# THE POSITIVE DISPLACEMENT METHOD FOR CALIBRATION OF GAS FLOW METERS. THE INFLUENCE OF GAS COMPRESSIBILITY 

Carlos Pinho, ctp@fe.up.pt<br>CEFT-DEMEC, Faculdade de Engenharia, Universidade do Porto, Rua Dr. Roberto Frias, s/n, 4200-465, Porto, Portugal.

Abstract. An easy technique to calibrate small gas flow meters is the positive displacement method, whose application requires a simple setup and laboratory procedure. With this methodology there is an unknown gas flow coming from a given gas source that must be known by the action of a gas flow meter. The gas flow to be measured is sent to a reservoir with rigid walls and full of water. As gas enters the reservoir water flows out and the amount of water exiting the reservoir in a given time interval can be connected with the average gas flow in that same time interval. In simple terms the volume flow rate of water leaving the reservoir is equal to the gas volume flow rate entering it. The water being incompressible, the density variation is meaningless, however the same can not be assumed for the gas.
Considerations on the simple techniques to be used to minimize the importance of gas compressibility are presented in the paper.

Keywords: positive displacement, flow meters, calibration, compressibility effects

## 1. INTRODUCTION

A simple technique to calibrate small gas flow meters is the positive displacement method, whose application requires the setup shown in Fig. 1. With this methodology there is an unknown gas flow coming from a given source G, gas flow that must be known by the action of a gas flow meter MC. The flow-meter can be a rotameter, an orifice plate, a venturi or another flow measuring device. The pressure on the gas flow coming from the source G has to be adjusted to a previously defined value, the working pressure of the flow meter. This pressure must be height enough to assure that pressure drops downstream the flow-meter are unimportant for the measurement uncertainty, or in another words the gas absolute pressure while flowing through the gas meter must be well above the pressure drops to be expected. This pressure is regulated by means of the pressure regulator R and measured by pressure gauge M . After the flowmeter there is the gas flow control valve and beyond it there is the normal gas consumer installation or alternatively the calibration set-up. As the gas flow passing the flow-meter increases with the opening of the controlling valve V1 the absolute pressure of the gas reduces, unless the pressure reducer acts to compensate its decrease. In the case of semiautomatic gas pressure reducers they are manually adjusted to account for pressure variations with changes in the gas flow.

Measured gas flow can be sent either through valve V4, in the case of normal operating conditions, or through valve V2 under calibration conditions. In the present situations it is the calibration procedure that is under analysis and then the gas flow is sent to a reservoir with rigid walls D , inside which there is water. As gas enters the reservoir water is pushed out of it through pipe T provide valve V3, the water feeding valve, is closed. The amount of water exiting the reservoir can be easily weighted in a given time interval thus allowing the calculation of an average water mass and volume flow rate.

## 2. THE IMPORTANCE OF INITIAL THE GAS VOLUME

In simple terms the volume flow rate of water leaving the reservoir is equal to the gas volume flow rate entering it. The water being almost incompressible, the density variation is meaningless, however the same can not be assumed for the gas. The gas pressure inside the reservoir increases as the water level lowers and the liquid must be raised to a larger relative height before leaving the reservoir through the siphon shaped pipe T. To take into account gas compressibility effects its pressure is continuously measured through the manometer MU. Then, knowing the atmospheric pressure and the gas temperature, quite often assumed equal to ambient temperature, the thermodynamic state of the gas inside the reservoir is known and thus the gas density may be determined.

Considering ideal gas behaviour but being careful enough to consider simple compressibility effects through the compressibility factor (Sonntag et al., 1998 ), a simple equation to determine the gas mass flow rate dependent on the knowledge of the mass of water collected in a given time period can be deduced.

As said before the water volume flow rate can be easily calculated,

$$
\begin{equation*}
\dot{V}_{w}=\frac{m_{w}}{\Delta t \rho_{w}} \tag{1}
\end{equation*}
$$



Figure 1. Schematic layout of the experimental setup for calibration purposes
and for ideal gas behaviour, while considering at the same time the deviation of the actual gas from the ideal behaviour, through the compressibility factor,

$$
\begin{equation*}
p V=Z m R T \Leftrightarrow p \dot{V}=Z \dot{m} R T \tag{2}
\end{equation*}
$$

or,

$$
\begin{equation*}
\dot{V}_{g}=Z \dot{m}_{g} \frac{\bar{R}}{M_{g}} \frac{T_{g}}{p_{g}} \tag{3}
\end{equation*}
$$

equalizing equations (1) and (3) and considering that $T_{g}=T_{a}$ and that $p_{g}=p_{a}+\Delta p$

$$
\begin{equation*}
\dot{m}_{g}=\frac{m_{w}}{\rho_{w} \Delta t} \frac{1}{Z} \frac{M_{g}}{\bar{R}} \frac{\left(p_{a}+\Delta p\right)}{T_{a}} \tag{4}
\end{equation*}
$$

It is however important to analyse how rigorous is the assumption of equality of volume flows between the gas entering the reservoir and the liquid leaving it.

The gas filling and liquid leaving the reservoir is to be analysed in small steps, it is assumed that the gas is not soluble in the liquid and that the process follows a uniform regime procedure. So the gas mass variation inside the reservoir, i.e. , the control volume under analysis, is dependent upon the inlet gas flow rate,

$$
\begin{equation*}
d m_{g c v}=\dot{m}_{g} d t \tag{5}
\end{equation*}
$$

or,

$$
\begin{equation*}
d\left(V_{g} \rho_{g}\right)=\dot{m}_{g} d t \tag{6}
\end{equation*}
$$

whereas the variation of liquid mass contained in the same control volume is dependent upon the outlet mass flow rate,

$$
\begin{equation*}
d m_{w c v}=-\dot{m}_{w} d t \tag{7}
\end{equation*}
$$

As the liquid is incompressible,

$$
\begin{equation*}
d V_{w}=-\dot{V}_{w} d t \tag{8}
\end{equation*}
$$

If the reservoir has rigid walls

$$
\begin{equation*}
V_{w}+V_{g}=\text { constant } \Rightarrow d V_{w}+d V_{g}=0 \tag{9}
\end{equation*}
$$

and introducing Eq. (9) into Eq. (8),

$$
\begin{equation*}
d t=\frac{d V_{g}}{\dot{V}_{w}} \tag{10}
\end{equation*}
$$

Calculating the derivative of Eq. (6) and combining the result with Eq. (10),

$$
\begin{equation*}
\rho_{g} d V_{g}+V_{g} d \rho_{g}=\rho_{g} \dot{V}_{g} \frac{d V_{g}}{\dot{V}_{w}} \tag{11}
\end{equation*}
$$

and reworking this last equation,

$$
\begin{equation*}
\frac{d \rho_{g}}{\rho_{g}}=\left(\frac{\dot{V}_{g}}{\dot{V}_{w}}-1\right) \frac{d V_{g}}{V_{g}} \tag{12}
\end{equation*}
$$

which can be integrated between the following limits,

$$
\begin{array}{ll}
V_{g}=V_{g 1} & \rho_{g}=\rho_{g 1}  \tag{13}\\
V_{g}=V_{g 2} & \rho_{g}=\rho_{g 2}
\end{array}
$$

and consequently, if the gas evolution is isothermal, $p_{g 2} / p_{g 1}=\rho_{g 2} / \rho_{g 1}$,

$$
\begin{equation*}
\dot{V}_{g}=\dot{V}_{w}\left[1+\frac{\ln \left(p_{g_{2}} / p_{g 1}\right)}{\ln \left(V_{g 2} / V_{g 1}\right)}\right] \tag{14}
\end{equation*}
$$

It can be immediately deduced that if $V_{g 1} \rightarrow 0$, i.e., at the start of the calibration procedure the reservoir is full of water, $\dot{V}_{g} \rightarrow \dot{V}_{w}$. Then to minimize the calibration or measurement uncertainty of the gas volume flow entering the reservoir there are two possibilities, one is to start the calibration procedure with the reservoir full of water so that Eq. (14) can be safely used and the other is to know what is the initial value of the gas, i.e., the value of $V_{g 1} \neq 0$ for each calibration step. Of course the simplest experimental procedure will be fill up the reservoir of water at the beginning of each calibration step.

Figure 2 shows what happens when there are no precautions to minimize the initial gas volume inside the water reservoir during the calibration procedure.


Figure 2. Calibration plot for an orifice plate flow meter. Gas being measured, commercial propane. Grey symbols $V_{g 1} \neq 0$. Black symbols, $V_{g 1} \rightarrow 0$.

## 3. THE IMPORTANCE OF THE GAS COMPRESSIBILITY FACTOR

Another aspect to be accounted for is the need to consider the compressibility factor of the gas flowing in the flow meter undergoing the calibration procedure. Going back to Eq. (4) it is necessary to calculate the error obtained when for the sake of simplicity it is assumed, for the gas under consideration, a value of $Z=1$ instead of the correct value.

So there is the correct gas mass flow rate,

$$
\begin{equation*}
\dot{m}_{g}=\frac{m_{w}}{\rho_{w} \Delta t} \frac{1}{Z} \frac{M_{g}}{\bar{R}} \frac{\left(p_{a}+\Delta p\right)}{T_{a}} \tag{15}
\end{equation*}
$$

and the approximated gas flow,

$$
\begin{equation*}
\dot{m}_{g a p}=\frac{m_{w}}{\rho_{w} \Delta t} \frac{M_{g}}{\bar{R}} \frac{\left(p_{a}+\Delta p\right)}{T_{a}} \tag{16}
\end{equation*}
$$

The relative calibration error obtained through the use of Eq. (16) instead of Eq. (15) is determined by,

$$
\begin{equation*}
\Delta \dot{m}_{g}=\frac{\dot{m}_{g}-\dot{m}_{g a p}}{\dot{m}_{g}}=(Z-1) \tag{17}
\end{equation*}
$$

To evaluate the result of such approximation to the perfect gas behavior, errors for some gases that usually are used in the laboratory experiments are now quantified. The gases under evaluation are presented in Tab. 1.

For the calculation of the compressibility factor the following equations were used (Adebiyi, 2005)

$$
\begin{equation*}
Z=1+\frac{\beta_{1}\left(T_{r}\right)}{T_{r}} p_{r} \tag{18}
\end{equation*}
$$

is a truncated version of the pressure series virial form of the $\mathrm{p}-\mathrm{v}-\mathrm{T}$ equation of state. $\beta_{1}\left(T_{r}\right)$ is given by

$$
\begin{equation*}
\beta_{1}\left(T_{r}\right)=0.083-\frac{0.422}{T_{r}^{1.6}}+\left(0.139-\frac{0.172}{T_{r}^{4.2}}\right) \omega \tag{19}
\end{equation*}
$$

where $T_{r}=T / T_{c}$ and $p_{r}=p / p_{c}$ are the reduced temperature and pressure, while $T_{c}$ and $p_{c}$ are the critical temperature and pressure. Parameter $\omega$ is the acentric factor (Howell and Buckius, 1987), Tab. 1.

Table 1 - Properties of the gases under analysis

| Gas | $\omega$ | References for $\omega$ | $\boldsymbol{T}_{\boldsymbol{c}}(\mathbf{K})$ | $\boldsymbol{p}_{\boldsymbol{c}}(\mathbf{P a})$ |
| :--- | :---: | :--- | :---: | :---: |
| Air | 0.078 | Adebiyi (2005) | 132.6 | $3.77 \times 10^{6}$ |
| CO | 0.051 | Chouaieb et al.(2004) | 132.8 | $3.49 \times 10^{6}$ |
| $\mathrm{CO}_{2}$ | 0.225 | Xiang and Deiters (2008) | 304.13 | $7.38 \times 10^{6}$ |
| $\mathrm{CH}_{4}$ | 0.011 | Xiang and Deiters (2008) | 190.56 | $4.60 \times 10^{6}$ |
| $\mathrm{C}_{3} \mathrm{H}_{8}$ | 0.1524 | Chouaieb et al. (2004) | 369.85 | $4.25 \times 10^{6}$ |
| $\mathrm{C}_{4} \mathrm{H}_{10}$ | 0.2 | Xiang and Deiters (2008) | 425.13 | $3.80 \times 10^{6}$ |
| $\mathrm{~N}_{2}$ | 0.038 | Xiang and Deiters (2008) | 126.19 | $3.40 \times 10^{6}$ |
| $\mathrm{O}_{2}$ | 0.0222 | Chouaieb et al. (2004) | 154.58 | $5.04 \times 10^{6}$ |

Figures 3 to 6 present calibration error values when the compressibility factor Z, calculated according to Eqs. (18) and (19), using data from Tab. 1, is replaced by 1 for the several gases under analysis. In such circumstances gases are being considered as having ideal behavior at ambient temperatures in the 0 to $45^{\circ} \mathrm{C}$ range.

These errors are meaningless for air, $\mathrm{CO}, \mathrm{CO}_{2}, \mathrm{~N}_{2}, \mathrm{O}_{2}$ and even for $\mathrm{CH}_{4}$, but are above $1 \%$ for propane $\left(\mathrm{C}_{3} \mathrm{H}_{8}\right)$ and butane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$. In Fig. 3 there is a comparison for 8 gases at 1 atm and for temperatures going from 0 to $45^{\circ} \mathrm{C}$. The worst situation is for propane and butane. Figures 4 and 5 show what happens for air and methane, in the 0.8 to 1.2 atm range, and it is evident that both gases can be assumed as ideal. The same happens for oxygen, nitrogen, carbon monoxide and carbon dioxide, whose plots are similar to that for air.

For propane, and above all for butane, the assumption of ideal gas behavior leads to large errors in the calibration procedure, Fig. 5 and Fig. 6.

So, although the positive displacement method is a very simple one to calibrate gas flow meters, some precautions need to be taken when working with higher molecular weight gases, and the true value of the gas compressibility factor must be adopted.


Figure 3. Calibration errors when real gases are supposed to have ideal gas behavior at 1 atm.


Figure 4. Calibration errors for air at several pressures and temperatures.


Figure 5. Calibration errors for methane at several pressures and temperatures.


Figure 6. Calibration errors for butane at several pressures and temperatures.

## 4. CONCLUSIONS

The positive displacement method is a very simple methodology to calibrate gas flow meters. However some simple precautions need to be taken to minimize calibration errors.

First of all, to minimize the gas compressibility effects the initial gas volume in the water and gas collecting tank must very small. Secondly, the real value of the gas compressibility factor must be taken into account, primarily when working with higher molecular weight gases, as the displacement from ideal gas conditions can lead to serious calibration errors.

Using these very simple experimental precautions can lead to successful calibration procedures.

## 5. REFERENCES

Adebiyi, G. A., 2005, "Formulations for the Thermodynamic Properties of Pure Substances", Journal of Energy Resources Technology, March, Vol. 127, pp. 83-87.
Chouaieb, O., Ghazouani, J. and Bellagi, A., 2004, "Simple correlations for saturated liquid and vapour densities of pure fluids", Thermochimica Acta, 424, pp. 43-51.
Howell, J. R. and Buckius, R. O., 1987, " Fundamentals of Engineering Thermodynamics", Ed. McGraw-Hill Book Company, New York, U.S.A.
Sonntag, R. E., Borgnake, C. and Van Wylen, G.J., 1998, "Fundamentals of Thermodynamics", Ed. John Wiley and Sons New York, U.S.A.
Xiang, H. W. and Deiters, U. K., 2008, "A new generalized corresponding-states equation of state for the extension of the Lee-Kesler equation to fluids consisting of polar and larger nonpolar molecules", Chemical Engineering Science, 63, pp. 1490-1496.

## 6. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.

