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NUMERICAL SIMULATION OF MAGNETIC FLUID FLOWS BY A FINITE VOLUME METHOD

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Abstract. *Pressure-driven flow of a magnetic fluid in cylindrical tubes is investigated using numerical simulations. The simulations are based on a Finite Volume Method adapted for incorporating a magnetization evolution equation. The magnetostatic limit of the Maxwell's equations is treated in terms of a Poisson equation numerically integrated. Suitable boundary conditions for velocity, magnetization and field intensity on the pipe wall are described. Results are obtained for velocity and magnetization profiles under several conditions of the identified physical parameters of the flow. The simulations are verified by comparison between numerical results and asymptotic theory, and they show good agreement.*

Keywords*. Ferrohydrodynamics, Magnetic Fluids, Computer Simulations, Finite Volume Methods, Laminar Pipe Flow.*

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1. Introduction

Many applications in chemical engineering, fluid mechanics and biology involve suspensions of magnetic particles immersed in a liquid. The behavior of magnetic motion is difficult to predict due to the complex coupling between the magnetic and hydrodynamic forces acting on the particles. The problem falls naturally into two parts: that of finding the magnetic field, and that of determining the fluid motion. Numerical solutions of the flow of a magnetic fluid have been developed and applied to this problem. Recently Shumacher et al. (2003) have examinated the laminar and turbulent pipe flow with an imposed linearly polarized, oscillating, magnetic field. The flow stability of magnetic fluid between two rotational cylinders have been investigated by Chang et al. (2002). Rinal & Zahn (2002) investigated the effect of the angular viscosity on a magnetic fluid pipe flow when a rotational magnetic field is applied. Additional investigations including analytical solutions and experimental studies were proposed by Kamyiama & Koike (1992). Some numerical techniques, notably finite elements or finite volume (Rosensweig, 1997) have reproduced the behavior of a magnetic homogeneous fluid.

The aim of the present paper is to perform numerical simulations of laminar flows of a magnetic fluid based on the solution of the coupled magnetic-hydrodynamic governing equations. The flow phenomenon is brought out by the computer simulations and confirmed by recent asymptotic theory (Cunha & Sobral, 2004). We present results for velocity and pressure profiles and for the friction factor of the flow under several conditions of the Reynolds number and the magnetic pressure coefficient.

2. Statment of the problem and governing equations

Dilute magnetic fluids made of colloidal superparamagnetic particles behave without hysteresis, since no interactions among neighboring particles are present. Thus, the local magnetization is allowed to be instantaneously orientated in the direction of the local magnetic field. In the remainder of this work, it shall be considered that local magnetization $\mathbf{M}$ is collinear with the local magnetic field intensity $\mathbf{H}$, what implies $\mathbf{M}\times\mathbf{H} = \mathbf{0}$. The starting point is the non-dimensional coupled magneto-hydrodynamic equation to describe the motion of a magnetic fluid. The dimensionless continuity and momentum equations for an incompressible magnetic fluid are:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + C_{pm} \mathbf{M} \cdot \nabla \mathbf{H} \tag{2}$$

where ***u*** is the Eulerian velocity, *p* is the mechanical pressure, **M** the continuous magnetization of the fluid and **H** denotes the intensity of the magnetic field. The equations were made dimensionless by using *U* as a typical velocity scale, *R* (pipe radius) as the length scale and H_o, the intensity of the applied field, as a typical scale for magnetic quantities. In addition, time and pressure scales are *R/U*, ρU^2 respectively. The identified physical parameters are then the Reynolds number (Re) and the magnetic pressure coefficient (C_{pm}), say Re = RU/ν and $C_{pm} = \mu_o H_o^2/\rho U^2$. Here μ_0 is the vacuum magnetic permeability. While the Reynolds number measures the relative intensity of the inertial and viscous mechanisms of momentum transport, the magnetic pressure coefficient states for the relative importance of the magnetic pressure compared to the dynamical pressure of the flow, i.e. magnetic effect relative to hydrodynamic effect in the flow field.

The magnetostatic equations are:

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

and:

$$\nabla \times \mathbf{H} = \mathbf{0}, \tag{4}$$

where **B** is the induced field defined as $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$. The magnetic field can be written in terms of a magnetic potential ϕ as $\mathbf{H} = \nabla \phi$. So from the definition of **B** and Eq. (3) we have in dimensionless terms a Poisson's equation for the magnetic potential:

$$\nabla^2 \phi = -\nabla \cdot \mathbf{M}. \tag{5}$$

Now, an evolution equation for the local magnetization **M** is needed. In this work, we propose a magnetization equation slightly different from the one proposed by Shliomis & Morosov (1972), valid for the regime of dilute magnetic fluids with small magnetization. It is considered a quasi-steady regime for the magnetization, where changes in this quantity occur instantaneously compared with a typical time scale of the flow. That means, significant changes on the magnetization of the fluid are consequence of flow vorticity only, namely

$$\omega (\mathbf{W} \times \mathbf{M}) = \mathbf{M} - \mathbf{M}^0, \tag{6}$$

where ω is the magnetic dimensionless frequency given by:

$$\omega = \frac{\tau_S}{L/U}, \tag{7}$$

W is the flow vorticity and the vector quantity $\mathbf{M^0}$ is the equilibrium magnetization for a quiescent magnetic fluid. For a dilute magnetic fluid, the equilibrium magnetization is collinear with the applied field $\mathbf{H^0}$, and $\mathbf{M^0}$ can be determined by the Langevin function discussed in reference (Rosensweig ,1997).

The fluid velocity on the fixed walls vanishes. The magnetic boundary conditions are Neumann condition for the prescribed magnetic field. The continuity of the normal component of the magnetic induction vector and the continuity of the tangential component of the field intensity vector on the boundaries of the computational domain are also considered.

3. Numerical procedure

We now briefly summarize the sequence of steps that are necessary to perform our numerical simulations. We use a finite volume method on a two-dimensional Eulerian grid to solve potential and hydrodynamic equation of the flow in Cartesian coordinate system. We solve the magnetic fluid momentum equation coupled with the magnetic-potential equations for velocity, field and magnetization. The effective viscosity of the dilute magnetic field is evaluated by a combination of an $O(\phi)$ contribution as a pure hydrodynamic effect due to particle stresslet exerted on the fluid (Batchelor, 1970) and a magnetization effect $O(\phi)$ based on the theoretical prediction of Shliomis (1972).

The governing equations are solved simultaneously by the following numerical procedure. A regular grid of quadrilateral 2D control volumes is used to solve the velocity and pressure fields of the flow. The upwind difference scheme (UDS) is used to predict the convective fluxes at the control volume faces and central difference scheme (CDS) to interpolate the difusive fluxes (Ferziger & Peric, 1997). The velocity at volume control centers is calculated by solving the algebraic system of the discretized form of the momentum governing equation given by:

$$A_P^{u_i} u_{i,p}^m = \sum_k A^u{}_k u^m{}_{i,k} = Q_{u_i}{}^{m-1} - (\delta p^{m-1} / \delta x_i)_P \tag{8}$$

for any grid node. Here *A* represents a generic matrix coefficient, the superscripts *m-1* and *m*, respectively, denote the values of the velocity at iteration iter=*m-1* and *iter=m*. The subscript *P* denotes the center of an arbitrary control volume, *k* denotes the neighboring points of the discretized equation, *m* represents the current iteration and $Q_{u_i}^{m-1}$ is the source term.

The problem of the numerical solution for Eqs.(1) and (2) is the absence of an evolution equation for the pressure. Solving Eq. (8) the continuity equation is not fulfilled, so a pressure correction equation is needed to ensure mass conservation. The corrected velocity field not satisfy the momentum equation and an iteration process is performed. The velocity-pressure couple is solved using the SIMPLE algorithm proposed by Patankar & Spalding (1972).

Time evolution is made using the Euler implicit procedure and the unsteady derivative term is added to the source $Q_{u_i}^{m-1}$. This procedure leads to a slight change in the velocity of the fluid when choosing the time step Δt sufficiently small compare with the relevant diffusive time D^2/ν of the flow or a characteristic period of the flow $1/\nu_f$. In dimensionless terms this condition is given by:

$$\Delta t << \min\left(\mathrm{Re}^{-1}, \frac{1}{n_f}\frac{U}{D} \right) \tag{9}$$

where n_f is the forcing frequency of an oscillatory magnetic field.

The magnetic force contributions are treated in this work explicitly as a source contribution in the Eq.(8). The magnetic field gradient is interpolated by using CDS on the control volume faces. The source term is simply given in a discretized form by:

$$Q_{u_i}^{m-1} = C_{pm} M_i (\partial H_j / \partial x_i) \tag{10}$$

The Poisson discretized equation of the magnetic potential ϕ is given by:

$$\frac{\delta}{\delta x_i}\left(\frac{\delta \phi_m}{\delta x_i} \right) = -\frac{\delta M_i}{\delta x_i} \tag{11}$$

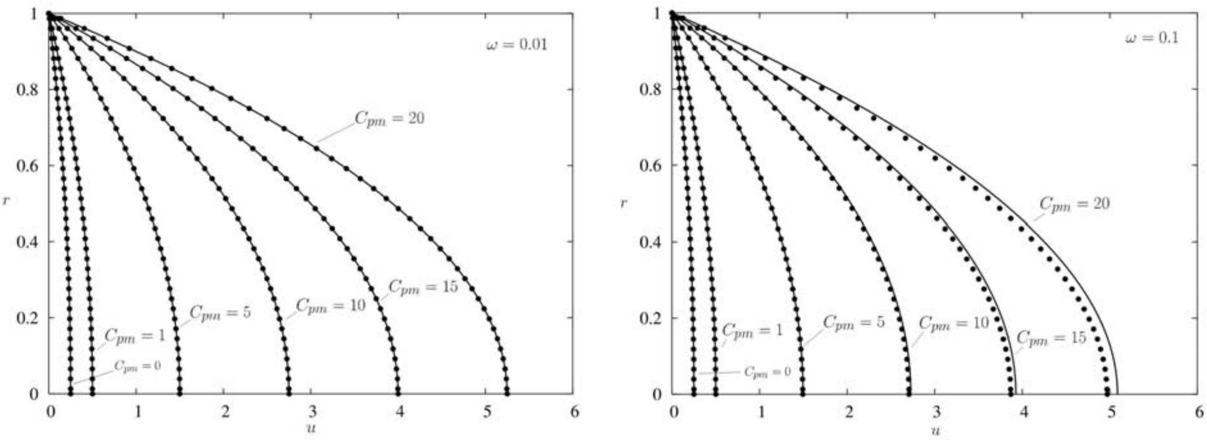
The magnetization M_i is coupled with the flow given in Eq.(6). This require iterative process of solving a linear system on the grid control volumes since the vorticity $\mathbf{\Omega}$ and $\mathbf{M^0}$ are known at each step of the calculation.

The maximum of control volumes we considered is 18000 (300x60 control volumes). More details of the numerical scheme is given in Ramos (2004).

4. Numerical results

In order to check the convergence of the numerical scheme we perform two simulations with different grid size and time steps. In the first simulation there are 40x100 control volumes in the r and z direction (for a pipe with radio and length set equal 1 and 10 dimensionless units). A 80x200 grid was used to compare solutions. The numerical results obtained with the same boundary conditions formulas have shown a negligible influence of the mesh size used. So the 40x100 grid was picked up to be carried out in linear regimes. In the other hand we use a 60x300 grid (1x20 dimensionless units) to simulate flows under unsteady conditions and non-linear regimes. That bigger mesh offers a best range to observe the flow instabilities.

The numerical scheme is testing by comparing velocity profile given by computations with a recent asymptotic prediction described in Cunha & Sobral (2004). The numerical domain in the present simulations have z, r dimensions equal to 10 and 1, respectively. Fig. 1a and Fig. 1b compare the velocity profile in the pipe for Re=1 for several values of magnetic parameter C_{pm} with theory. The values of the dimensionless magnetic relaxation time were **w**=0.01 in Fig. 1a and **w**=0.1 in Fig. 1b. To make this comparison possible specified pressure and magnetic gradients were imposed and equal to unit. We see that they are in excellent agreement in the range of validity of the predictions. It is seen a slightly discrepancy for high C_{pm} when ω is great (ω=0.1). For superparamagnetic fluids the magnetization relaxation time is naturally small, here we can observe despite high ω a good agreement between results.



a) ω = 0.01 b) ω = 0.1

Figure 1. Pipe velocity profile for Re=1, **w**=0.01 (a) e **w**=0.1 (b) , *dp/dz* =1 and *dH/dz* =1for several values of C_{pm}. Full lines represent the theory and points the numerical solution.

In addition, Fig. 2 presents the maximum velocity of the same flow as a function of the small parameter $\varepsilon = (\omega^2 \text{ Re } C_{pm})/8$ defined in Cunha and Sobral´s theory (Cunha & Sobral 2004). The results are in excellent agreement in the wholly range in which the theory makes sense (small ε) and, thus, validating the present numerical procedure.

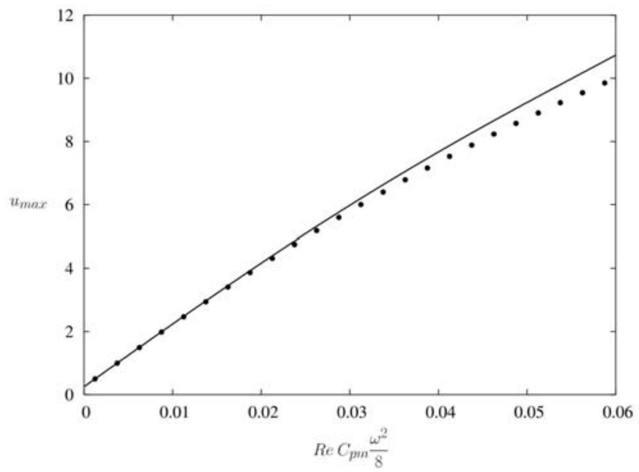


Figure 2. Maximum velocity in the pipe as function of $(\omega^2 \text{ Re } C_{pm})/8$. Full line represents the theory and points the numerical solution.

We have demonstrated that the phenomenon of drag reduction by magnetic action can be also exhibited by our numerical simulations for moderate and large values of Re and C_{pm}. This agrees with what has been predicted by the mentioned theory (Cunha & Sobral, 2004). Fig. 3 shows the effect of the Cpm parameter over the friction factor defined as:

$$f = \frac{2}{\text{Re}} \left(\frac{\partial u}{\partial r} \right)_{r=1} \tag{12}$$

In the absence of applied magnetic field (C_{pm} = 0) the friction factor corresponds to the Poiseuille flow, i. e., $f = 16\text{Re}^{-1}$. It is seen that this power law dependence breaks even at small values of the magnetic parameter. We can see that for Re = 10 and C_{pm} = 0.1 the friction factor is reduced around 30%.

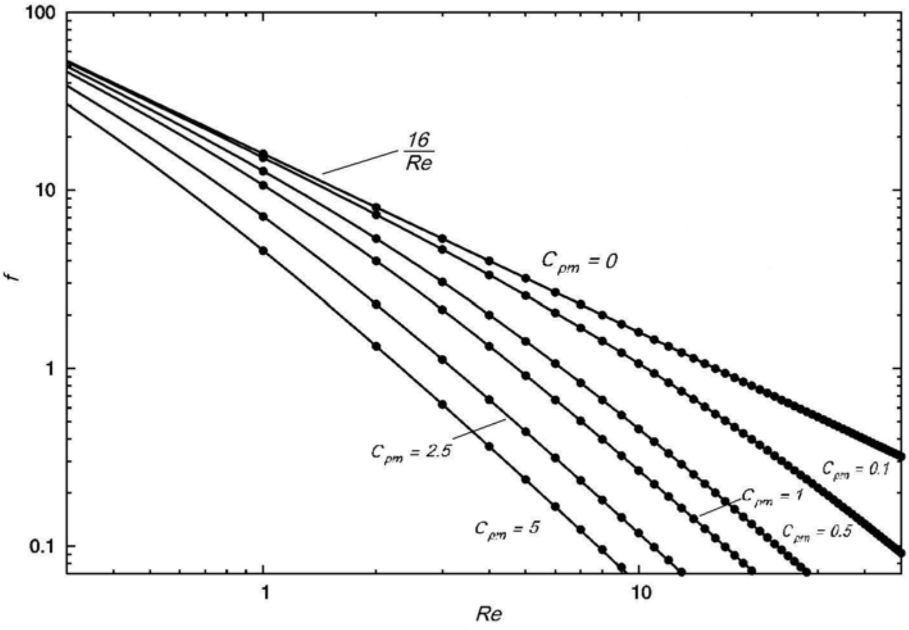


Figure 4. Moody's diagram comparing the drag reduction for several values of C_{pm} at ω=0. Full lines represent the theory and points the numerical solution.

4.1 Oscillatory Magnetic Field

The numerical scheme to calculate velocity and magnetization fields in regimes of the flow where the mentioned theory cannot be applied is now figure out. In these regimes a powerful magneto-hydrodynamic coupling is present since the full set of equations is considered, Eqs. (1)-(6). An external linear magnetic field is applied in *z* directions at specified locations of the flow. The field is made equal zero from $z = 0$ to $z = z_0$ and equal $H_0\,(1+ z - z_0)$ to $z > z_0$. This condition is applied at the flow boundaries (walls and/or symmetry lines). To guarantee the magnetostatic law in the fluid, crosswise components of ***H*** and ***M*** appear. So 2D magnetic field, magnetization and magnetic force arise into the flow. A prescribed velocity profile is used as inlet condition in the pipe and zero gradient of velocity is imposed at outlet boundary. The pressure is made equal zero in the pipe exit.

A sequence of the pressure response of a flow undergoing an oscillatory harmonic external magnetic fields is shown in Figs. 5-6. The pressure history sign at $z = 18$ and $r = 0$ is plotted for several values of C_{pm}. It is seen that for low magnetic action the pressure response is harmonic and periodic. At high C_{pm}'s, however the response of the flow no more harmonic with the applied field, but is still periodic.

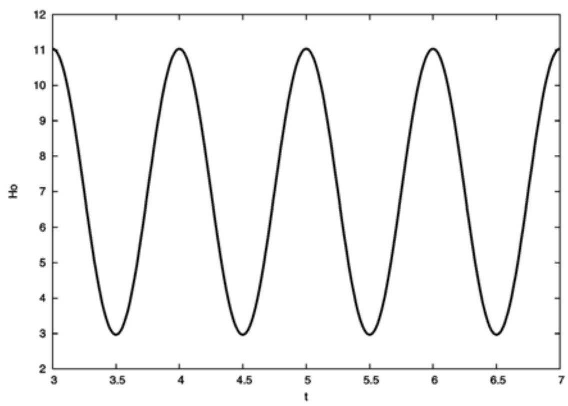


Figure 5. Harmonic applied magnetic field at at $z = 18$ and $r = 0$. Dimensionless oscillating frequency $n_f = 1$.

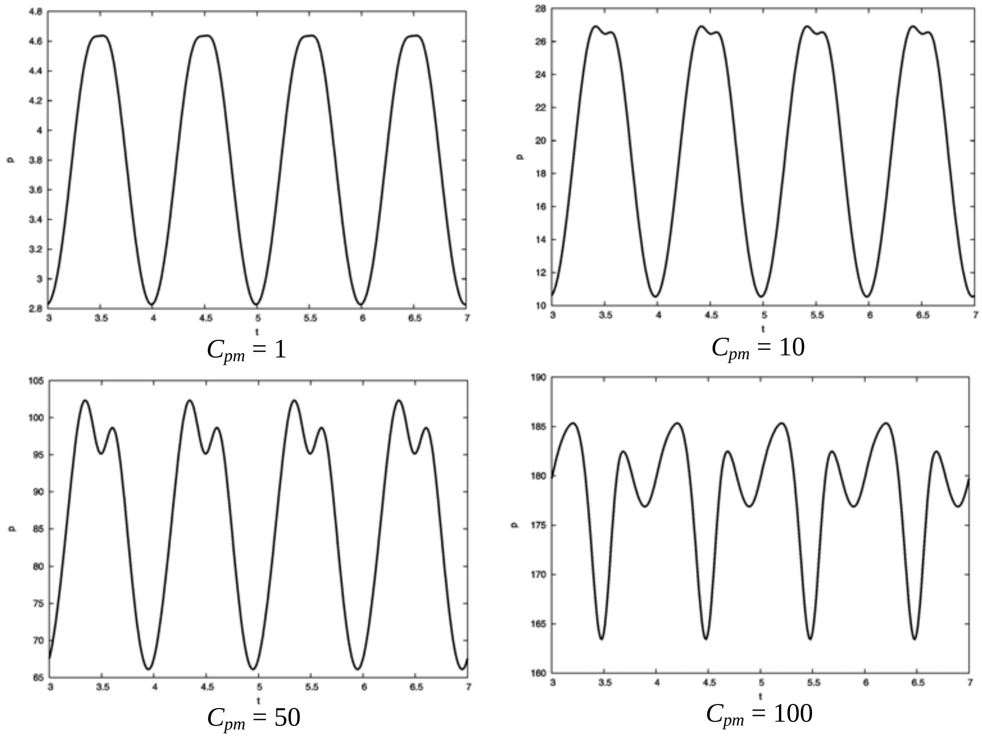


Figure 6. Time evolution of the pressure for several C_{pm} at z = 18 and r = 0, n_f = 1. Fluid characteristics: Particle volume concentration ϕ^C=0.01, dimensionless magnetization saturation M_S = 10 and ω = 0.01.

The phase diagrams for pressure is plotted in Fig 7. For fully harmonic regimes the phase diagram is equivalent to a circular curve. Some non-linearity appears at moderate magnetic pressure coefficients. At large C_{pm}'s non-harmonic flow behavior is evident. These diagrams are showing a sequence of flow bifurcation with the increasing of the magnetic parameter.

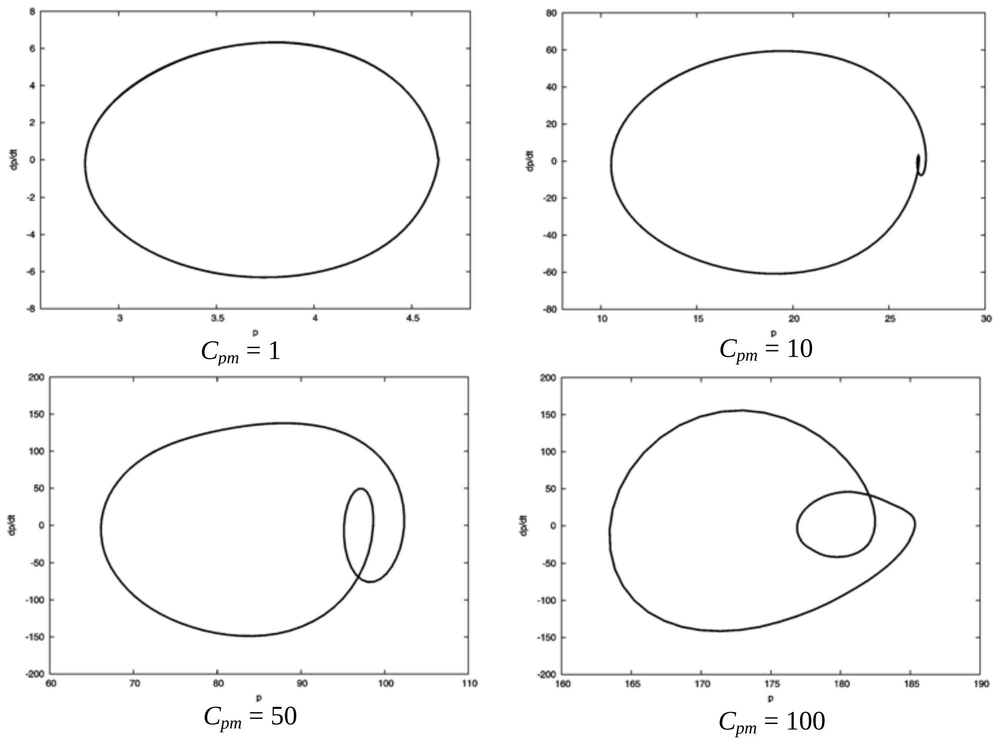
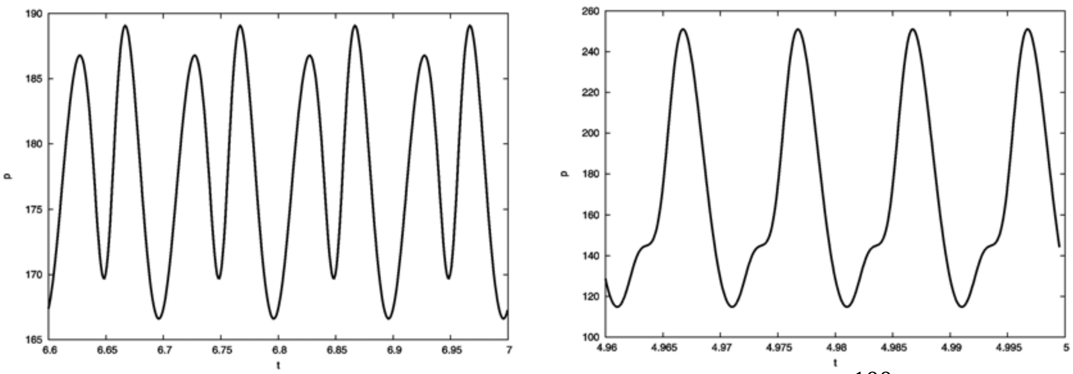


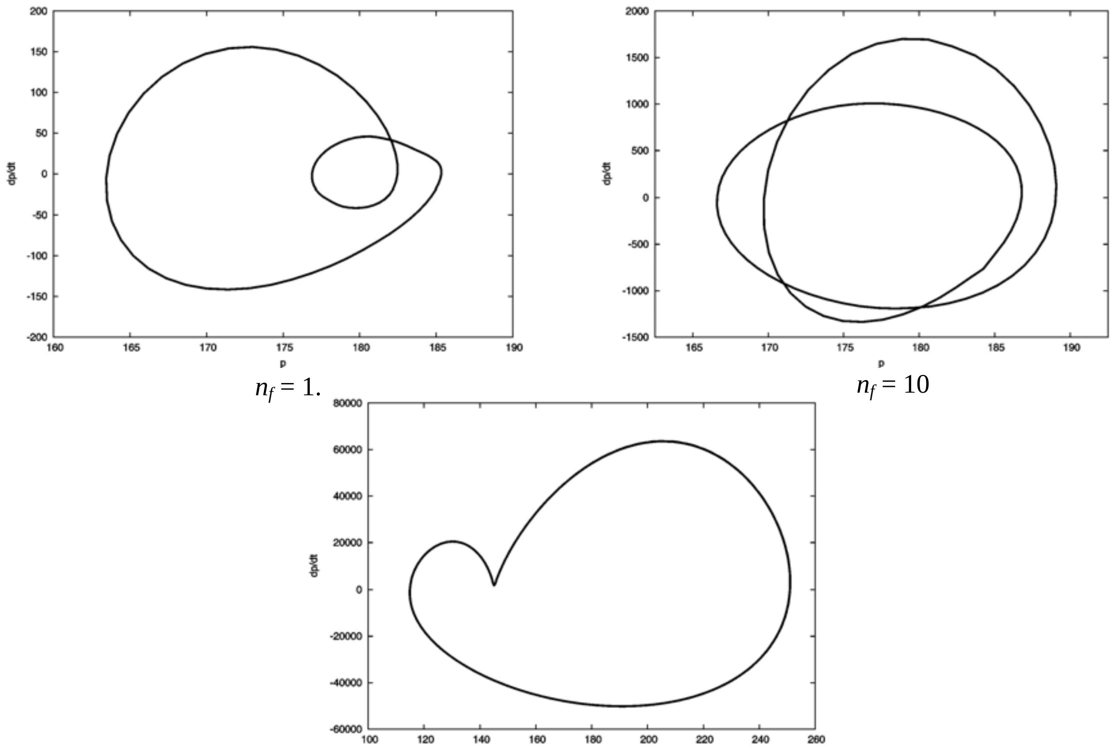
Figure 7. Pressure phase diagram at position z = 18 and r = 0, n_f = 1, ϕ^C=0.01, M_S = 10 and ω=0.01.

Fig. 8-9 show the fluid response at a higher oscillating frequency. It is seen that the steady state response of the pressure in a point (r = 0, z = 18) of the flow is not harmonic as the applied field, but is still periodic. At high frequencies however, typically n_f=100 and C_{pm}=100, the response is quite nonlinear, pointing out the complex coupled flow that is present in this regime.



n_f = 10 n_f = 100

Figure 8. Time evolution of the pressure for C_{pm} = 100, at z = 18 and r = 0, ϕ^C=0.01, M_S = 10 and ω = 0.1.



n_f = 1. n_f = 10

n_f = 100

Figure 7. Pressure phase diagram at position z = 18 and r = 0, C_{pm} = 100, ϕ^C=0.01, M_S = 10 and ω = 0.01.

The results reported in this paper demonstrate that numerical simulation of the coupled magnetic-hydrodynamic flow of a magnetic fluid is a good way to simulate such complex fluid motion. The efficiency of the high-order numerical scheme used in this paper is confirmed for a large range of Reynolds number and the magnetic pressure coefficient. The numerical scheme was able to capture the nonlinear effect and tendency of instability on the laminar flow of magnetic fluid. Simulations results are in good agreement with asymptotic predictions. A fully coupled efficient algorithm based on the numerical technique presented here is the subject of our future investigation.

5. References

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