Development of a Code for BiGlobal Linear Stability Analysis of Compressible Flows: Application to Leading-Edge Boundary Layer

Elmer Mateus Gennaro, elmer@sc.usp.br
University of São Paulo

José Miguel Pérez Pérez, jomipp@torroja.dmt.upm.es
Daniel Rodríguez Álvarez, dani@torroja.dmt.upm.es
Vassilis Theofilis, vassilis@aero.upm.es
Universidad Politécnica de Madrid

Marcello A. Faraco de Medeiros, marcello@sc.usp.br
University of São Paulo

Abstract. *BiGlobal linear stability analysis has been shown to provide useful insight in flows over or through complex geometries and is a promising path in devising theoretically founded flow control strategies. The present work seeks the development of a computational code capable of performing BiGlobal linear stability analyses of compressible flows. The aim is to investigate the computational challenges associated with problems of the linear stability and propose alternative numerical methods. The ideas developed are applied to analyze the effect of compressibility in attachment-line boundary global flow instability.*

Keywords: *BiGlobal linear instability analysis, generalized eigenproblem, swept Hiemenz flow*

1. INTRODUCTION

Investigation of linear instability mechanisms is essential for the understanding of the transition process from laminar to turbulent flow. Many studies over several decades have reported results in simple one-dimensional basic flows, such as boundary layers or shear layers. However, most flows of practical engineering significance remain unexplored. The reason is that the underlying basic state of most practical flows depends in an inhomogeneous manner on more than one spatial direction and the cost of performing a complete parametric instability analysis can be formidable.

Linear stability analysis is often based on decomposing the flow into a steady and an unsteady parts, so-called basic and perturbation flow respectively. If a basic flow is established, the Navier-Stokes equations can be written in terms of disturbance variables and linearized for small-amplitude disturbances. If the basic flow depends in an inhomogeneous manner on two spatial directions, the disturbance can be considered periodic in the third direction, along which Fourier modes can be introduced. The problem becomes an eigenvalue problem with an eigenfunction that depends on the other two directions. In view of the linearization an exponential function is also considered in time, and the time coefficient is the eigenvalue. Because the eigenfunction depends on two directions, this problem is called BiGlobal linear stability analysis.

In principle, all these assumptions lead to a problem easier to solve than the direct numerical simulation (DNS), but the large size of the discretized matrices makes the numerical solution challenging. The most effective techniques to solve generalized eigenproblem are based on subspace projection-iterative methods such as the Arnoldi iteration which is based on the Krylov-subspaces (Saad, 2000). The Arnoldi method delivers a window of the eigenspectrum, but it favors the eigenvalues with the largest modulus, thus inversion of the matrix is required in order to introduce an eigenvalue shift towards the interested part of spectrum.

From a numerical point of view, the aim of the present contribution is to develop a computational code capable of performing BiGlobal linear stability analyzes of compressible flows exploiting sparsity that may exist in a given problem. As an application, the algorithm and codes developed, will then be used to perform BiGlobal linear instability analysis of compressible swept attachment-line boundary layer flow.

From a physical point of view, instability of the flow near the leading edge of swept wings has a great engineering significance. This is mainly because this kind of instability promotes the growth of disturbances which can be convected downstream, having direct influence on the transitional process from laminar to turbulent flow on the wing surface, which, in turn, affects aerodynamic performance. Study of instability mechanisms in this flow can provide an insight in the aerodynamic design of wings.

Hall et al. (1984) were the first to study the linear stability of the incompressible swept attachment-line boundary layer adopting the swept Hiemenz basic flow and the Görtler-Hämmerlin (1955) similarity model for the perturbations. Lin and Malik (1996) used a two-dimensional representation of the perturbations around the Hiemenz flow. They went on to include curvature and concluded that its effect is stabilizing (Lin and Malik, 1997).

Theofilis et al. (2003) performed temporal BiGlobal analysis of the incompressible swept Hiemenz flow and proposed a polynomial model to describe the chordwise dependence of the amplitude functions, reducing the cost of performing global analysis without loss of physical information in the linear regime.

Study of compressibility effects on the leading edge boundary layer was first introduced in a global analysis context by Theofilis, Fedorov and Collis (2004) who solved a dense BiGlobal eigenvalue problem. The objective here is to complete this study by performing a full parameter scan by a new computationally more efficient approach which involves a sparse eigenproblem solver.

2. THEORY

2.1 Basic Flow

The leading-edge flow in the vicinity of the attachment line of a swept wing is treated as a compressible stagnation line flow, with a constant non zero free-stream velocity component along the attachment line. We consider a Prandtl number equal to 0.72. The basic flow in normal and tangential directions is based on the solution for the two dimensional compressible stagnation flow proposed by Cohen and Reshotko (1956), which also assumes a two dimensional stream-function that depends on the tangential and the normal to the wall directions

$$\psi(\xi,\eta) = \nu\xi f(\eta)\,. \tag{1}$$

where ξ and η are respectively, x and y dimensionless variables scaled by $\Delta = \left(\frac{\nu}{S}\right)^{1/2}$. ν is the kinematic viscosity and S is the local strain rate of the boundary layer basic flow, $S = \left(\frac{dU_e}{dx}\right)_{x=0}$. The spanwise velocity, W, is taken to be dependent only on η.

The compressible laminar boundary layer can be transformed to the incompressible case using Illingworth-Stewartson transformation defined by:

$$X = \int_0^x \lambda \frac{p_e c_e}{p_0 c_0} dx\,, \quad Y = \frac{a_e}{a_0}\int_0^y \lambda \frac{\rho}{\rho_0} dy\,, \tag{2}$$

where p is the pressure, a is the sound velocity, subscript o indicates free-stream stagnation values and subscript e refers to the outer edge of boundary layer.

The velocity profiles and temperature are solutions of the ordinary differential equations:

$$V' = -U + V\frac{T'}{T} \tag{3}$$

$$U'' = \frac{1}{\mu}\left(\frac{U^2 + VU'}{T} - 1 - \frac{\partial\mu}{\partial T}T'U'\right) \tag{4}$$

$$W'' = \frac{1}{\mu}\left(\frac{VW'}{T} - \frac{\partial\mu}{\partial T}T'W'\right) \tag{5}$$

$$T'' = \frac{Pr}{\mu}\left(-\frac{\partial\mu}{\partial T}\frac{T'^2}{Pr} + \frac{T'V}{T} - (\gamma-1)Ma^2\mu W'^2\right) \tag{6}$$

$$\tag{7}$$

subject to the boundary conditions:

$$U(0) = W(0) = 0,\; V(0) = -C_q T_w \tag{8}$$

$$U(\infty) = W(\infty) = T(\infty) = 1 \tag{9}$$

where C_q is a suction parameter and T_w is the wall temperature.

System (3-9) was solved using a shooting method and the results are compared with those obtained by Theofilis et al. (2004) in Tab. 1.

2.2 BIGLOBAL LINEAR STABILITY ANALYSIS

The BiGlobal linear stability analysis is based on describing the flow variables as

$$\mathbf{q}(x,y,z,t) = \mathbf{Q}(x,y) + \epsilon\tilde{\mathbf{q}}(x,y,z,t), \quad \epsilon \ll 1, \tag{10}$$

where $\mathbf{Q} = (\bar{U},\bar{V},\bar{W},\bar{T},\bar{P})^t$. Moreover, $\bar{U} = xU(y)$, $\bar{V} = V(y)$, $\bar{W} = W(y)$, $\bar{T} = T(y)$ and $\tilde{\mathbf{q}} = (\tilde{u},\tilde{v},\tilde{w},\tilde{\theta},\tilde{p})^t$ represents the basic and disturbance flow, respectively, where the chordwise, wall-normal and spanwise (along the attachment line) spatial coordinates are denoted by x, y, and z, respectively and the spanwise direction, z, is taken homogeneous, while curvature is neglected.

Table 1. Dependence of basic flow shear-stress and wall-temperature on Ma

Ma	$U_0{}'(0)$	$W_0{}'(0)$	$T_0(0)$
10^{-5}	1.232588	0.570465	1.000000
0.25	1.228483	0.566095	1.010746
0.50	1.216689	0.553511	1.042990
0.75	1.198610	0.534132	1.096741
1.00	1.176141	0.509881	1.172016
2.00	1.076630	0.399360	1.688629
3.00	1.000514	0.308888	2.550336

Substituting the disturbance flow quantities into the compressible continuity, Navier-Stokes and energy equations and linearizing about $\mathbf{Q}$, we obtain a system of partial-differential equations in the terms of disturbances variables. The eigenmodes are introduced as

$$\tilde{\mathbf{q}}(x,y,z,t) = \hat{\mathbf{q}}(x,y)\exp(i\beta z - \omega t) + c.c, \tag{11}$$

and indicate that the disturbances evolve with complex frequency ω, real wavenumber $\beta = 2\pi/L_z$, where L_z is the wavelength along the homogeneous direction. Moreover $\hat{\mathbf{q}}(x,y)$ denotes the amplitude functions $(\hat{u},\hat{v},\hat{w},\hat{\theta},\hat{p})^t$ with $c.c$ the respective conjugate complex. In a temporal analysis, a positive value of the amplification/damping rate ω_i indicates growth and the mode is temporally unstable, while $\omega_i < 0$ indicates damping, and the mode is stable. Thus the problem becomes a two-dimensional generalized eigenvalue problem. It can be written in the form

$$\mathcal{L}\hat{\mathbf{q}} = \omega\mathcal{R}\hat{\mathbf{q}}, \tag{12}$$

where the matrices $\mathcal{L}$ and $\mathcal{R}$ are shown in detail by Theofilis & Colonius (2004) and de Vicente et al. (2008), subject to the boundary conditions:

- at the wall, no-slip and $\frac{\partial p}{\partial y} = 0$;
- at the far-field, a fast decay is assumed for the disturbances in the normal direction and the homogeneous direction Dirichlet condition imposed.
- at the $x = \pm L_x$ for all variables, linear extrapolation from the interior of the computational domain.

The equation of state linearized gives

$$\frac{\hat{p}}{\bar{p}} = \frac{\hat{\rho}}{\bar{\rho}} + \frac{\hat{T}}{\bar{T}}. \tag{13}$$

The viscosity and thermal conductivity are taken as functions of temperature alone and the resulting linearized expressions resulting are:

$$\hat{\mu} = \frac{d\bar{\mu}}{dT}\hat{T}, \quad \hat{\kappa} = \frac{d\bar{\kappa}}{dT}\hat{T} \tag{14}$$

3. METHODOLOGY

3.1 Chebyshev spectral collocation method

The Chebyshev spectral collocation method can be described as an approximation based on Chebyshev polynomials to an unknown partial differential equation (PDE) solution. The domain of interest can be discretized using Chebyshev-Gauss-Lobatto (CGL) points defined as

$$\xi_j = \cos\left(\frac{\pi j}{N}\right), \quad j = 0,1,\ldots,N. \tag{15}$$

These CGL points are extrema of the N^{th} order Chebyshev polynomial for $\xi \in [-1,1]$

$$T_k(\xi) = \cos(k\theta), \quad \theta = \arccos(\xi), \quad k = 0,1,2,3,\ldots \tag{16}$$

A function $u(\xi)$ can be approximated by a Chebyshev series

$$u_N(\xi) = \sum_{k=0}^{N} a_k T_k(\xi). \tag{17}$$

where a_k are the Chebyshev coefficients defined by

$$a_k = \frac{2}{N}\frac{1}{c_k}\sum_{i=0}^{N}\frac{u(\xi_i)T_k(\xi_i)}{c_i} \tag{18}$$

and

$$c_i = \begin{cases} 2 & \text{if } i = 0, N \\ 1 & \text{otherwise} \end{cases}$$

The p^{th}-order derivate of the u can be obtained as

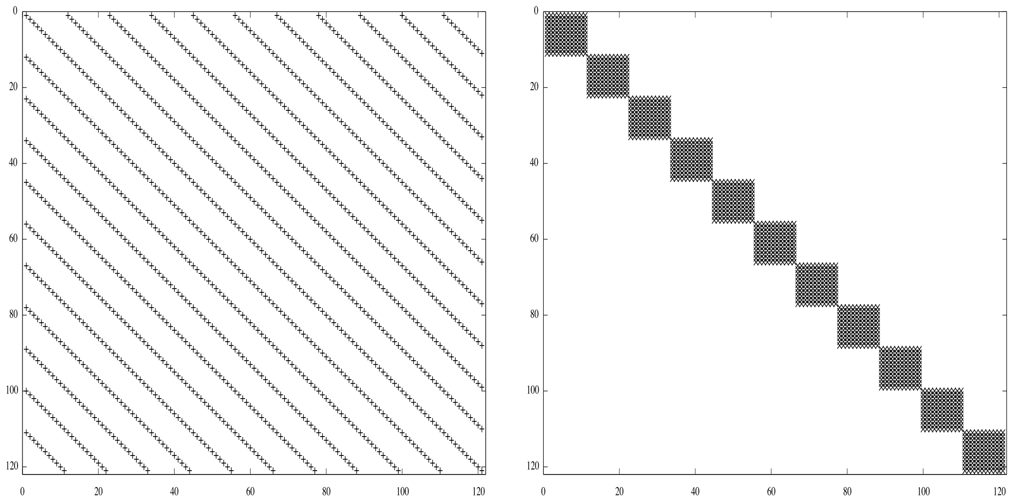
$$\frac{d^p u(\xi)}{d\xi^p} = \sum_{k=0}^{N} a_k \frac{d^p T_k(\xi)}{d\xi^p}. \tag{19}$$

Thus, the values of the derivative $\frac{d^n u(\xi)}{d\xi^n}$, with $n \in \{1, \dots, p\}$ at the CGL points can be computed by $\mathcal{D}^n u$, and the entries for the Chebyshev differentiation matrix are defined as:

$$(\mathcal{D}_N)_{ij} = \begin{cases} \frac{2N^2+1}{6} & \text{if } i,j = 0 \\ -\frac{2N^2+1}{6} & \text{if } i,j = N \\ \frac{-\xi_j}{2(1-\xi_j^2)} & \text{if } j = 1, \dots, N-1 \\ \frac{c_i}{c_j}\frac{(-1)^{i+j}}{(\xi_i - \xi_j)} & \text{if } i \neq j,\ \ i,j = 1, \dots, N-1 \end{cases} \tag{20}$$

The Chebyshev approximations for derivative over 2D grids can be formed using the Kronecker tensor product (Trefethen, 2000). The first order derivative with respect to x and y at the grid points can be computed by $\mathcal{D}_x = \mathcal{D} \otimes \mathcal{I}$ and $\mathcal{D}_y = \mathcal{I} \otimes \mathcal{D}$. Using the same technique, the second order derivative are compute by $\mathcal{D}_{xx} = \mathcal{D}^2 \otimes \mathcal{I}$, $\mathcal{D}_{yy} = \mathcal{I} \otimes \mathcal{D}^2$ and for the cross-derivative $\mathcal{D}_{xy} = \mathcal{D}_x \times \mathcal{D}_y$.

Figures (1) show the structure of the Chebyshev differentiation matrix for the first order derivative with respect to x and y over a 2D grid.



(a) $\mathcal{D}_x$ (b) $\mathcal{D}_y$

Figure 1. Structure of the Chebyshev differentiation matrix using $N_x = N_y = 10$ Chebyshev-Gauss-Lobatto points.

Applying the spectral discretization to the linear operator, $\mathcal{L}$ and $\mathcal{R}$ can be written in a matrix form in which many elements are equal to zero. Figure 2 shows the structure of the elements for a EVP problem discretized in Chebyshev-Gauss-Lobatto points.

3.2 GRID STRETCHING

Simple transformations can be used to cluster grid points in regions of large gradients, where more resolution is required, as the case is in boundary layers. In this work the stretching adopted for the wall-normal y-direction was

$$y = \frac{L(1-\xi)}{1+s+\xi} \tag{21}$$

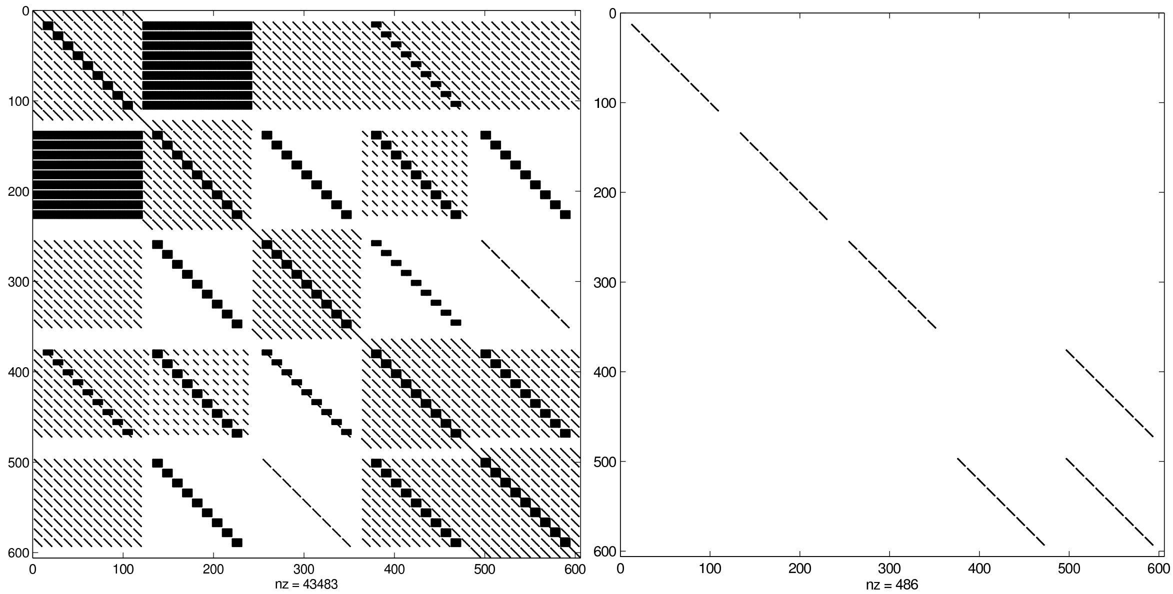


Figure 2. The matrix forms $\mathcal{L}$ and $\mathcal{R}$ of the compressible BiGlobal EVP (12) for $N_x = 10$ and $N_y = 10$.

where the y_{GL} are the Chebyshev Gauss-Lobatto points, the stretching parameters s and L are determined by the domain length in y-direction and control the refinement.

For spatial derivative on y-direction, we first compute the derivative by Chebyshev approximations over 2D grids, and then the transformation is performed to obtain the derivatives in the physical domain.

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial y} \tag{22}$$

where

$$\frac{\partial \xi}{\partial y} = \frac{-2L - 2s}{(L+y)^2}. \tag{23}$$

3.3 Arnoldi Method

Out of several possible approaches for the solution of linear systems, Krylov subspace iteration is used to perform the approximate solution of $Ax = b$. Considering A no-singular matrix, x^* the exact solution and the minimal polynomial associated to A such as $q(A) = 0$ is

$$\alpha_0 I + \alpha_1 A + \ldots + \alpha_m A^m, \tag{24}$$

where $\alpha_0 \neq 0$ (A is no-singular),

$$A^{-1} = -\frac{1}{\alpha_0}\sum_{j=0}^{m-1} \alpha_{j+1} A^j. \tag{25}$$

On the order hand, the exact solution of the linear system is given by:

$$x^* = A^{-1}b \tag{26}$$

$$= \left(-\frac{1}{\alpha_0}\sum_{j=0}^{m-1} \alpha_{j+1} A^j\right) b \tag{27}$$

Thus, writing x^* as a linear combination of $(b, Ab, A^2b, \ldots, A^{m-1}b)$, one can use $K_m(A, r_0)$ as the search space to the approximate solution.

Suppose $x_0 \neq 0$, then

$$x^* = A^{-1}b \tag{28}$$

$$= A_{-1}(Ax_0 + r_0) \tag{29}$$

$$= x_0 + A^{-1}r_0 \tag{30}$$

therefore $x^* \in x_0 + K_m(A, r_0)$.

The Arnoldi algorithm is a method to compute the orthonormal base $\{v_1, v_2, \ldots, v_j\}$ of the Krylov subspace, $K_j(A, v_1)$. We applying the Gram-Schmidt process to orthogonalization. Arnoldi algorithm:

```
Choose a vector v_1 of norm 1
For j = 1, 2, ..., m Do:
    Compute w_ij := A v_j
    For i = 1, ..., j Do:
        h_ij = (w_j, v_i)
        w_j = w_j - h_ij v_i
    EndDo
    h_{j+1,j} = ||w_j||_2
    v_{j+1} = w_j / h_{j+1,j}
EndDo
```

4. NUMERICAL ASPECTS

Discretization of equation (12) by Chebyshev Gauss-Lobatto points in both the x and y directions leads to a generalized matrix eigenvalue problem, suitable for numerical solution. The Arnoldi method was used to compute the eigenvalues and corresponding eigenvectors. Two different algorithms were tested for the associated LU factorization, namely, the LAPACK routine (ZGETRF) which is based on dense serial linear algebra and the MUMPS package, which performs the LU factorization by storing the matrices in sparse format.

The ZGETRF routine computes an LU factorization of a general matrix using pivoting with row interchanges. MUltifrontal Massively Parallel Solver (MUMPS) (Amestoy et al., 2001) is a package for solving linear equation systems of the form $\bar{\mathcal{L}}\hat{\mathbf{q}} = \bar{\omega}\hat{\mathbf{q}}$ that is based on a multifrontal technique which performs a direct factorization $\bar{\mathcal{L}} = LU$, where $\bar{\mathcal{L}} = \mathcal{L} - \sigma\mathcal{R}$ is a complex or real sparse matrix input in assembled format, that can be asymmetric, positive definite symmetric, or general symmetric, where σ is a complex and $\bar{\omega} = \omega - \sigma$. Fill reducing ordering for sparse matrices within MUMPS is provided by the package METIS. The linear system is solved in three steps: analysis, factorization and solution. The original matrix and the factors are mapped during the analysis step that returns estimates of the memory needed. The numerical factorization is then performed on a sequence of dense factorizations. The factor matrices are kept to be used during solution step. The codes were run on a desktop computer featuring a 3.06GHZ Intel Core 2 Duo processor 8Gb 1067 MHZ DDR3 of RAM.

4.1 PERFORMANCE

We compared the CPU time and memory required for each approach for the LU decomposition, namely, LAPACK (dense) and MUMPS (sparse, serial). Also compared was the average CPU time in seconds for each Arnoldi iteration. The results are summarized in Tab. 2 for different resolutions.

Table 2. Comparison of the CPU time for LU factorization using MUMPS (using METIS ordering) and ZGETRF

	LU decomposition		**Memory**		**Arnoldi iteration**	
Resolution	ZGETRF (s)	MUMPS (s)	ZGETRF (Gb)	MUMPS (Gb)	ZGETRF (s)	MUMPS (s)
10×10	0.11	0.088	0.01	0.0058	0.017	0.00408
20×20	4.31	3.12	0.15	0.093	0.13	0.045
30×30	53.71	28.01	0.68	0.41	0.661	0.209
40×40	267.90	146.06	2.10	1.3	1.72	0.66
50×50	823.37	541.19	5.04	3.3	3.82	1.56
60×60	-	1696.32	10.32	6.72	-	3.35

Most of the operations of a LU decomposition are of the type:

$$a(i,j) = a(i,j) - (a(i,k) * a(k,j))/a(k,k), \tag{31}$$

so, if $a(i,k)$, $a(k,j)$ and $a(k,k)$ are nonzero, the update $a(i,j)$ will be nonzero as well even if it were zero in the original matrix. One such coefficient is called a "fill-in coefficient" and the amount of fill-ins depends on many aspects. Nevertheless, it depends mostly on the order in which the pivots are eliminated by ordering method. For this reason, at the end of the analysis phase, MUMPS returns estimates of the memory needed. For example, for a problem of resolution 60×60 the number of nonzero entries is 3.2×10^6, a number well below the estimated number of non-zero entries by MUMPS, which was 2.5×10^7. As seen in the results of Table 2, even though MUMPS performs operations of the kind (31), there exist reductions in both memory and CPU time requirements, as compared to those of the dense LAPACK routines.

5. RESULTS

In this section results of the BiGlobal linear stability analysis in an attachment-line boundary layer flow are shown and briefly discussed. As verification, we compared with the incompressible result for the Görtler-Hämmerlin (GH) mode presented by Lin and Malik (1996) and the results are shown in the Tab. 3. Running the present compressible code at Mach number $Ma = 0.02$ a good agreement was obtained for the leading Görtler-Hämmerlin (GH) eigenvalue.

Table 3. Grid refinement history in the numerical solution for incompressible attachment-line boundary flow at $Re = 800$ and $\beta = 0.255$.

	Görtler-Hämmerlin mode		**A1 mode**	
Resolution	c_r	c_i	c_r	c_i
08×48	0.35844146	0.0058556	0.3573547	0.0043549
16×48	0.35844146	0.0058556	0.3580007	0.0042997
24×48	0.35844146	0.0058556	0.3458880	0.0043912
32×48	0.35844146	0.0058556	0.3580521	0.0041461
48×48	0.35844146	0.0058556	0.3579257	0.0040956
Lin and Malik, (1996)	0.35840982	0.0058325	0.3579170	0.0040988
Theofilis et al., (2003)	0.35840980	0.0058327	0.3579195	0.0040987

Figure (3) presents amplification rate $c_i = \omega_i/\beta$ results obtained by the asymptotic analysis and numerical solution of the BiGlobal eigenvalue problem at the two Mach numbers discussed by Theofilis et al. (2004), superimposed upon presently-obtained results. The main conclusion that can be reached is that the asymptotic analysis of the reference work and the present high-resolution numerical solutions of the eigenvalue problem are in excellent agreement, while the previous numerical solutions were under-resolved. The agreement with the analytical results further underlines the quality of the present implementation of the numerical BiGlobal eigenvalue problem solution.

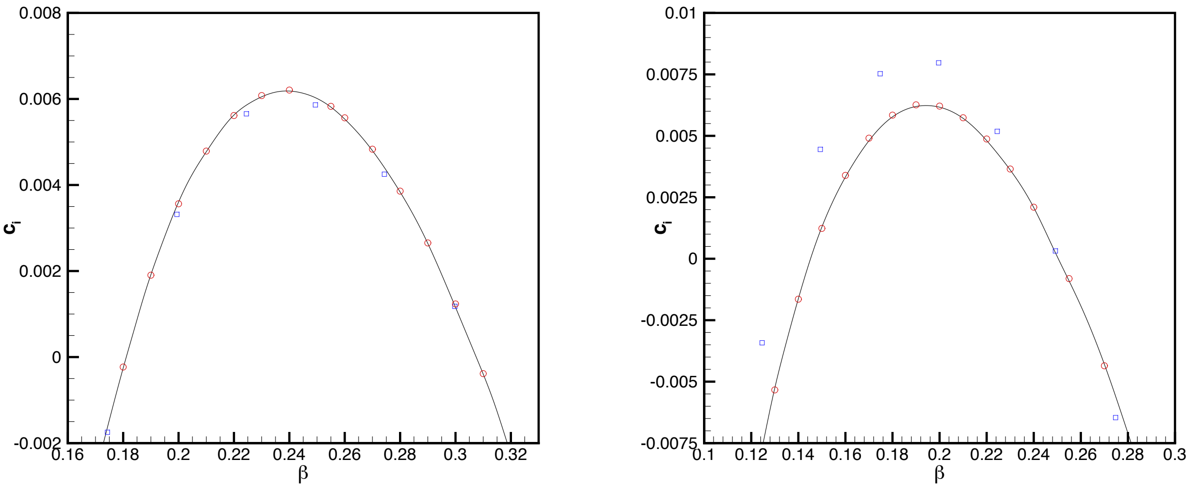
Figures (4) presents temporal amplification rate results obtained numerically using the present code at two Reynolds numbers $Re = 800$ and $Re = 1500$ and for a range of subsonic Mach numbers (note that the Mach number in the present work is defined using the velocity component along the attachment line). As already seen in Theofilis et al. (2004), the amplification rate increases with increasing Reynolds number and decreases as the Mach number increases. Besides that, the range of unstable modes vary significantly with Mach number, instability waves typically having longer wavelengths as Ma increases. Finally, neutral curves summarizing this effect can be seen in Fig. (5) at two Mach numbers. The critical conditions at $Ma = 0.02$ and 0.90 are $(Re_c \approx 583, \beta \approx 0.288)$ and $(Re_c \approx 564.5, \beta \approx 0.216404)$, respectively.

6. FINAL REMARKS

An algorithm for the numerical solution of the compressible BiGlobal linear eigenvalue problem has been developed. It features matrix storage and inversion in sparse serial format and has been shown to provide $O(35\%)$ savings in both CPU time and memory required. Further improvements may be obtained by parallelization, which is presently being considered. The code developed has been utilized to complete the parametric analysis of global instability in compressible swept Hiemenz flow. The efficiency of the the developed algorithm permitted obtaining neutral curves of this flow at two Mach numbers for the first time.

7. Acknowledgments

The authors acknowledged the financial supports from the CNPq, CAPES and MICINN , under Grant number # TRA2009 - 13648, entitled "Metodologías computacionales para la predicción de inestabilidaes globales hidrodiámicas y aeroacústicas de flujos complejos".



(a) $Ma = 0.50$ (b) $Ma = 0.90$

Figure 3. Dependence of c_i on β for mode GH at (a) $Ma = 0.5$ and (b) $Ma = 0.9$, $Re = 800$. Solid line obtained by asymptotic analysis and square symbols by BiGlobal analysis from (Theofilis, Fedorov and Collis, 2004). Circle symbols are results of this present work.

8. REFERENCES

Amestoy, P. R., Duff, I. S., Koster, J. and L'Excellente J.-Y., 2001, "A fully asynchronous multifrontal solver using distributed dynamic scheduling", SIAM Journal of Matrix Analysis and Applications, Vol 23, No 1, pp 15-41.

Cohen, C. B. and Reshotko, E., 1956, " Similar solutions for the compressible laminar boundary layer with heat transfer and pressure gradient", NACA Report 1293.

Görtler, H., 1955, "Dreidimensionale Instabilität der ebenen Staupunktströmung gegenüber wirbelartigen Störungen", *50 Jahre Grenzschichtforschung*, pp. 304-314, Vieweg und Sohn.

Hämmerlin, H., 1955, "Zur Instabilitätstheorie der ebenen Staupunktströmung, *50 Jahre Grenzschichtforschung*, pp. 315-327, Vieweg und Sohn.

Hall, P., Malik, M. R. and Poll, D. I. A., 1984, "On the Stability of an Infinitive Swept Attachment Line Boundary Layer", Proc. R. Soc. Lond. A, Vol. 395, 229-245.

Lin, R.S. and Malik, M.R., 1996, "On the Stability of Attachment-Line Boundary Layers. Part 1. The Incompressible Swept Hiemenz Flow", J. Fluid Mech., Vol. 311, pp. 239-255.

Lin, R.S. and Malik, M.R., 1997, "On the Stability of Attachment-Line Boundary Layers. Part 2. The Effect of Leading-Edge Curvature", J. Fluid Mech., Vol. 333, pp. 125-137.

Saad, Y., 2000, "Iterative Methods for Sparse Linear Systems", SIAM, 3rd.

Theofilis, V. and Colonius, T., 2004, "Three-dimensional Instabilities of Compressible Flow Over Open Cavities: Direct Solution of the BiGlobal Eigenvalue Problem", AIAA Paper 2004-2544.

Theofilis, V., Fedorov, A., Obrist, D. and Dallmann, U. , 2003, "The Extended Görtler-Hämmerlin model for Linear Instability of Three-Dimensional Incompressible Swept Attachment-Line Boundary Flow ", Journal of Fluids Mechanics, Vol.487, pp. 271-313.

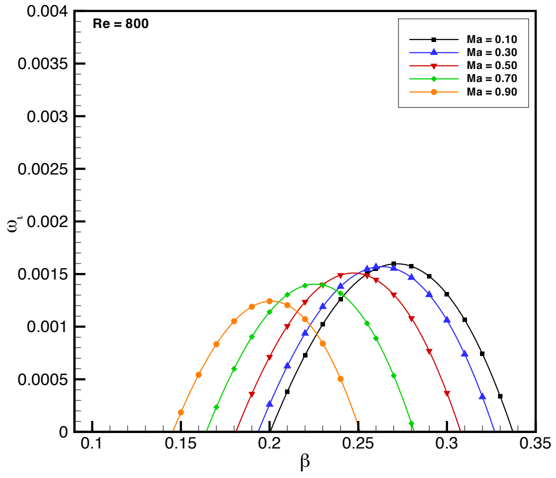
Theofilis, V., Fedorov, A. and Collis, S. S., 2004, " Leading-edge boundary layer flow - Prandtl's vision, current developments and future perspectives", IUTAM Symposium on One Hundred Years of Boundary Layer Research, Vol.129.

Trefthen, L. N., 2000, "Spectral elements in MatLab", SIAM.

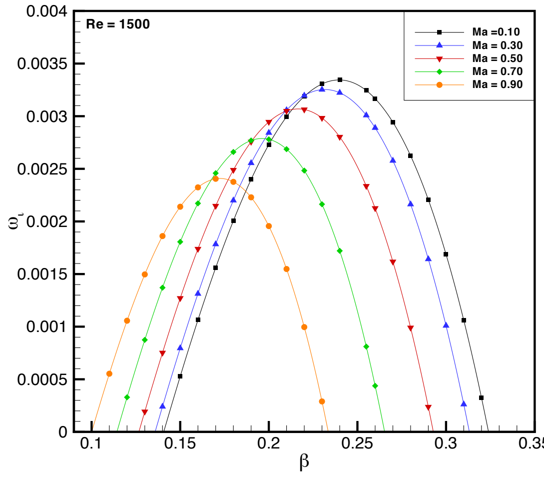
Vicente, J., Rodríguez, D., González, L. and Theofilis, V., 2008, "High-Order Numerical Methods for BiGlobal Flow Instability Analysis and Control", Turbulência, Vol. 6, No. 2, ABCM, 123p.

9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.



(a)



(b)

Figure 4. Temporal amplification rate as a function of the β for (a) $Re = 800$ and (b) $Re = 1500$.

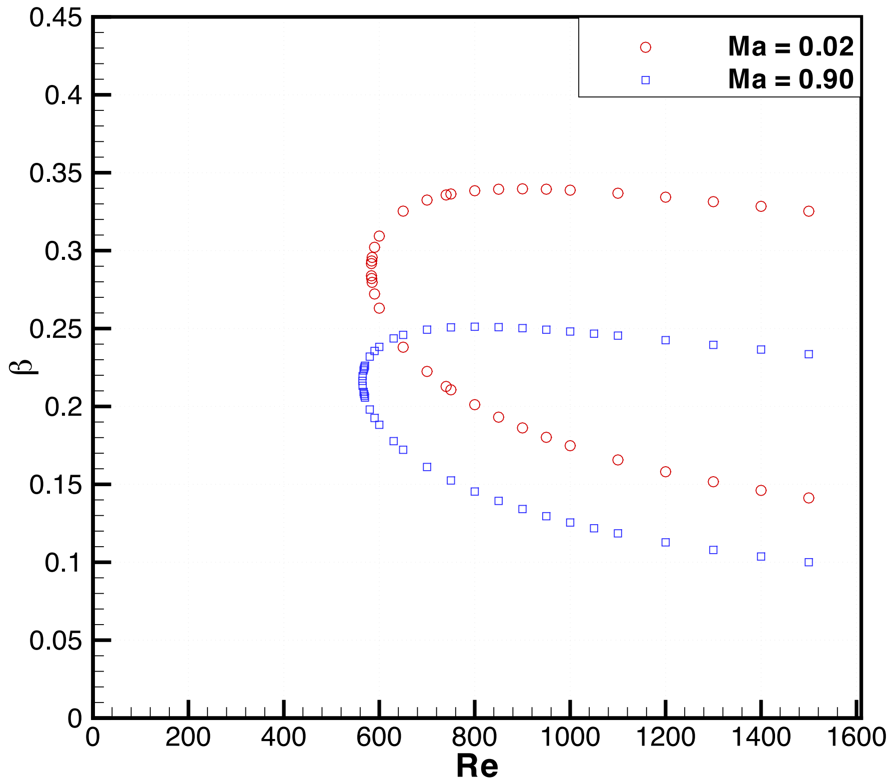


Figure 5. Neutral curves in two Mach numbers.