

## ANALYSIS OF ANGULAR HEAT CONDUCTION IN ROTARY HEAT REGENERATORS

**M. C. Reis, marcelloreis@vm.uff.br**

**L. A. Sphaier, lasphaier@mec.uff.br**

Laboratório de Mecânica Teórica e Aplicada, Programa de Pós-Graduação em Engenharia Mecânica, Departamento de Engenharia Mecânica, Universidade Federal Fluminense

***Abstract.** Heat regenerators can be found in a considerable number of engineering applications, and are either used as pair of fixed matrices or as single rotary matrix. The thermal design of these devices is usually done considering models that rely on well-established simplifying assumptions. While most of these assumptions comprise reasonable considerations, some of them could lead to noticeable errors on some occasions. One such assumption is that there is no heat transfer between adjacent channels within the regenerator matrix. While this is quite reasonable for fixed-bed exchangers, this might not be a good choice for rotary exchangers on some occasions. Since rotary matrices can operate between two process streams presenting a large temperature difference between them, a large temperature gradient may develop within the plane normal to the flow direction, especially in the angular direction. This paper proposes a new model for simulating rotary heat regenerators, taking into account this previously unconsidered matrix heat conduction effect. A numerical solution of a test case with angular heat conduction is carried-out. With this solution, a parametric analysis is performed, showing how the effects that gradually increasing the angular heat conduction can affect the temperature distributions within the matrix and regenerator outlet.*

***Keywords:** regenerator, heat exchanger, computational simulation, porous medium*

### 1. INTRODUCTION

The need for sustainable development and the growing energy savings requirements have increased the necessity for more effective heat exchangers. As a result, the thermal design for these heat transfer devices must take into consideration details previously seen as of minor importance. Regenerative heat exchangers have a substantial number of applications in several processes where indirect heat transfer between two process streams with a compact construction is required. The traditional thermal design of these exchangers are based on solving a system of two energy transfer equations, one for the process streams and the other one for the solid matrix. Different types of formulations are found in the literature (de Monte, 1999; Saastamoinen, 1999; Larsen, 1967), some with more details than others. Nevertheless, apparently all previous formulations model problem as that of heat transfer in a single independent channel. The problem with these formulations, is that they cannot account for temperature gradients (and consequently heat transfer) in the direction perpendicular to that of the fluid flow. As this may be unimportant for switching fixed-bed exchangers, it may be problematic for rotary devices. Under this scenario, this investigation proposes a mathematical formulation for heat regenerators that actually takes into account radial and angular temperature gradients within a rotary exchanger.

## 2. PROBLEM FORMULATION

The usual simplifying assumptions for heat regenerators are (Kays and London, 1998; Shah and Sekulic, 2002):

1. The regenerator operates under quasi-steady-state or regular periodic-flow conditions (i.e., having constant mass flow rates and inlet temperatures of both fluids during respective flow periods).
2. Heat loss to or heat gain from the surroundings is negligible (i.e., the regenerator outside walls are adiabatic).
3. There are no thermal energy sources or sinks within the regenerator walls or fluids.
4. No phase change occurs in the regenerator.
5. The velocity and temperature of each fluid at the inlet are uniform over the flow cross section and constant with time.
6. The analysis is based on average and thus constant fluid velocities and the thermo-physical properties of both fluids and matrix wall material throughout the regenerator (i.e., independent on time and position).
7. The heat transfer coefficients ( $h_h$  and  $h_c$ ) between the fluids and the matrix wall are constant (with position, temperature and time) throughout the exchanger.
8. Longitudinal heat conduction in the wall and the fluids are negligible.
9. The temperature across the wall thickness is uniform at cross section and the wall thermal resistance is treated as zero for transverse conduction in the matrix wall (in the wall thickness direction).
10. No flow leakage and flow bypassing of either of the two fluid streams occurs in the regenerator due to their pressure differences. No fluid carryover leakage (of one fluid stream to the other fluid stream) occurs of the fluids trapped in flow passages during the switch from hot to cold period, and vice versa, during matrix rotation or valve switching.
11. The surface area of the matrix as well as the rotor mass is uniformly distributed.
12. The time required to switch the regenerator from the hot to cold gas flow is negligibly small.
13. Heat transfer caused by radiation within the porous matrix is negligible compared with the convective heat transfer.
14. Gas residence (dwell) time in the matrix is negligible relative to the flow period.

The formulation employed in this work considers all these simplifications, but one. The channels are no longer considered to be independent since there can be heat transfer within the matrix in the transversal directions. Since this consideration does not affect the energy transfer within the fluid flow, the same equation used in (Shah and Sekulic, 2002) is herein employed:

$$\rho c_p \left( \frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = \frac{h P_w}{A_f} (T_s - T). \quad (1)$$

For the heat conduction within the matrix a different balance is considered. The traditional model employed in heat regenerators studies is based on the following equation for the energy transfer within the matrix:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = - \frac{h P_w}{A_s} (T_s - T), \quad (2)$$

In this study, since local conduction within the matrix is considered, the previous equation must be modified to include these effects, leading to:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = \nabla \cdot (\mathbf{K}_s \cdot \nabla T_s) - \frac{h P_w}{A_s} (T_s - T), \quad (3)$$

where  $\mathbf{K}_s$  is a thermal conductivity tensor. Considering the matrix as an orthotropic medium and assuming that the components of  $\mathbf{K}_s$  are invariable with position, equation (3) is reduced to:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = k_r \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_s}{\partial r} \right) + k_\theta \frac{1}{r^2} \frac{\partial^2 T_s}{\partial \theta^2} + k_z \frac{\partial^2 T_s}{\partial z^2} - \frac{h P_w}{A_s} (T_s - T). \quad (4)$$

The boundary conditions for this problem are given by:

$$T = T_{in} \quad \text{for} \quad z = 0, \quad \left. \frac{\partial T_s}{\partial z} \right|_{z=0} = \left. \frac{\partial T_s}{\partial z} \right|_{z=L} = 0, \quad (5)$$

$$\left. \frac{\partial T_s}{\partial \theta} \right|_{\theta=0} = \left. \frac{\partial T_s}{\partial \theta} \right|_{\theta=2\pi}, \quad T_s|_{\theta=0} = T_s|_{\theta=2\pi}, \quad |T_s|_{r=0}| < \infty, \quad \left. \frac{\partial T_s}{\partial r} \right|_{r=R} = 0, \quad (6)$$

and the initial conditions are given by

$$T = T_s = T_0 \quad \text{for} \quad t = 0. \quad (7)$$

Although a transient single-blow can lead to useful information regarding the effects of heat transfer in the matrix, the actual operation of a regenerator is periodic (also known as a quasi steady-state), the inlet conditions are modified to give, for a parallel-flow arrangement:

$$T = T_{in}^A \quad \text{for} \quad z = 0 \quad \text{and} \quad 0 < \theta \leq \theta_{AB}, \quad (8)$$

$$T = T_{in}^B \quad \text{for} \quad z = 0 \quad \text{and} \quad \theta_{AB} < \theta \leq 2\pi, \quad (9)$$

and the velocity  $v_z$  can assume different values for each process stream:

$$v_z = \frac{\dot{m}_A''}{\rho_A} \quad \text{for} \quad z = 0 \quad \text{and} \quad 0 < \theta \leq \theta_{AB}, \quad (10)$$

$$v_z = \frac{\dot{m}_B''}{\rho_B} \quad \text{for} \quad z = 0 \quad \text{and} \quad \theta_{AB} < \theta \leq 2\pi, \quad (11)$$

where  $\dot{m}''$  are the mass fluxes in the direction of the flow and  $\rho$  is the specific mass of each process stream.

By observing at the resulting formulation one can see that the model resembles formulations used for heat transfer in porous medium. In fact, the proposed approach to the given problem treats the regenerator matrix as a porous medium composed of cylindrical pores. The advantage of the current formulation is that it allows heat transfer interactions among adjacent regenerator channels.

### 3. NORMALIZATION

#### 3.1 Dimensionless parameters

The dimensionless parameters involved in this work are traditional parameters employed in heat exchangers (Shah and Sekulic, 2002) and transient heat transfer (Özişik, 1993) analyses. The heat capacity ratio is defined as the ratio below:

$$C_r^* = \frac{C_r}{C_{\min}}, \quad (12)$$

where fluid capacity rate and the matrix capacity rate are given by:

$$C = \dot{m} c_p, \quad C_r = \frac{\rho_s c_s A_s L}{\tau}, \quad (13)$$

in which  $C_{\min}$  is the minimum capacity of the two process streams. The number of transfer units is defined as:

$$NTU = \frac{h A_s}{C_{\min}} \quad (14)$$

Another parameter is the dimensionless dwell time:

$$\tau_{dw}^* = \frac{\tau_{dw}}{\tau} = \frac{L}{v_z \tau} \quad (15)$$

where the dwell time  $\tau_{dw}$  represents the time it takes for a fluid particle to cross the regenerator length.

Since different conductivities can be employed for each direction (and hence three different diffusivities), different Fourier numbers arise. These are defined as:

$$Fo_r = \frac{\alpha_r \tau}{(D/2)^2}, \quad Fo_\theta = \frac{\alpha_\theta \tau}{(D/2)^2}, \quad Fo_z = \frac{\alpha_z \tau}{L^2} \quad (16)$$

where

$$\alpha_r = \frac{k_r}{\rho_s c_s}; \quad \alpha_\theta = \frac{k_\theta}{\rho_s c_s}; \quad \alpha_z = \frac{k_z}{\rho_s c_s} \quad (17)$$

### 3.2 Dimensionless formulation

In order to normalize the mathematical formulations employed in this work the following dimensionless variables are introduced:

$$r^* = \frac{r}{D/2}, \quad z^* = \frac{z}{L}, \quad t^* = \frac{t}{\tau} \quad (18)$$

$$T_s^* = \frac{T_s - T_{\min}}{T_{\max} - T_{\min}}, \quad T^* = \frac{T - T_{\min}}{T_{\max} - T_{\min}} \quad (19)$$

Introducing the dimensionless parameters in equation (4) leads to the following normalized equation for the solid matrix:

$$\frac{\partial T_s^*}{\partial t^*} = Fo_r \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial T_s^*}{\partial r^*} \right) + Fo_\theta \frac{1}{r^{*2}} \frac{\partial^2 T_s^*}{\partial \theta^2} + Fo_z \frac{\partial^2 T_s^*}{\partial z^{*2}} - \frac{NTU}{C_r^*} (T_s^* - T^*) \quad (20)$$

Introducing the dimensionless variables in equation (1) leads to the following normalized equation for the fluid stream:

$$\left( \frac{\partial T^*}{\partial t^*} \tau_{dw}^* + \frac{\partial T^*}{\partial z^*} \right) = NTU (T_s^* - T^*) \quad (21)$$

The normalized boundary conditions are given by:

$$T^* = T_{in}^* \quad \text{for} \quad z^* = 0, \quad \frac{\partial T_s^*}{\partial z^*} \Big|_{z^*=0} = \frac{\partial T_s^*}{\partial z^*} \Big|_{z^*=1} = 0, \quad (22)$$

$$\frac{\partial T_s^*}{\partial \theta} \Big|_{\theta=0} = \frac{\partial T_s^*}{\partial \theta} \Big|_{\theta=2\pi}, \quad T_s^*|_{\theta=0} = T_s^*|_{\theta=2\pi}, \quad |T_s^*|_{r^*=0}| < \infty, \quad \frac{\partial T_s^*}{\partial r^*} \Big|_{r^*=1} = 0 \quad (23)$$

and the normalized initial condition is given by:

$$T^* = T_0^* \quad \text{for} \quad t^* = 0. \quad (24)$$

In order to simulate the operation of the regenerator between two process streams, the inlet temperature can vary with time and angular position, such that:

$$T_{in}^* = \begin{cases} T_{in,A}^*, & \text{for } 0 \leq \theta < \theta_{AB} \\ T_{in,B}^*, & \text{for } \theta_{AB} \leq \theta < 2\pi \end{cases} \quad (25)$$

### 3.3 Performance assessment

The performance of heat regenerators is generally assessed by means of heat transfer effectiveness. For balanced and symmetric exchangers, two different expressions can be written:

$$\epsilon_1 = \frac{-\dot{Q}_A}{\dot{Q}_{\max}} = \frac{T_{in}^A - T_{out,av}^A}{T_{in}^A - T_{in}^B} \quad (26)$$

$$\epsilon_2 = \frac{\dot{Q}_B}{\dot{Q}_{\max}} = \frac{T_{out,av}^B - T_{in}^B}{T_{in}^A - T_{in}^B} \quad (27)$$

Naturally, once a periodic operation regime has been reached  $\dot{Q}_A + \dot{Q}_B = 0$  and both expressions lead to the same value.

### 4. TEST-CASE

Since this paper is an initial study aimed at investigating the effects of transversal matrix conductivity, a simpler case with heat conduction only in the angular direction is considered. Under this consideration, the matrix equation is reduced to the following form:

$$\frac{\partial T_s^*}{\partial t^*} = Fo_\theta \frac{1}{r^{*2}} \frac{\partial^2 T_s^*}{\partial \theta^2} - \frac{NTU}{C_r^*} (T_s^* - T^*) \quad (28)$$

in addition for the sake of simplicity, since the problem has no dependence in the radial coordinate, a unitary radius is considered, leading to the simpler form:

$$\frac{\partial T_s^*}{\partial t^*} = Fo_\theta \frac{\partial^2 T_s^*}{\partial \theta^2} - \frac{NTU}{C_r^*} (T_s^* - T^*) \quad (29)$$

In order to compare the solution of the test-case problem with the model used in previous studies, the following simplified equation for the case with no heat conduction in the matrix is considered:

$$\frac{\partial T_s^*}{\partial t^*} = -\frac{NTU}{C_r^*} (T_s^* - T^*) \quad (30)$$

which naturally corresponds to the case with  $Fo_\theta = 0$ .

### 5. NUMERICAL SOLUTION

Equations (21) and (29) are spatially discretized using the Finite Volumes Method. Second-order central differences are used for the diffusive terms, leading to the following discretized equation for the matrix:

$$\frac{dT_{SP}}{dt} = Fo_\theta^* \frac{T_{SN} - 2T_{SP} + T_{SS}}{\Delta\theta^2} - \frac{NTU}{C_r} (T_{SP} - T_P), \quad (31)$$

This equation is valid for all volumes; however, some considerations must be taken into account when considering volumes adjacent to the boundaries  $\theta = 0$  and  $\theta = 2\pi$ . Due to the periodicity conditions, the following relation holds:

$$T_{SS} \text{ for volume adjacent to } \theta = 0 \text{ equals } T_{SN} \text{ for volume adjacent to } \theta = 2\pi \quad (32)$$

For the fluid stream both second-order central differences and first- and second-order upwind schemes were considered. When CDS is used the fluid equations lead to the equations below.

- For volumes adjacent to the channel entrance:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{T_E + T_P - 2T_{in}}{2\Delta z} = NTU (T_{SP} - T_P), \quad (33)$$

- For volumes adjacent to the channel outlet:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{T_P - T_W}{\Delta z} = \text{NTU} (T_{sP} - T_P), \quad (34)$$

- For internal volumes:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{T_E - T_W}{2 \Delta z} = \text{NTU} (T_{sP} - T_P), \quad (35)$$

For a first-order upwind scheme the equations below are obtained.

- For volumes adjacent to the channel entrance:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{2(T_P - T_{in})}{\Delta z} = \text{NTU} (T_{sP} - T_P), \quad (36)$$

- For internal volumes:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{T_P - T_W}{\Delta z} = \text{NTU} (T_{sP} - T_P), \quad (37)$$

For a second-order upwind scheme the equations below are obtained.

- For volumes adjacent to the channel entrance:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{2(T_P - T_{in})}{\Delta z} = \text{NTU} (T_{sP} - T_P), \quad (38)$$

- For volumes adjacent to the one next to the channel entrance:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{3T_P - 5T_W + 2T_{in}}{\Delta z} = \text{NTU} (T_{sP} - T_P), \quad (39)$$

- For internal volumes:

$$\frac{dT_P}{dt} \tau_{dw} + \frac{3T_P - 4T_W + T_{WW}}{2 \Delta z} = \text{NTU} (T_{sP} - T_P), \quad (40)$$

## 5.1 Average outlets

The average temperatures used for assess the regenerator effectiveness are obtained from the following expressions:

$$T_{out,av}^A(\theta) = \frac{1}{1/2} \int_0^{1/2} T(1, \theta, t^*) dt^* \quad (41)$$

$$T_{out,av}^B(\theta) = \frac{1}{1/2} \int_{1/2}^1 T(1, \theta, t^*) dt^* \quad (42)$$

## 6. RESULTS AND DISCUSSION

A numerical solution to these equations using the CDS implementation for heat conduction within the matrix and three different discretization schemes for discretizing the fluid equations was implemented in the Mathematica framework. The time integration was performed using a readily available ODE system integrator, similar to the procedure described in (Sphaier and Worek, 2008). The chosen ODE integrator for this work was the Mathematica function NDSolve.

### 6.1 Numerical convergence analysis

This first section presents numerical results for determining an adequate grid size for evaluating the numerical solutions. In order to validate the 2D numerical solution, the solution of a one dimensional problem, without angular dependence is used. Then the 2D numerical solution code is executed for  $Fo_\theta = 0$ , which should lead to the same solution as the grid is refined. The parameter  $\epsilon_m$  is the average value between  $\epsilon_1$  and  $\epsilon_2$ . Tables 1 through 4 display comparison results between 1D and 2D solutions, gradually refining the grid in the angular direction. As can be seen, the 2D solutions gradually progress to the effectiveness values calculated with the 1D solution as the  $\theta$ -grid is refined.

Table 1. Comparisons of 2D solution with 1D solution for  $Fo_\theta = 0$ ,  $NTU = 1$  and  $C_r = 1$ .

$C_r$	NTU	$\tau$	Fo	imax	jmax	$\epsilon_1$	$\epsilon_2$	$\epsilon_m$
1	1	0.01	0	10	10	0.315147	0.370054	0.342601
1	1	0.01	0	10	20	0.292199	0.353987	0.323093
1	1	0.01	0	10	40	0.280852	0.346167	0.313509
1	1	0.01	0	10	80	0.276232	0.344335	0.310283
1	1	0.01	0	10	100	0.275530	0.344407	0.309969
1D						0.273006	0.344950	0.308978

Table 2. Comparisons of 2D solution with 1D solution for  $Fo_\theta = 0$ ,  $NTU = 3$  and  $C_r = 1$ .

$C_r$	NTU	$\tau$	Fo	imax	jmax	$\epsilon_1$	$\epsilon_2$	$\epsilon_m$
1	3	0.01	0	10	10	0.42031	0.527688	0.473999
1	3	0.01	0	10	20	0.410284	0.53079	0.470537
1	3	0.01	0	10	40	0.405274	0.5323	0.468787
1	3	0.01	0	10	80	0.4029	0.533291	0.468095
1	3	0.01	0	10	100	0.402471	0.533577	0.468024
1D						0.400889	0.534742	0.467815

Table 3. Comparisons of 2D solution with 1D solution for  $Fo_\theta = 0$ ,  $NTU = 5$  and  $C_r = 1$ .

$C_r$	NTU	$\tau$	Fo	imax	jmax	$\epsilon_1$	$\epsilon_2$	$\epsilon_m$
1	5	0.01	0	10	10	0.452457	0.572006	0.512232
1	5	0.01	0	10	20	0.445938	0.57968	0.512809
1	5	0.01	0	10	40	0.442609	0.583342	0.512976
1	5	0.01	0	10	80	0.440948	0.585158	0.513053
1	5	0.01	0	10	100	0.440624	0.585534	0.513079
1D						0.439412	0.586985	0.513198

Table 4. Comparisons of 2D solution with 1D solution for  $Fo_\theta = 0$ ,  $NTU = 10$  and  $C_r = 1$ .

$C_r$	NTU	$\tau$	Fo	imax	jmax	$\epsilon_1$	$\epsilon_2$	$\epsilon_m$
1	10	0.01	0	10	10	0.479098	0.60104	0.540069
1	10	0.01	0	10	20	0.4754	0.61155	0.543475
1	10	0.01	0	10	40	0.473417	0.616537	0.544977
1	10	0.01	0	10	80	0.472395	0.618956	0.545675
1	10	0.01	0	10	100	0.472187	0.619431	0.545809
1D						0.471399	0.621226	0.546313

After presenting the convergence analysis, simulation results are performed to investigate the effect of gradually increasing the angular heat conduction in the matrix. Figures 1, 2 and 3 illustrate the effects of varying some of the dimensionless parameters on the outlet temperature (for fixed position within the matrix) and average matrix temperature within an full cycle, composed of a hot period ( $0 \leq t \leq 1/2$ ) followed by a cold period ( $1/2 \leq t \leq 1$ ). Figure 1 represents the case with no angular conduction in the matrix ( $Fo_\theta = 0$ ). As a result

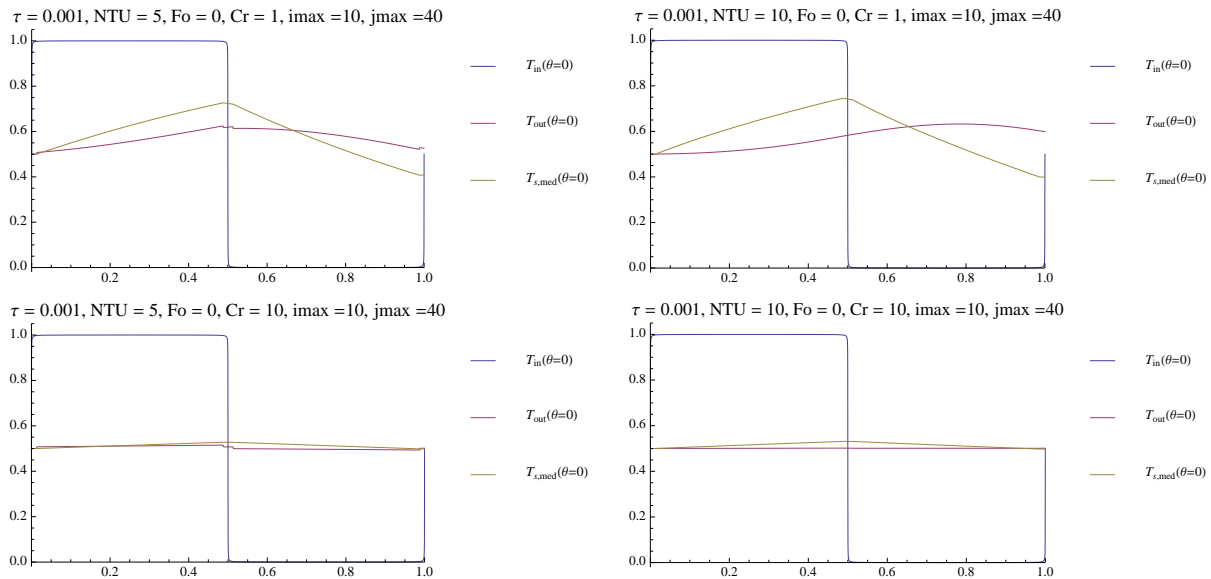


Figure 1. Variation of outlet temperature and average matrix temperature within a operating period with NTU and  $C_f$  for  $Fo_\theta = 1$ .

there is no heat transfer interaction among adjacent regenerator channels. As can be seen, when varying the NTU causes a large heat exchange between matrix and fluid, which contributes for larger variations in matrix temperatures. This will naturally lead to a larger energy exchange effectiveness since more heat could be exchanged between the hot and cold streams. Increasing the heat capacity rate, causes the matrix temperature to present smaller variation because the relative thermal inertia of the matrix is larger.

Figure 2 presents the same results for  $Fo_\theta = 1$ . Comparing the results with the previous figure, one can observe that the presence of matrix heat conduction causes the temperature variations in the matrix to be less prominent; however, the effects are small for this value of  $Fo_\theta$ .

Figure 3 presents the same results for  $Fo_\theta = 10$ . Comparing the results with the previous figures, one can observe that the even larger presence of angular matrix conduction leads to significantly less variation in matrix temperature variations.



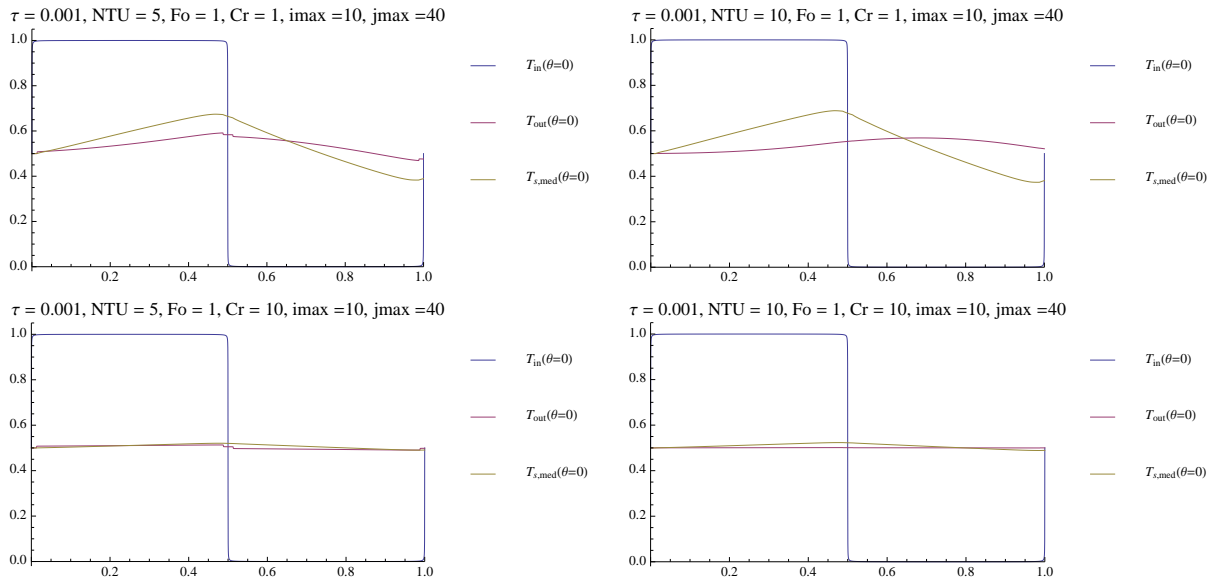


Figure 2. Variation of outlet temperature and average matrix temperature within a operating period with NTU and  $C_f$  for  $Fo_\theta = 1$ .

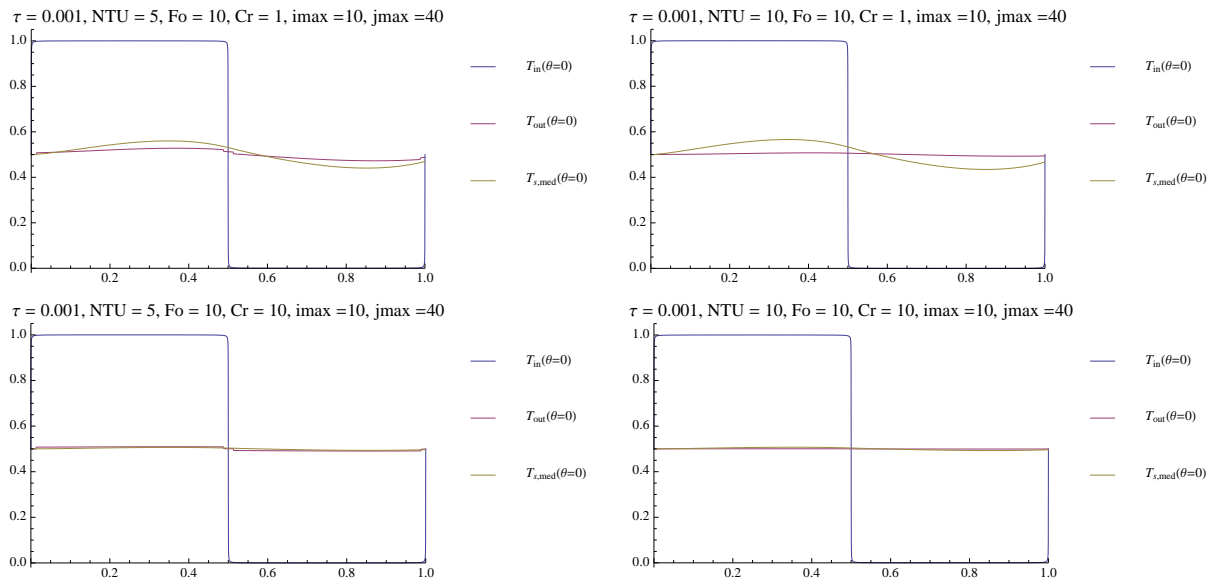


Figure 3. Variation of outlet temperature and average matrix temperature within a operating period with NTU and  $C_f$  for  $Fo_\theta = 1$ .

## 7. CONCLUSIONS

This paper presented a formulation for analyzing the effects of matrix heat conduction (in all three directions) in rotary regenerative heat exchangers. The formulation was normalized and the effects of heat conduction in the three directions of the exchanger lead to the definition of three Fourier numbers. A test case with only angular heat conduction was numerically simulated for conduction an initial investigation on the effects of matrix heat conduction in regenerators. This direction was chosen because there is tendency to develop a temperature gradient in this direction. The numerical simulation was constructed using the finite volumes method with different differencing schemes for advective and diffusive terms, and implemented in the Mathematica system. The results provide an indication to the effects that angular heat conduction have on temperature distributions in heat regenerators.

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