of discrete ordinates, the equation of radiation transport is substituted by a set of M discrete equations for a finite number of directions $\Omega_m$, and each integral is substituted by a quadrature series (Fiveland W., 1984),

$$(\Omega_m.\nabla)\boldsymbol{I}(\boldsymbol{r},\Omega_m) = -\beta\boldsymbol{I}(\boldsymbol{r},\Omega_m) + \boldsymbol{S}(\boldsymbol{r}) + \frac{\sigma}{4\pi}\sum_{m=1}^{M}\boldsymbol{w}_m\boldsymbol{I}(\boldsymbol{r},\Omega_m) \text{ with } \quad \boldsymbol{S}_m = \frac{\sigma}{4\pi}\sum_{m=1}^{M}\boldsymbol{w}_m\boldsymbol{I}(\boldsymbol{r},\Omega_m) \tag{4}$$

This angular approximation transforms the original equation into a set of coupled differential equations, with $\beta=(\kappa+\sigma)$ as the extinction coefficient; $S_m$ represents the entering scattering source term, $\boldsymbol{w}_m$ are the ordinates weight, $M$ is the number of directions $\Omega_m$ of the angular quadrature and $\boldsymbol{I}_m$ is obtained by solving the radiative transport equation in discrete ordinates.

In the Cartesian ordinate system, the two-dimensional radiative transport equation in the $\boldsymbol{m}$ direction for an emitting, absorbing and scattering medium is

$$\mu_m \frac{d\boldsymbol{I}_m}{dx} + \xi_m \frac{d\boldsymbol{I}_m}{dy} = \beta\boldsymbol{I}_m + \boldsymbol{S}(x,y) + \boldsymbol{S}_m \tag{5}$$

where $\mu_m, \xi_m$, are the directional cosine of $\Omega_m$. The boundary condition in discrete ordinates for the case analyzed in this work can be written as

$$\begin{aligned} &\boldsymbol{I}_m = 0; \quad \mu_m > 0 \text{ in } x=0; \\ &\boldsymbol{I}_m = 0; \quad \mu_m < 0 \text{ in } x=x_L; \\ &\boldsymbol{I}_m = 0; \quad \xi_m > 0 \text{ in } y=0; \\ &\boldsymbol{I}_m = 0; \quad \xi_m < 0 \text{ in } y=y_L \end{aligned} \tag{6}$$

The direct problem is solved using the numerical method outlined in Ismail and Salinas (2004). The multidimensional non-linear high order scheme of Balsara (2001), the so-called genuinely multidimensional (GM) is used here in the spatial discretization. Validation of the model and numerical method used for the direct problem can be found in Ismail and Salinas (2004).

### 2.2 Inverse Problem

For the inverse problem, the source term distribution or the temperature distribution are unknown, but the other quantities in Eqs. (4) and (6) are known. Measured exit radiative intensities at the center of the wall boundaries are considered available. In the inverse analysis, the source term distribution is estimated by the measured data of exit radiative intensities. The source term can be represented by a polynomial as

$$\boldsymbol{S}(\boldsymbol{x},\boldsymbol{y}) = \sum_{\boldsymbol{q}=0}^{\boldsymbol{M}}\sum_{\boldsymbol{r}=0}^{\boldsymbol{N}}\boldsymbol{a}_{\boldsymbol{qr}}\,\boldsymbol{f}_{\boldsymbol{q}}(\boldsymbol{x})\boldsymbol{g}_{\boldsymbol{r}}(\boldsymbol{y}) \tag{7}$$

where $\boldsymbol{f}_q$ (x) e $\boldsymbol{g}_r$ (y) are basic functions, M and N is the order of the source term polynomial expansion. The inverse radiation problem can be formulated as an optimization problem. We wish to minimize the objective function

$$\begin{aligned} \boldsymbol{J}(\tilde{\boldsymbol{a}}) = &\sum_{\mu_i<0}\boldsymbol{w}_i[\boldsymbol{I}_1(\boldsymbol{x}_0,0{,}5\boldsymbol{y}_L,\mu_i,\tilde{\boldsymbol{a}})-\boldsymbol{Y}_1(\mu_i)]^2 + \sum_{\mu_i>0}\boldsymbol{w}_i[\boldsymbol{I}_2(\boldsymbol{x}_L,0{,}5\boldsymbol{y}_L,\mu_i,\tilde{\boldsymbol{a}})-\boldsymbol{Y}_2(\mu_i)]^2 + \\ &\sum_{\xi_i<0}\boldsymbol{w}_i[\boldsymbol{I}_3(0{,}5\boldsymbol{x}_L,\boldsymbol{y}_0,\xi_i,\tilde{\boldsymbol{a}})-\boldsymbol{Y}_3(\xi_i)]^2 + \sum_{\xi_i>0}\boldsymbol{w}_i[\boldsymbol{I}_4(0{,}5\boldsymbol{x}_L,\boldsymbol{y}_L,\xi_i,\tilde{\boldsymbol{a}})-\boldsymbol{Y}_4(\xi_i)]^2 \end{aligned} \tag{8}$$

where $Y_1(\mu_i)$, $Y_2(\mu_i)$, $Y_3(\xi_i)$ and $Y_4(\xi_i)$ are measured exit radiation intensities at the boundaries in the points $(x, y)$: $(x_0, y_L/2)$, $(x_L, y_L/2)$, $(x_L/2, y_0)$ and $(x_L/2, y_L)$ respectively, where $x_0 = 0$ and $y_0 = 0$; $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$ e $\boldsymbol{I}_4$, are estimated exit radiation intensities at $(x_0, y_L/2)$, $(x_L, y_L/2)$, $(x_L/2, y_0)$ and $(x_L/2, y_L)$, respectively, for the estimated vector $\tilde{\boldsymbol{a}} = (a_{00}, a_{10}, \ldots, a_{MN})^T$. Then the problem is to find the vector $\tilde{\boldsymbol{a}}$ which minimizes the function $\boldsymbol{J}$. The computational algorithm of this minimization procedure consists of two main modules: the direct radiation computation and the search modules. As explained previously, for the first module the Discrete Ordinates Method (DOM) with the multidimensional spatial scheme is employed in this work, while for the latter the Conjugate Gradient Method (CGM) is used as the basic search method to minimize the function $\boldsymbol{J}$.

### 2.3 Conjugate Gradient Method of Minimization

The minimization of the objective function with respect to the desired vector is the most important procedure in solving the inverse problem. The Conjugate Gradient Method for determining unknown temperature distribution is used in this work. Iterations are built in the following manner (Li and Ozisik, 1992; Liu et al., 2000): $\boldsymbol{a}^{k+1} = \boldsymbol{a}^k - \alpha^k \boldsymbol{d}^k$, where $\alpha^k$ is the step size, $\mathbf{d}^k$ is the direction vector of descent given by $\mathbf{d}^k = \nabla \boldsymbol{J}(\boldsymbol{a}^k) + \beta^k \mathbf{d}^{k-1}$ and the conjugate coefficient $\beta^k$ is determined from

$$\beta^{\boldsymbol{k}} = \frac{\nabla \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}}) \nabla \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{a}^{\boldsymbol{k}})}{\nabla \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}-1}) \nabla \boldsymbol{J}^{\boldsymbol{T}}(\boldsymbol{a}^{\boldsymbol{k}-1})} \qquad \beta^0 = 0 \tag{9}$$

where the row vector $\nabla \boldsymbol{J}$ defined by

$$\nabla \boldsymbol{J} = \left( \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{00}}, \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{10}}, \ldots\ldots\ldots, \quad \frac{\partial \boldsymbol{J}}{\partial \boldsymbol{a}_{\boldsymbol{MN}}} \right) \tag{10}$$

is the gradient of the objective function. Its components are defined as

$$\begin{aligned} \frac{\partial \boldsymbol{J}(\widetilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{qr}}} = {} & 2 \sum_{\mu_i < 0} \boldsymbol{w}_{\boldsymbol{i}} [\boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_{\boldsymbol{L}}, \mu_{\boldsymbol{i}}, \widetilde{\boldsymbol{a}}) - \boldsymbol{Y}_1(\mu_{\boldsymbol{i}})] \frac{\partial \boldsymbol{I}_1(\boldsymbol{x}_0, 0.5\boldsymbol{y}_{\boldsymbol{L}}, \mu_{\boldsymbol{i}}, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{qr}}} + \\ & 2 \sum_{\mu_i > 0} \boldsymbol{w}_{\boldsymbol{i}} [\boldsymbol{I}_2(\boldsymbol{x}_{\boldsymbol{L}}, 0.5\boldsymbol{y}_{\boldsymbol{L}}, \mu_{\boldsymbol{i}}, \widetilde{\boldsymbol{a}}) - \boldsymbol{Y}_2(\mu_{\boldsymbol{i}})] \frac{\partial \boldsymbol{I}_2(\boldsymbol{x}_{\boldsymbol{L}}, 0.5\boldsymbol{y}_{\boldsymbol{L}}, \mu_{\boldsymbol{i}}, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{qr}}} + \\ & 2 \sum_{\xi_i < 0} \boldsymbol{w}_{\boldsymbol{i}} [\boldsymbol{I}_3(0.5\boldsymbol{x}_{\boldsymbol{L}}, \boldsymbol{y}_0, \xi_{\boldsymbol{i}}, \widetilde{\boldsymbol{a}}) - \boldsymbol{Y}_3(\xi_{\boldsymbol{i}})] \frac{\partial \boldsymbol{I}_3(0.5\boldsymbol{x}_{\boldsymbol{L}}, \boldsymbol{y}_0, \xi_{\boldsymbol{i}}, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{qr}}} + \\ & 2 \sum_{\xi_i > 0} \boldsymbol{w}_{\boldsymbol{i}} [\boldsymbol{I}_4(0.5\boldsymbol{x}_{\boldsymbol{L}}, \boldsymbol{y}_{\boldsymbol{L}}, \xi_{\boldsymbol{i}}, \widetilde{\boldsymbol{a}}) - \boldsymbol{Y}_4(\xi_{\boldsymbol{i}})] \frac{\partial \boldsymbol{I}_4(0.5\boldsymbol{x}_{\boldsymbol{L}}, \boldsymbol{y}_{\boldsymbol{L}}, \xi_{\boldsymbol{i}}, \tilde{\boldsymbol{a}})}{\partial \boldsymbol{a}_{\boldsymbol{qr}}} \end{aligned} \tag{11}$$

In principle, the step size of the $k$ th iteration $\alpha^k$ can be determined by minimizing the function, $\boldsymbol{J}(\boldsymbol{a}^k - \alpha^k \mathbf{d}^k)$, for the given $\boldsymbol{a}^k$ and $\boldsymbol{d}^k$ in the following manner:

$$\frac{\partial \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}} - \alpha^{\boldsymbol{k}} \boldsymbol{d}^{\boldsymbol{k}})}{\partial \alpha^{\boldsymbol{k}}} = 0 \tag{12}$$

Since $\boldsymbol{J}(\mathbf{a}^k - \alpha^k \mathbf{d}^k)$ is the implicit function of $\alpha^k$, the exact step is difficult to solve. We make the first-order Taylor expansion of the function with respect to $\alpha^k$. Using Eq. (19), we have in discrete ordinates

$$\alpha^{k} = \Big\{ \sum_{\mu_i<0} \boldsymbol{w_i}[\boldsymbol{I}_1(\boldsymbol{x}_0,0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}) - \boldsymbol{Y}_1(\mu_i)][\nabla \boldsymbol{I}_1(\boldsymbol{x}_0,0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}] +$$
$$\sum_{\mu_i>0} \boldsymbol{w_i}[\boldsymbol{I}_2(\boldsymbol{x_L},0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}) - \boldsymbol{Y}_2(\mu_i)][\nabla \boldsymbol{I}_2(\boldsymbol{x_L},0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}] +$$
$$\sum_{\xi_i<0} \boldsymbol{w_i}[\boldsymbol{I}_3(0.5\boldsymbol{x_L},\boldsymbol{y}_0,\xi_i,\tilde{\boldsymbol{a}}^{k}) - \boldsymbol{Y}_3(\xi_i)][\nabla \boldsymbol{I}_3(0.5\boldsymbol{x_L},\boldsymbol{y}_0,\xi_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}] +$$
$$\sum_{\xi_i>0} \boldsymbol{w_i}[\boldsymbol{I}_4(0.5\boldsymbol{x_L},\boldsymbol{y_L},\xi_i,\tilde{\boldsymbol{a}}^{k}) - \boldsymbol{Y}_4(\xi_i)][\nabla \boldsymbol{I}_4(0.5\boldsymbol{x_L},\boldsymbol{y_L},\xi_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k} \Big\}$$
$$\Big/ \Big\{ \sum_{\mu_i<0} \boldsymbol{w_i}[\nabla \boldsymbol{I}_1(\boldsymbol{x}_0,0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}]^2 + \sum_{\mu_i>0} \boldsymbol{w_i}[\nabla \boldsymbol{I}_2(\boldsymbol{x_L},0.5\boldsymbol{y_L},\mu_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}]^2 +$$
$$\sum_{\xi_i<0} \boldsymbol{w_i}[\nabla \boldsymbol{I}_3(0.5\boldsymbol{x_L},\boldsymbol{y}_0,\xi_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}]^2 + \sum_{\xi_i>0} \boldsymbol{w_i}[\nabla \boldsymbol{I}_4(0.5\boldsymbol{x_L},\boldsymbol{y_L},\xi_i,\tilde{\boldsymbol{a}}^{k}).\boldsymbol{d}^{k}]^2 \Big\} \tag{13}$$

where the row vector

$$\nabla \boldsymbol{I} = \left( \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_{00}}, \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a}_1 0}, ........, \quad \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{a_{MN}}} \right) \tag{14}$$

is the sensitivity coefficient vector, which is essential in the solution procedure of the inverse problem.

### 2.4 Sensitivity Problem

To obtain the sensitivity coefficients, we substitute Eq. (7) into Eq. (5) and differentiate the direct problem defined by Eqs. (4) and (5) with respect to $\boldsymbol{a}_{qr}$. The equations of sensitivity coefficients can be written as

$$\mu \frac{\partial}{\partial \boldsymbol{x}} \left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) + \xi \frac{\partial}{\partial \boldsymbol{y}} \left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) + \beta \left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) = \boldsymbol{f_q}(\boldsymbol{x}) \boldsymbol{g_r}(\boldsymbol{y}) + \frac{\omega}{4\pi} \int_{4\pi} \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega')}{\partial \boldsymbol{a_{qr}}} \boldsymbol{d}\Omega' \tag{15}$$

where $q$ = 1, 2,……., M and $r$ = 1, 2,……., $N$

With the boundary conditions

$$\left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) = 0 \qquad x = 0, \quad \mu > 0\,; \quad \left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) = 0 \qquad x = x_L, \quad \mu < 0$$
$$\left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) = 0 \qquad y = 0, \quad \xi > 0\,; \quad \left( \frac{\partial \boldsymbol{I}(\boldsymbol{x},\boldsymbol{y},\Omega)}{\partial \boldsymbol{a_{qr}}} \right) = 0 \qquad y = y_L, \quad \xi < 0 \tag{16}$$

A similar numerical iteration procedure as the direct problem is used for the solution of the sensitivity problem and will not be repeated here. Because the sensitivity coefficient vector $\nabla \boldsymbol{I}$ is independent of the vector $\boldsymbol{a}$, the estimation of the source term distribution is linear, and it is only necessary to solve it once at first.

### 2.5 Stopping criterion and error estimation

The stopping criterion of the iteration is selected in the following manner: if the problem contains no measurement error, the following condition

$$\boldsymbol{J}(\mathbf{a}^{k+1}) < \delta^* \tag{17}$$

is used for terminating the iterative process, where $\delta^*$ is a small specified positive number; otherwise, it is used in the following two conditions

$$\boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}}) - \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1}) < \delta_1^* \ \text{ and } \boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1)} = 8\pi\sigma_0^2 \tag{18}$$

where $\delta_1^*$ is a small specified positive number, $\sigma_o$ is the standard deviation (Li, 1997; Alifanov, 1974). Written in discrete ordinates is

$$\boldsymbol{J}(\boldsymbol{a}^{\boldsymbol{k}+1}) < \sum_{\mu_m<0} \sigma_{0\boldsymbol{W},\boldsymbol{m}}^2 + \sum_{\mu_m>0} \sigma_{0\boldsymbol{E},\boldsymbol{m}}^2 + \sum_{\xi_m<0} \sigma_{0\boldsymbol{S},\boldsymbol{m}}^2 + \sum_{\xi>0} \sigma_{0\boldsymbol{N},\boldsymbol{m}}^2 \tag{19}$$

When Eq. (17) or Eq. (18) is satisfied, then it is used as the stopping criterion. After several numerical experiments, the value of $\delta_1^*$ is selected as $10^{-5}$. To examine the accuracy of the estimation by using the multidimensional spatial scheme and high order angular quadratures for DOM, the relative error and the rms error defined as (Li, 1997) will be used.

$$\text{Relative error} = \frac{\boldsymbol{S}_{estimated}(x,y) - \boldsymbol{S}_{exact}(x,y)}{\boldsymbol{S}_{exact}(x,y)} \tag{20.a}$$

$$\text{rms error} = \left\{ \frac{1}{x_L y_L} \int_0^{y_L} \int_0^{x_L} \left[\boldsymbol{S}_{estimated}(x,y) - \boldsymbol{S}_{exact}(x,y)\right]^2 dxdy \right\}^{1/2} \tag{20.b}$$

The computational algorithm for the solution of the inverse radiation problem follows the method outlined in Li and Ozisik (1993) and in Li (1997).

## 3. RESULTS AND DISCUSSION

Based on the theoretical and numerical analysis described earlier, a computer code has been developed to solve the inverse radiation problem of the source term in two-dimensional rectangular media by knowing the exit radiation at points in different positions on the boundaries. To examine the accuracy of the method presented in this paper, two different test cases are considered. In the first case, four measurement points are considered and assuming the measurement data exit radiation intensities have errors or do not have errors, the temperature profile is determined. Different angular quadratures of DOM are tested and their accuracy on the estimation is presented. In the second case, the effects of the number of measurement points and their positions on the estimation are analyzed. Results for one, two and four measurement points are compared. The optical thickness of the slab is chosen to be 1.0. To simulate the measured exit radiation intensities, $\boldsymbol{Y}_i$, containing measurement errors, random errors of standard deviation $\sigma_o$ are added to the exact exit radiation intensities computed from the solution of the direct problem. Thus we have

$$(\boldsymbol{Y}_i)_{\text{measured}} = (\boldsymbol{Y}_i)_{\text{exact}} + \sigma_o\,\zeta \qquad i = 1, 2, 3, 4 \tag{21}$$

where $\zeta$ is a normal distribution random variable with a zero mean and unit standard deviation. There is a 99% probability of $\zeta$ lying in the range $-2.567 < \zeta < 2.567$ (Li and Ozisik, 1992). For all the results presented in this work, it is assumed that the exit radiation intensities are available at the quadrature points for Sn (Balsara, 2001), LC11 (Koch et al.,2004) and Tn (Thurgood et al., 1995) angular quadratures of DOM. In this work, the Discrete Ordinates Scheme with Infinitely Small Weight (DOS + ISW) (Li et al., 2003) is used to calculate the $(\boldsymbol{Y}_i)_{\text{exact}}$ values. In that method, one or more new discrete directions are added to an existing discrete ordinate quadrature set, and weights associated with these new directions are set infinitely small. The new discrete direction(s) and the original quadrature set make up a new discrete ordinate quadrature set. The $(\boldsymbol{Y}_i)_{\text{exact}}$ values are calculated using the ($ISW_{S4}$ + $ISW_{S6}$ + $ISW_{LC11}$) + Tn6 angular quadrature. First we consider a source term expressed as a polynomial of degree 4 as in Li (1997),

$$S(x,y) = 1 + 6x + 3y + 4x^2 + 2y^2 + 5x^4y^4, \qquad \text{W/m}^2 \tag{22}$$

The estimate values of the source term by inverse analysis are calculated. Figures 1 shows the characteristic convergence for the solution of the inverse problem. The value of $\delta_1^*$ is selected being equal to $10^{-5}$ after several numerical experiments using the graphics of convergence. With measurement errors, $\sigma_o$ = 0.12, little observable difference was found between the exact values of the source term and the estimated values when the LC11 angular quadrature is used.

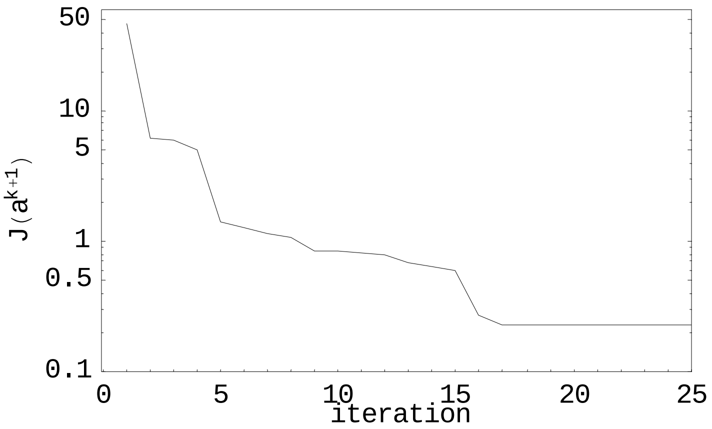


Figure 1. Characteristic convergence profile of the function objective for $\sigma_o = 0.12$

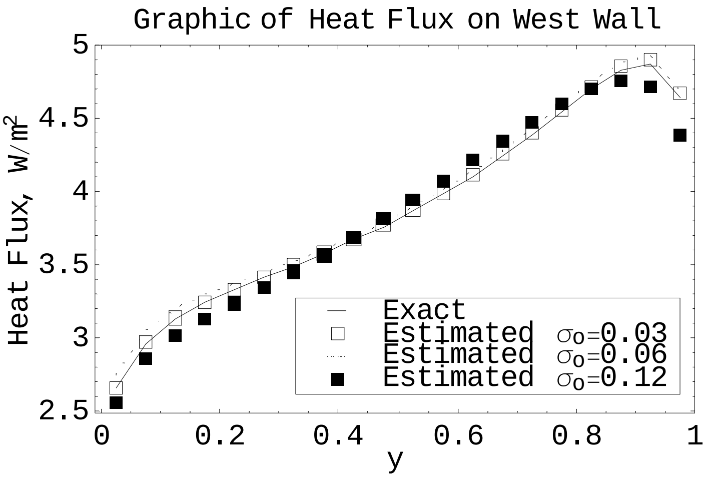


Figure 2. Comparison of the estimation of the heat flux on the west wall using simulated measured exit radiation intensity data with angular quadratures LC11, and $\sigma_o$ = 0.03, 0.06 and 0.12

To show the accuracy of the estimation more clearly, the heat flux on the west wall for $\sigma_o$ equal to 0.03, 0.06 and 0.12 is shown in Fig. 2. This heat flux is calculated using the estimated source term. All the estimations in this figure are for the LC11 quadrature. Clearly, the agreement between the exact and the estimated results for the heat flux on the west wall is good for $\sigma_o$ equal to 0.03 and 0.06. The accuracy decreases when a strong noise with $\sigma_o$ equal to 0.12 is used, but the results still have good approximation.

To examine the effects of the number of measurement points on the estimation several numerical experiments were realized. Figure 3 shows the comparison of the estimations using simulated measured exit radiation intensity data in one, two and four measurement points. A strong noise of $\sigma_o$ equal to 0.12 and center wall measurement points was used in these estimations. Fig. 3 shows the estimations of the radiative dissipation source at y = 0.5 for a different number of sensors or a different number of measurement points at the wall of the cavity. In estimations for two and four sensors, it is observed that similar accurate results are obtained, but for one sensor poor estimation is observed. To check the accuracy of the estimations a more exigent test was made, the estimation of the heat flux on the west wall that uses the intensities calculated near the frontier and near the corners of the cavity. The estimation for two and four sensors is accurate with the exact solution and between them it is close. It was observed that the estimation for one sensor is very poor. The figure of that estimation is not presented here for to sake brevity.

The rms errors of the inverse solution for four and two sensors are shown in Table 1 and compared with results in Li (1997). It can be seen that less rms errors are found in this work even when it used only two sensors to obtain measurement intensities.

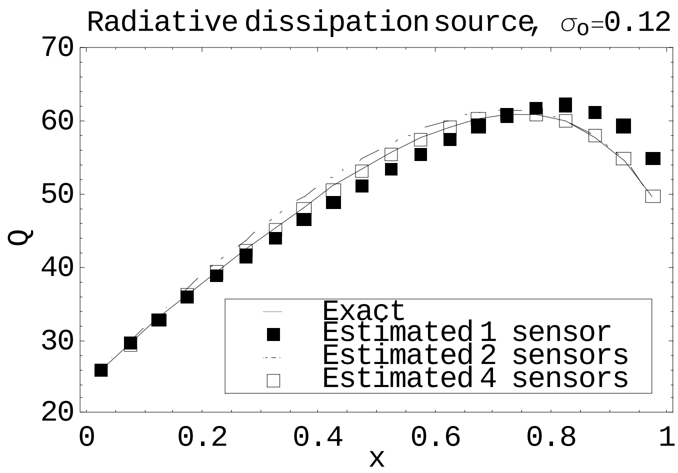


Figure 3. Comparison of the estimation using simulated measured exit radiation intensity data in one, two and four measurement points, for $\sigma_o$ = 0.12

Table 1 The rms error of the estimation for the source term S (x,y), with isotropic scattering, $\omega = 0.5$, $x_L = y_L = 1$

| $\sigma_o$ | This work<br>rms (2 sensors) | This work<br>rms (4 sensors) | Li work (Li, 1997)<br>rms (4 sensors) |
|---|---|---|---|
| 0 | 0.000 | 0.000 | 0.000 |
| 0.06 | 0.126 | 0.079 | 0.212 |
| 0.12 | 0.229 | 0.149 | 0.248 |

Figures 4 and 5 show the comparison of the estimation of the heat flux on the west wall using simulated measured exit radiation intensity data for two sensors at different positions at the two appositive vertical walls. Also, $\sigma_o$ equal to 0.12 is used. It is observed that for the case of symmetrical arrangement more accurate results are found than in the case of non-symmetrical arrangement. Also, in the case of non-symmetrical arrangement, the estimation is less accurate when the sensors are far from the center point of the walls and poor estimations are found when the sensors are near the corners. To examine the influence of the radiative properties on the inverse analysis, simulations for relative errors on the radiative properties were realized. Table 2 shows the effects on the estimations for non-uniform distribution of the absorption coefficient. In simulations two sensors were used. A uniform distribution of radiative properties was used to calculate the exact values, while a non-uniform distribution with 10% random error over a uniform distribution of radiative properties was used to calculate the estimated values (random errors values between -0.05 to 0.05 for the 0.5 value of absorption coefficient),. The rms errors of the inverse solution for non-uniform distribution of the absorption coefficient are higher than in uniform distribution, but it can be observed that for $\sigma_o$ equal to 0.06 and non-uniform absorption distribution the rms error in this work is less than the rms error in Li (1997). The rms error for $\sigma_o$ equal to 0.12 and non-uniform absorption distribution can still be considered acceptable.

Table 2 The rms error of the estimation for the source term ***S*** *(x,y)*, with isotropic scattering, $\omega = 0.5$, $x_L = y_L = 1$ and 10% error on the absorption coefficient

| $\sigma_o$ | rms (2 sensors)<br>$\kappa$ uniform | rms (2 sensors)<br>$\kappa$ non-uniform | rms (4 sensors)<br>Li (1997) |
|---|---|---|---|
| 0 | 0.000 | 0.128 | 0.000 |
| 0.06 | 0.126 | 0.202 | 0.212 |
| 0.12 | 0.229 | 0.314 | 0.248 |

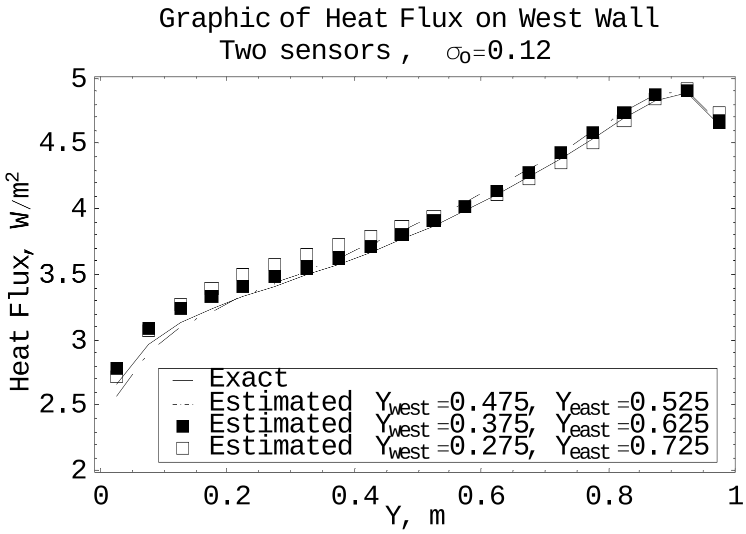


Figure 4. Comparison of the estimation of the heat flux using simulated measured exit radiation intensity data for two sensors at different positions with symmetric arrangement, for $\sigma_o = 0.12$.

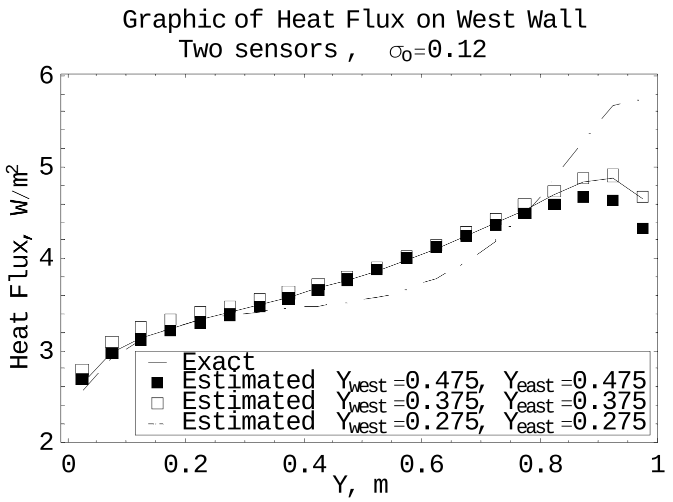


Figure 5. Comparison of the estimation of the heat flux using simulated measured exit radiation intensity data for two sensors at different positions with non-symmetric arrangement, for $\sigma_o = 0.12$.

## 4. CONCLUSIONS

An inverse method is presented for estimation of the temperature distribution for a gray emitting, scattering, two dimensional rectangular medium. The exit radiation intensities at the center of the bounding surfaces are assumed to be known. The direct and sensitive problems are solved using a high order multidimensional scheme for spatial discretization and the LC11 angular quadrature with 60 directions. The inverse problem is solved by using the conjugate gradient method. Noisy input data have been used to test the accuracy of the method used. The results show that the temperature profile can be estimated accurately even with noise data for a high order angular quadrature Tn6 or LC11 in DOM. The results show that the temperature profile can be estimated accurately when the exit radiation intensities at the center of two or four of the bounding surfaces are assumed to be known even with noise data for a high order angular quadrature Tn6 or LC11 in DOM. But the estimations of the temperature profile when the exit radiation intensities at the center of one of the bounding surfaces are assumed to be known are poor. Different arrangements for the sensors position are tested in the case of two sensors and it was found that the temperature profile can be estimated accurately when symmetrical arrangement is used even at the position of the sensors far from the center point of the walls. The results are more sensitive at the position for non-symmetrical arrangement and poor results were found at a position of the sensors far from the center point of the walls.

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