SOLUTION OF THE RADIATIVE ENCLOSURE WITH A HYBRID INVERSE METHOD

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Abstract. *This work applies the inverse analysis to solve a three-dimensional radiative enclosure – which the surfaces are diffuse-grays – filled with transparent medium. The aim is determine the powers and locations of the heaters to attain both uniform heat flux and temperature on the design surface. A hybrid solution that couples two methods, the generalized extremal optimization (GEO) and the truncated singular value decomposition (TSVD) is proposed. The determination of the heat sources distribuition is treated as an optimization problem, by GEO algorithm , whereas the solution of the system of equation, that embodies the Fredholm equation of first kind and therefore is expected to be ill-conditioned, is buid up throught TSVD regularization method. The results show that the hybrid method can lead to a heat flux on the design surface that satisfaies the imposed conditions with maximum error of less than 1,10%. The results illustrated the relevance of a hybrid method as a prediction tool.*

Keywords: *inverse analysis, GEO algorithm, TSVD regularization*

1. INTRODUCTION

Heat treatment is widely applied in the industry to restore or change materials properties. Such modifications results from procedures that occur at elevated temperatures. For instance, it can be mentioned the process named as annealing treatment, which request uniform temperature to avoid internal stress that may lead to warping or even cracking. To attain the suitable conditions controlled heat flux must be supplied on the material surface, thus both the temperature and the heat flux are prescribed.

In general, the radiant enclosure problem have been tackle by optimization or inverse methodologies; the optimization methods solves the problem implicitly by transforming it into a multivariable minimization problem, while inverse techniques solves the problem explicitly using regularization techniques (Daun and Howell, 2005).

Micro-genetic algorithm was applied in Safavinejad et al. (2008) to find the heaters setting that produce a desired heat flux and temperature distribution on the design surface. The same optimization algorithm was used in Safavinejad et al. (2009) to determine the optimal number and location of equally powered heaters.

Several works about inverse design are related to find powers of the heat sources in the radiant enclosure. Such problems explore various regularization methods. Bayat et al. (2010) applied the conjugated gradient method, Hoffmann et al. (2010) used truncated singular value decomposition, Rukolaine (2007) applied Tikhonov regularization. All the previous papers considered radiant enclosure with transparent media, but how showed in Pourshaghaghy et al. (2006), it is possible use regularization techniques when the media is participating.

When the problem of heaters distribution involves combined heat transfer, also is possible apply regularization. Mossi et al. (2008) used truncated singular value decomposition and Sarvari (2005) conjugated gradient method.

Some works was developed to solve inverse design applied to an illumination design problem in enclosure. Starting of fixed positions for the light sources, Schneider and França (2004) applied truncated singular value decomposition regularization. Cassol et al. (2008) solved a similar problem, however left unconstrained positions and luminous power of the light sources, and used generelized extremal optimization algorithm.

This paper considers the inverse analysis in a three-dimensional rectangular enclosure with diffuse-gray surfaces. The spatial distribution and powers of the heat sources are left unconstrained and two conditions are imposed on the design surface. The solution to this type of problem by conventional techniques is possible only with trial-and-error procedure. This work links two methods: GEO algorithm for optimization problem and TSVD to tackle the system of equations ill-conditioned. For the optimization problem, the adopted objective function is based on the minimization of the least-square of the deviations between the imposed and the actual heat flux on the design surface.

2. PHYSICAL AND MATHEMATICAL MODELING

2.1. Radiation exchange in enclosure

Siegel and Howell (2002) present several techniques to solve radiative transfer. The solution method in terms of the outgoing radiative flux – the sum of emitted and reflected radiation – is too convenient for radiation exchanges in an enclosure. To assure uniform boundary conditions each surface is subdivided into portions. It ought to write one equation for each surface element according to the boundary condition known – temperature Eq. (1) or net radiative heat flux Eq. (2).

$$q_{o,j} = \varepsilon_j e_{b,j} + (1-\varepsilon_j)\sum_{k=1}^{N} F_{j-k} q_{o,k} \tag{1}$$

$$q_{o,j} = q_{r,j} + \sum_{k=1}^{N} F_{j-k} q_{o,k} \tag{2}$$

where $q_{o,j}$ is the outgoing radiative flux or the radiosity. The term $e_{b,j} = \sigma T_j^4$ is the black body emissive power (σ is the Stefan-Boltzmann constant). The view factor between two elements F_{j-k} is the fraction of energy that become from an element and is interpreted by other element. If the surfaces are diffuse-gray ε_j is the hemispherical total emissivity, that for convenience will called emissivity. Finally, $q_{r,j}$ is the net radiative heat flux.

2.2. Problem definition

The Figure 1 shows a rectangular enclosure with diffuse-gray surfaces. The bottom surface – where the heat flux and temperature are prescribed – is termed design surface. The heaters are located in the top wall and the remainders are insulated. The length, width and height are designated by *L, W* and *H*, respectively.

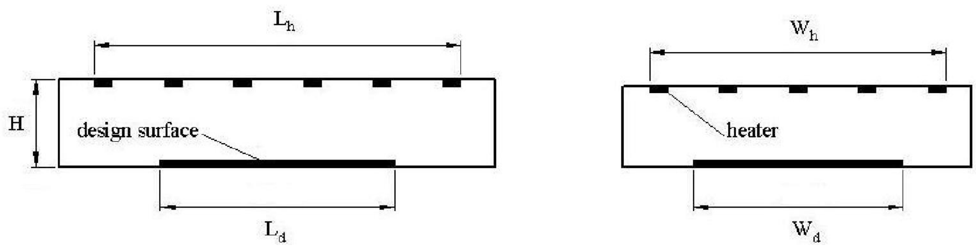


Figure 1. Views of the enclosure

In the Figure 2, the three dimensional enclosure was divided into finite-sized square elements $\Delta x = \Delta y = \Delta z$. But only one-quarter of the domain was presented.

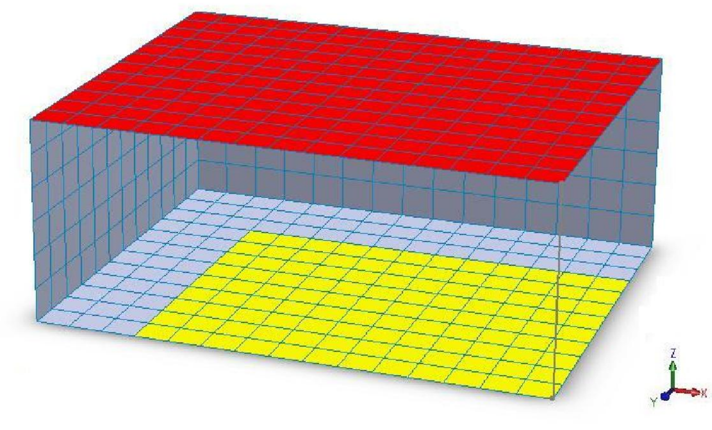


Figure 2. Discretization of the computational domain

In this analysis, it is prescribed on the design surface uniform heat flux, $q_{prescribed}$(W/m²), and uniform temperature, $T_{prescribed}$ (K). The task consists of finding the power and position of each heater on the top surface to attain the both conditions on the design surface. Thus, there is one surface where the elements have two boundary conditions. Whether a conventional technique is single out, the trial-and-error procedure must be applied and the forward solution will be run oftentimes without guarantee to find out a good answer.

The rationale aforementioned bolsters the use of an inverse method. Nevertheless, it is necessary cope with special solutions techniques. The Equation (1) and Equation (2) are the background to the solution and, for generality, it will rewrite in dimensionless form:

$$Q_{o,j} = \varepsilon_j t_j^4 + (1-\varepsilon_j)\sum_{k=1}^{N} F_{j-k} Q_{o,k} \tag{3}$$

$$Q_{o,j} = Q_{r,j} + \sum_{k=1}^{N} F_{j-k} Q_{o,k} \tag{4}$$

where the dimensionless heat flux and temperature are given by $Q = q/\sigma T_{prescribed}^4$ and $t = T/T_{prescribed}$, respectively. The temperature, termed prescribed, is the temperature on the design surface.

The combination of Equations (3) and (4) yields a relation to the radiosity on each element *jd* on the design surface that relates the emissive power and the net radiative flux.

$$Q_{o,jd} = t_{jd}^4 + \frac{(1-\varepsilon_{jd})}{\varepsilon_{jd}} Q_{r,jd} \tag{5}$$

Next, Equation (4) is applied to each design surface element *jd* and then rearranged to provide a system of equations to the dimensionless radiosity on the heat source elements *jh*.

$$\sum_{jh} F_{jd-jh} Q_{o,jh} = (Q_{o,jd} - Q_{r,jd}) - \sum_{jw} F_{jd-jw} Q_{o,jw} \tag{6}$$

2.3. The generalized extremal optimization algorithm

In this work the search to the positions of each heater is treated as an optimization problem that minimize an objective function, Eq. (7), which is based on the minimization of the least-square deviation between the specified heat flux on design surface, $Q_{r,prescribed}$, and one obtained from a given configuration of the heat sources, $Q_{r,jd}$. To handle with this task it was choose a stochastic algorithm: the generelized extremal optimization. To the aim in this work it will presented only more pertinent aspects about GEO, however it is possible find a good explanation about one in De Souza et al. (2003).

$$F_{lsq} = \sqrt{\sum_{jd} (Q_{r,prescribed} - Q_{r,jd})^2} \tag{7}$$

In the GEO method the *N* variables of the optimization problem are encoded by only one binary string and the bits are randomly aligned forming an initial bit configuration like in the Fig. 3. A preliminary objective function is calculated.

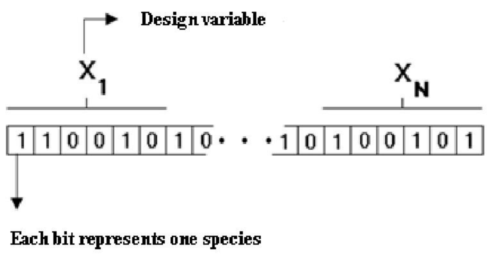


Figure 3. Representation of the binary string. In this case each variable is represented by six bits

The Equation (8) defines the number of bits *m* to attain a desired precision *q* for each design variable.

$$2^m \geq \frac{x_i^u - x_i^l}{q} + 1 \tag{8}$$

where x_i^u and x_i^l are the upper and lower bounds, respectively, of the variable *i*, with $i = 1, N$.

The physical value of each design variable, in this case the position of the heaters, in a configuration is given by Eq. (9), where I_i is an integer number obtained in the transformation of the variable i from its binary form to a decimal representation.

$$x_i = x_i^l + \left(x_i^u - x_i^l\right)\frac{I_i}{2^m - 1} \tag{9}$$

A bit is flipped (0 to 1 or 1 to 0), therefore a new configuration is created and the positions of the heaters are updated. The objective function value is calculated and compared to the old one. When all the bits are mutated every one of them is ranked according to the change in the objective function. The lowest rank corresponds to the least change in the objective function and the highest one is attached to the best change. The bits are returned to its original value and it is calculated the probability of a bit mutates, by the Eq. (10). The aforementioned process is repeated until a given stopping criterion is reach. Then, the best value for the objective function is found and the corresponding configuration is returned.

$$P(k) \propto k^{-\tau} \tag{10}$$

where τ is an adjustable parameter and *k* rely on the rank of the bit.

2.4. The truncated singular value decomposition

The numerical discretization of the domain and application of Eq. (6) lead to a system of equations that can be represented by:

$$\mathbf{Ax} = \mathbf{b} \tag{11}$$

where the matrix **A** is formed by the view factors between the design surface elements and the heat sources elements F_{jd-jh} . The solution vector **x** represents the unknown radiosity on each heat source. The vector **b** contain the terms of right side of Eq. (6). The system of linear equations involves the Fredholm integral equation of the first kind that is ill-conditioned.

How discussed in França et al. (2002), the solution of the equations system by conventional methods, such as Gaussian elimination or LU decomposition will return an unrealistic answer or no physical solution. To tackle with this, it is necessary the application of some regularization method. In the singular value decomposition the matrix **A** is decomposed in three other one.

$$\mathbf{A} = \mathbf{UWV}^{\mathbf{T}} \tag{12}$$

The matrixes **U** and **V** are orthogonal and the **W** is a diagonal matrix formed by the singular values σ_i . With this, the solution vector will be:

$$\mathbf{x} = \sum_{j=1}^{N}\left(\frac{b_k u_{kj}^T}{\sigma_j}\right)v_j \tag{13}$$

If the equation system is singular, some singular values of matrix **A** will be null. The TSVD implementation eliminate the terms related to the null singular values of the linear combination. Thus, the solution vector will be:

$$\mathbf{x} = \sum_{j=1}^{p}\left(\frac{b_k u_{kj}^T}{\sigma_j}\right)v_j \tag{14}$$

where $\sigma_{p+1},...,\sigma_n$ are nulls and p is a regularization parameter. The matrix **A** was replaced for $\mathbf{A}_p$ and is introduced a residual error in the solution $r = |\mathbf{A}_p\mathbf{x} - \mathbf{b}|$.

More information about TSVD method and other regularization techniques are available in Hansen (1998).

3. SOLUTION PROCEDURE

The first task in the solution procedure is the view factor calculation. Next following steps are applied:

1) Run the GEO algorithm to generate a preliminary spatial distribution for the heaters.
2) The TSVD method writes the matrix **A** and applies singular value decomposition.
3) Solve the radiosities according to Eq. (3), Eq. (4) and Eq. (5).
4) Write vector **b** – right side of Eq. (6).
5) Solve vector **x** with TSVD regularization.
6) Repeat steps (4) to (6) to minimize the norm $|\mathbf{A}_p\mathbf{x} - \mathbf{b}|$.
7) Run the forward problem and use the objective function value – Eq. (7) – as input to GEO algorithm.
8) Return to GEO algorithm and generate a new spatial distribution to the heat sources.
9) Use result of previous step as input to TSVD in step (2).
10) Repeat steps (2) to (9) until find the stop criterion into GEO.

3.1. Verification of the solution

It was mentioned in the TSVD section that only p terms are kept in the solution, as consequence an error is introduced in the solution. The error is defined as a relation between the specified heat flux and the calculated heat flux on each element, jd, on the design surface.

$$\gamma_{jd} = \left| \frac{Q_{r,prescribed} - Q_{r,jd}}{Q_{r,prescribed}} \right| \tag{15}$$

4. RESULTS AND DISCUSSION

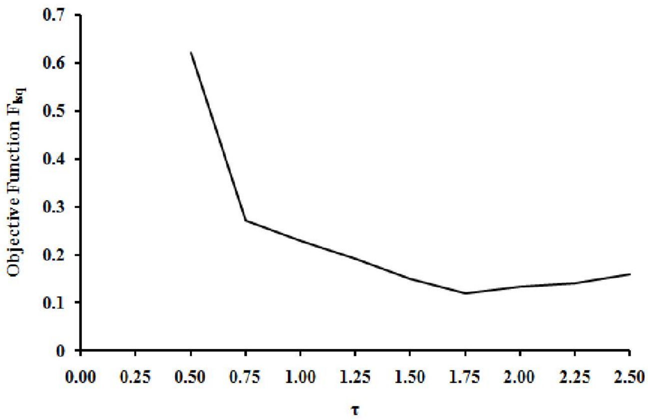
The enclosure in this problem is the same studied in Schneider and França (2004) and Cassol et al (2008). The aspect ratio of the enclosure base is $W/L = 0{,}8$; the dimensionless height is $H/L = 0{,}2$. The design surface must not to cover the entire extension of the base, since the portions close to the corners would be mainly affected by the reflections from the side walls, not from the heat flux from the heat source elements on the top surface. Therefore, the design surface dimensions are taken as $W_d/L = 0{,}6$ and $L_d/L = 0{,}8$. The emissivities of the design surface, of the heat sources and of the walls are $\varepsilon_d = 0{,}9$, $\varepsilon_h = 0{,}9$ and $\varepsilon_w = 0{,}5$, respectively.

The boundary conditions on the design surface elements are the net heat flux $q_{prescribed} = -3{,}22 \times 10^3$ W/m² and temperature $T_{prescribed} = 673$ K. In the dimensionless form: $Q_{r,jd} = -0{,}277$ and $t_{jd} = 1{,}00$. Thus, two boundary conditions are applied on the design surface. The remains elements are kept insulated. Traditionally this type of problem is solved by trial-and-error procedure.

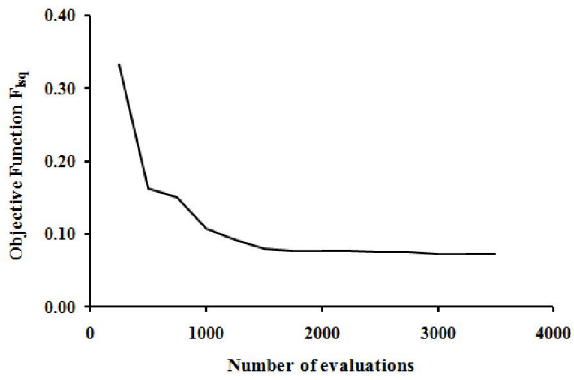
Due geometry symmetry only one quarter of the enclosure was solved and the adopted number of division in the x, y and z directions are 15, 12 e 6, respectively. The heating is provided by a set of ten heaters for each quarter of the domain. This leads to a total of jh = 10 elements on the heater surface while the number of elements on the design surface is jd = 108. Thus, the system of equations formed by Eq. (6) will be composed by M = 108 equations and N = 10 unknowns. Therefore, the resulting system of equations is over-constrained. Furthermore, the system of equations is ill-conditioned because embodies the Fredholm equation of first kind. The TSVD regularization is suitable to tackle with all this problems.

The GEO algorithm scatters the heaters on the top surface to attain uniform heat flux on the design surface. This method, coupled with TSVD regularization avoid trial-and-error procedure, because GEO finds a heat sources distribution and TSVD gives the best power for each configuration of the heaters.

The Figure 4(a) shows the search for the best adjustable parameter (τ) for this problem. The algorithm was run 10 times for each τ and stopped after 2000 evaluations of the objective function. It was selected the value $\tau = 1.75$ because it showed the lowest values for the error function F_{lsq}. The Figure 4(b) shows that after 3000 evaluations, the objective function value no change meaningful.



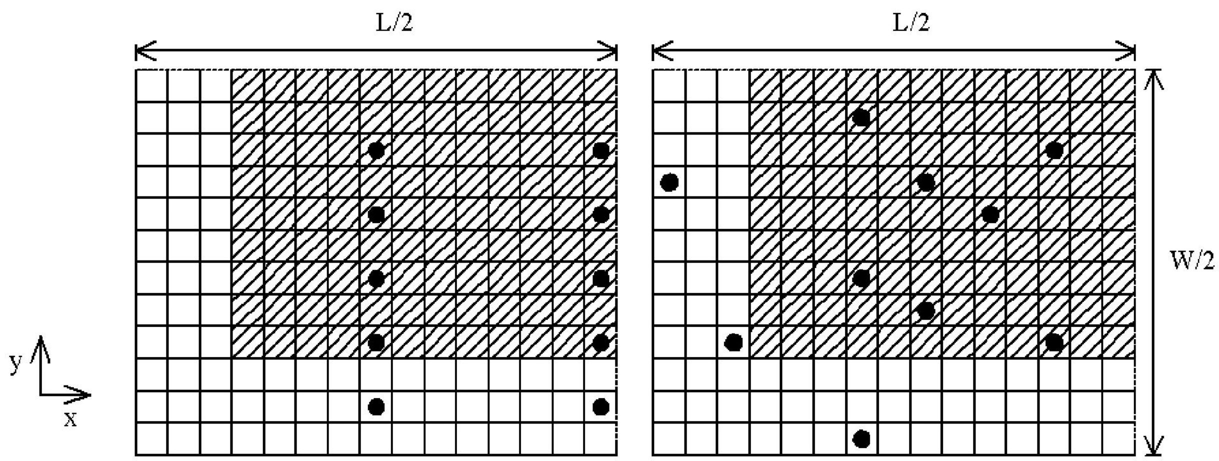
(a)



(b)

Figure 4. (a) Average of best results for 10 independent runs of GEO for different τ. Each run stopped after 2000 evaluations of the objective function F_{lsq}. (b) Lowest values of F_{lsq} for different numbers of evaluations with $\tau = 1{,}75$.

The Figure 5 shows the spatial distribution of the heaters (circular dots) in the top surface. The shaded area represents the design surface (bottom surface) and the dashed lines indicate symmetry. In the Figure 5 (a) the distribution of the heater sources were obtained manually and the powers were obtained only by TSVD regularization. The scatter shape was obtained by hybrid method.



(a) (b)

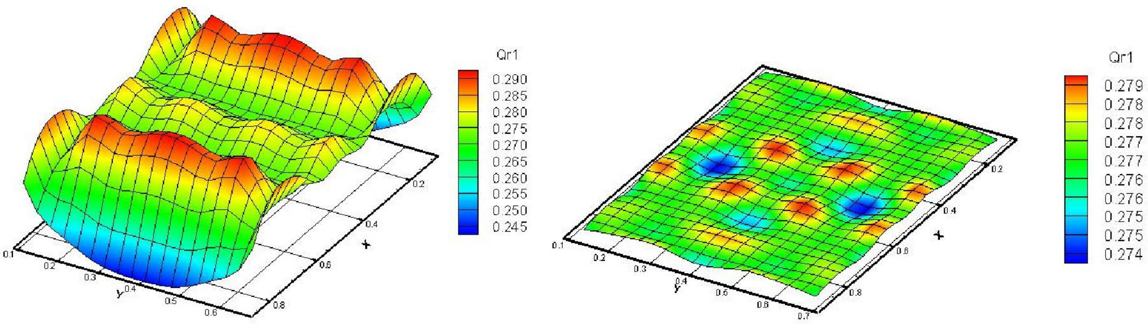
Figure 5. Location of the design surface (shaded area) and heat source elements (circular dots) in one quarter of bottom and top surfaces. Dashed lines indicate symmetry. (a) Heaters manually distributed (b) Heaters placed by hybrid method

Table 1 presents the distribution of the heaters and the required dimensionless net heat flux on each heater – obtained by hybrid method – to find the uniform heat flux on the design surface. The Figure 6 shows the dimensionless net heat flux on the design surface. The hybrid solution was capable of satisfy the heat flux on the design surface with a maximum and average errors of less than 1,10% and 0,24 %, respectively, which would be very difficult to obtain using a trial-and-error approach. When the heaters are placed manually, the maximum and average errors are 11,87% and 2,71%, respectively. This indicates the usefulness of the hybrid method as a designing tool.

Table 1. Required dimensionless net heat flux on the heat source elements (hybrid method)

Heat source element (*jh*)	Element in the x direction	Element in the y direction	$Q_{r,jh}$
1	9	5	4.6400
2	13	4	2.3446
3	1	9	2.8324
4	7	1	2.1297
5	9	9	3.1540
6	7	11	1.3587
7	7	6	1.8179
8	11	8	0.7523
9	13	10	5.3858
10	3	4	5.4839

Observing Figure 6, indeed, the hybrid method gives better solution than manual placement of the heaters. When the results were compered with the target value ($Q_{r,jd} = -0{,}277$), it is possible to verify that hybrid solution provide uniform heat flux on design surface with the smallest error. Furthermore, in this case, the hybrid method gives a non-regularized solution.



(a) (b)

Figure 6. Dimensionless net heat flux on design surface (target value – 0,277). (a) Manual placement (*p* = 9). (b) Hybrid solution (*p* = 10).

5. CONCLUSIONS

This paper presents a hybrid method that couples the generalized extremal algorithm (GEO) with the truncated singular value decomposition (TSVD) to solve the radiant enclosure. The spatial distribution of heaters was encountered by GEO algorithm and the power required on each heater to attain the desired conditions on the design surface was found by TSVD regularization. Despite the simplifications employed in this work, the results illustrate the relevance of a hybrid method as a prediction tool. The hybrid solution was capable of satisfy the heat flux on the design surface with a maximum and average errors of less than 1,10% and 0,24 %, respectively.

Other advantage of the hybrid method is that one provides solutions with physical meaning (radiosity to be positive) and according to practical requirements (net heat flux positive on the heat sources). The adopted methodology is suitable to tackle with the mean difficulties intrinsic to this type of problem: system of equations over specified and ill-conditioned.

In possible next steps, the proposed methodology can be applied to include surfaces that present both specular and diffuse reflections characteristics, to take in account the direction and/or wavelength dependency of surface emissivities. Moreover, it may be implemented to enclosures filled with an emitting, absorbing and scattering media.

6. REFERENCES

Bayat, N., Mehraban, S., Sarvari, S.M.H., 2010, “Inverse boundary design of a radiant furnace with diffuse-spectral design surface”, International Communications in Heat and Mass Transfer, Vol.37, pp. 103-110.

Cassol, F., Schneider, P.S., França, F.H.R., Sousa, F.L., Neto, A.J.S., 2008, “Inverse analysis applied to illumination design: determination of optimum locations of the light sources”, Proceedings of 12th Brazilian Congress of Thermal Sciences and Engineering, Belo Horizonte, Brazil.

De Sousa, F.L., Ramos, F.M., Paglione, P., Girardi, R.M., 2003, “New Stochastic Algorithm for Design Optimization”, AIAA Journal, Vol.41, No. 9, pp. 1808-1818.

Daun, K.J., Howell, J.R., 2005, “Inverse design methods for radiative transfer systems”, Journal of Quantitative Spectroscopy & Radiative Transfer, Vol.93, p. 43-60.

França, F.H.R., Howell, J.R., Ezekoye, O.A., Morales, J.C., 2002, “Inverse Design of Thermal Systems with Dominant Radiative Transfer”, Advances in Heat Transfer, Vol.36, pp. 1-30.

Hansen, P.C., 1998, Rank-Defficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion. SIAM Monographs on Mathematical Modeling and Computation.

Hoffmann, R.S., Seewald, A., Schneider, P.S., França, F.H.R., 2010, “Inverse design of thermal systems with spectrally dependent emissivities”, International Journal of Heat and Mass Transfer, Vol.53, pp. 931-939.

Mossi, A.C., Vielmo, H.A., França, F.H.R., Howell, J.R., 2008, “Inverse design involving combined radiative and turbulent convective heat transfer”, International Journal of Heat and Mass Transfer, Vol.51, pp. 3217-3226.

Pourshaghaghy, A., Pooladvand, K., Kowsary, F., Karimi-Zand, K., 2006, “An inverse radiation boundary design problem for an enclosure filled with an emitting, absorbing, and scattering media”, International Communications in Heat and Mass Transfer, Vol.33, pp. 381-390.

Rukolaine, S.A., 2007, “Regularization of inverse boundary design radiative heat transfer problems”, Journal of Quantitative Spectroscopy & Radiative Transfer, pp. 171-195.

Safavinejad, A., Maruyama, S., Mansouri, S.H., Sakurai, A., 2008, “Optimal Boundary Design of Radiant Enclosures Using Micro-Genetic Algorithm”, Journal of Thermal Science and Technology, Vol.3, No. 2, pp. 179-194.

Safavinejad, A., Maruyama, S., Mansouri, S.H., Sakurai, A., 2009, “Optimal number and locations of heaters in 2-D radiant enclosures composed of specular and diffuse surfaces using micro-genetic algorithm”, Applied Thermal Engineering, Vol.29, pp. 1075-1085.

Sarvari, S.M.H., 2005, “Inverse determination of heat source distribution in conductive-radiative media with irregular geometry”, Journal of Quantitative Spectroscopy & Radiative Transfer, Vol.93, pp. 383-395.

Siegel, R., Howell, J.R., 2002, “Thermal Radiation Heat Transfer”. Ed. Taylor & Francis, New York, 867 p.

Schneider, P.S., França, F.H.R, 2004, “Inverse Analysis applied to an illumination design”, Proceedings of 10th Brazilian Congress of Thermal Sciences and Engineering, Rio de Janeiro, Brazil.

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