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UNIFORM SHEAR FLOW AROUND A CIRCULAR CYLINDER AT SUB-CRITICAL REYNOLDS NUMBER

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Abstract. *The uniform flow around a circular cylinder has been widely investigated both experimentally and numerically due the practical applications. However, in some particular cases, the structures are subject to non-uniform flow, as the case of suspended bridges and offshore pipelines near a sea-bed, which are immersed in a boundary layer. This flow has received some attention in the last decades, however, the results presented still have some discrepancies. The incompressible Navier-Stokes equations for two-dimensional uniform shear flow around a horizontal circular cylinder are solved using a compact finite difference scheme and virtual boundary technique to model the presence of the obstacle. The uniform shear flow around a circular cylinder is simulated for Reynolds number from* 60 *to* 200 *and shear parameter up to* 0.20*. The numerical results indicate that vortex shedding persists at the shear parameters up to* 0.20 *for the Reynolds number range, therefore, no evidence of vortex shedding suppression was confirmed. The drag coefficient decreases with the increase of the shear parameter or Reynolds number. The transverse force acts from high velocity side towards low velocity in the shear flow and it is approximately proportional to the shear parameter.*

keywords: *shear flow, turbulence, vortex shedding, virtual boundary method, direct numerical simulation*

1. Introduction

The uniform flow around a circular cylinder constitutes a classical problem of fluid mechanics, and has been widely investigated both experimentally and numerically due to theoretical and practical importance (Zdravkovich, 1997). However, there are some particular applications where the incoming flow is a non-uniform flow, which may influence the vortex-shedding behaviour and the aerodynamic forces acting upon it. The simplest case is the uniform shear flow, where the velocity profile has a linear distribution of the longitudinal velocity component along the transverse direction, yielding a constant velocity gradient and vorticity.

This kind of problem has attracted research rather recently, the principal reason being the difficult of generating an uniform shear flow in laboratory (Zdravkovich, 1997). Most of the previous works which dealt with this type of flow were developed using numerical simulation as can be seen in the literature (Jordan and Fromm, 1972; El-Refaee and El-Taher, 1985; Ayukawa et al., 1993 and Lei et al., 1993). However, in the lasts decades some experimental studies were also developed (Kiya et al., 1980; Kwon et al., 1992 and Sumner and Akosile, 2003).

The main feature of the uniform stream flow is the velocity gradient, defined as $K = du(y)/dy$ (where $u(y)$ is the streamwise velocity and y is the cross stream coordinate) characterised by a dimensionless shear parameter, $\beta = KD/U_c$, where D is the diameter of the cylinder and U_c is the velocity in the center of the cylinder.

There are two basic uniform shear flow configurations depending on the cylinder orientation relative to velocity profile in the shear flow: a) an axial shear flow, where the velocity profile varies linearly along the span of the cylinder (Fig. $1a$); and b) a planar shear flow, where the velocity profile varies across the diameter of the cylinder (Fig. $1b$);

The first case is a common feature in many engineering applications such as marine risers, offshore platforms and heat exchangers. In this sense, a number of experimental and numerical studies has been motivated. Important effects in axial shear flow includes cellular vortex shedding, spanwise variation of the base pressure and drag force coefficient,

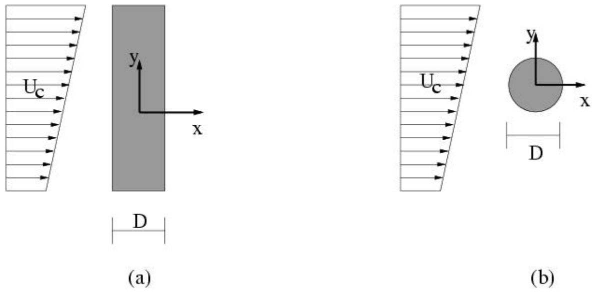


Figure 1: Circular cylinder in a uniform shear flow: (a) axial shear and (b) planar shear flow.

and a lower critical Reynolds number (Sumner and Akosile, 2003). A mechanism to explain the formation of the cellular vortex shedding has been recently suggested by Silvestrini and Lamballais, 2004.

The second case (Fig. 1b) has received less attention, although it also has many practical applications such as marine pipelines near a sea bed and suspended bridges (both immersed in a boundary layer). Analysis of the uniform flow is also important to study boundary layer transition induced by small bluff bodies as small horizontal circular cylinders. The principal feature of this flow is the difference of velocities on both sides of the cylinder. The vorticity generated by the cylinder in the high velocity side (negative vorticity) has the same sign as the free stream vorticity and opposite sign in the low-velocity side (positive vorticity). This difference is expected to promote separation on the high-velocity side and to postpone it on the low-velocity side.

The first two dimensional numerical study of the uniform shear flow was obtained for Reynolds number 400 and shear parameter $\beta = 0.05$ by Jordan and Fromm, 1972. As result, an average lift force towards the low side of the free stream was observed. Others numerical studies have confirmed this behaviour (El-Refaee and El-Taher, 1985 and Lei et al., 1993). The magnitude of the mean lift forces increasing with the shear parameter has been observed in these studies. The behaviour of the Strouhal number and the average drag coefficient have been described by Lei et al., 1993. It was shown that both parameters decreased as shear parameter increases.

Relatively few experimental investigation of a circular cylinder in a uniform shear flow has been conducted. The first experimental study has been conducted in the low sub-critical Reynolds number ($Re = 35 - 1500$) and shear parameter up to $\beta = 0.25$ (Kiya et al., 1980). The results showed that the critical Reynolds number increased approximately linearly with the shear parameter. A vortex shedding suppression has been also observed in the range of Reynolds number $40 - 220$ at large shear parameter. The Strouhal number increases with the shear parameter, this trend is divergent from the results obtained in numerical studies (El-Refaee and El-Taher, 1985 and Lei et al., 1993). Kwon et al., 1992, have also found that Strouhal number increases with the shear parameter, although no comments were done about vortex shedding suppression. For all experimental studies, the mean drag coefficient decreases as the Reynolds number and the shear parameter increase. The same behaviour has been also observed in the numerical study of Lei et al., 1993.

Recently, Sumner and Akosile, 2003, have extended the experimental studies to high sub-critical Reynolds number, i.e., $Re = 4.0 \times 10^4$-9.0×10^4, at low to moderate shear parameter ($\beta = 0.02 - 0.07$). For these shear parameters, the Strouhal number has shown a small decrease, almost negligible, indicating no dependence between Strouhal number and shear parameter. Others results indicated that as mean base pressure coefficient increases, the mean drag force coefficient decreases, and a small mean lift force directed towards the low-velocity side increases when the shear parameter increases. Theses results agree with the previous numerical study of Lei et al., 1993.

The divergence in the results observed in the literature between experimental and numerical studies, principally concerning the behaviour of the Strouhal number (see Tab. 2, in section 4), has motivated the present work. The aim of this work is to study the influence of the shear parameter (β) in the behaviour of Strouhal number and hydrodynamics forces act upon a circular cylinder. This study was undertaken for a Reynolds number range between $Re = 60 - 200$ and for shear parameter up to 0.20.

2. Numerical Methods

The incompressible Navier-Stokes equations,

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{\omega} \times \vec{u} = -\vec{\nabla} p + \nu \nabla^2 \vec{u} + \vec{f} \tag{2}$$

were directly solved in a non-staggered uniform mesh. In this equation, $p(\vec{x}, t)$ is the modified pressure field while $\vec{u}(\vec{x}, t)$ and $\vec{\omega}(\vec{x}, t)$ are the velocity and vorticity field, respectively and $\vec{f}(\vec{x}, t)$ is an additional force included in the momentum equations to model the circular cylinder. The spatial derivatives have been evaluated using a sixth-order compact finite difference scheme, except near the inflow and outflow boundaries where single side schemes were employed for x-derivative calculation (Lele, 1992). A third-order low-storage Runge-Kutta has been used for the time integration (Williamson, 1980).

The immersed boundary method have been used to model the circular cylinder. This method consists in adding an external force to the momentum equation (Eq. 2). This approach allows the imposition of the no-slip condition at the surface of the cylinder. Various formulations have been proposed in the literature (Linnick and Fasel, 2003). In the present, the feedback force methodology proposed by Goldstein et al., 1993, has been adopted. This force term is given by

$$\overrightarrow{f} = \alpha \int_0^t \overrightarrow{u}(x_s, t)dt + \beta \overrightarrow{u}(x_s, t) \tag{3}$$

where α and β are negative constants. More details about the numerical code can be found in Lardeau et al., 2002, and Silvestrini and Lamballais, 2002, and about the immersed boundary methods, can be found in Lamballais and Silvestrini, 2002.

The computational flow configuration that has been used is schematically showed in Fig. 2, where the cylinder axis is developed normal to the xy plane. At the inflow section, a Dirichlet condition was set using a mean velocity profile defined as

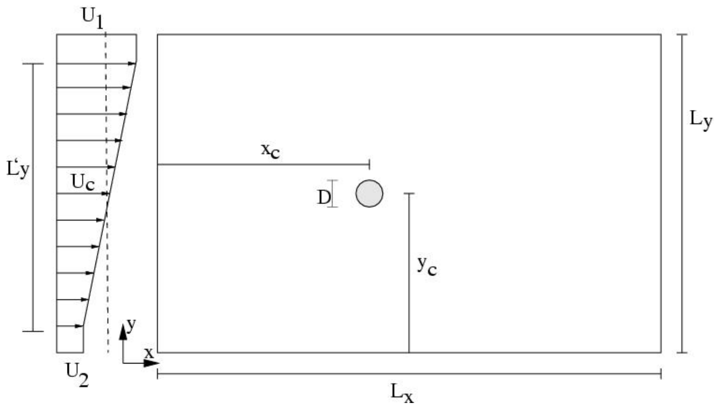


Figure 2: Numerical flow configuration

$$u(y) = \frac{U_1 + U_2}{2} + \frac{U_2 - U_1}{12}\frac{D}{L'_y} ln \left\{ \frac{cosh[\frac{6}{D}(y + \frac{L'_y}{2})]}{cosh[\frac{6}{D}(y - \frac{L'_y}{2})]} \right\} \tag{4}$$

where U_1 and U_2 is the higher and lower stream velocity, respectively, and $L_y^{'}$ is the width of the shear flow. At the outflow section a Neumann conditions was set while at the upper and down boundary a free-slip condition was used. Two non-dimensional parameters were used in the present work, the Reynolds number ($R_e = U_c D/\nu$), which is defined in terms of the diameter of the cylinder (D), the mean velocity of the approach flow at the center of the cylinder (U_c) and the kinematic viscosity (ν), and the shear parameter ($\beta = D/U_c du/dy$).

3. Validation of the Numerical Code

The numerical code, briefly described in section 2, was submitted to a several tests for uniform flow at Reynolds number 300 to define the necessary computational domain (Ribeiro et al., 2002). The optimal numerical parameters values (L_x,L_y, x_c, y_c and resolution) has been defined in these tests, in order to minimise the computational cost and obtain real values of the Strouhal number.

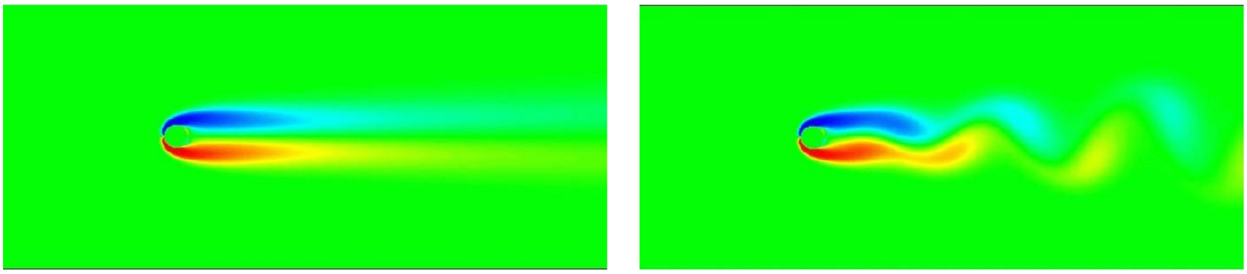
In the present work, the optimal parameters defined in the previous study, have been adopted and tested to validate the aerodynamic coefficients. The results obtained for two different resolutions at Reynolds number 300 an the reference values (Mital and Balachandar, 1997) are presented in the Tab. 1. The difference between the drag at high resolution and the reference value is less than 7%. The results have shown that it is necessary to employ higher mesh resolution ($D = 36\Delta$).

The ability of the numerical code to simulate the critical Reynolds number at uniform flow was also verified. Two simulation at $Re = 45$ and $Re = 50$ were performed. Figure 3 shows the vorticity field for these Reynolds number at $T = 350$. In Fig. $3a$ no vortex shedding can be observed, as was expected, while in Fig. $3b$ the vortex shedding is clearly observed.

Table 1: Strouhal number, drag and rms lift coefficient for different mesh resolution at Reynolds number 300.

$Resolution$	$< C_D >$	$C_L^{'}$	S_t
$D = 18\Delta$	1.689	0.7889	0.19139
$D = 36\Delta$	1.476	0.694	0.209
Mitall *et al.* 1997*	1.38	0.65	0.213

(*) reference value



(a) (b)

Figure 3: Vorticity field at dimensional time $T = 350$: a) $Re = 45$ and b) $Re = 50$

4. Results

In this section, the preliminary results obtained for a uniform shear flow at Reynolds number $Re = 60, 80, 100, 200$, and at shear parameter $\beta = 0.00, 0.05, 0.10, 0.15, 0.20$ are presented. The particular case $\beta = 0$ corresponds to the uniform flow, that is without shear. For all the simulations, the following configuration has been adopted: $Lx = 19D$, $Ly = 12D$, resolution 36Δ, $x_c = 8D$ and $y_c = 6D$.

Vortex Shedding

The vortex shedding suppression phenomenon is one of the divergence points between numerical and experimental studies, as can be seen in Table 2. Kiya et al., 1980, has observed that the vortex shedding suppression occurs at low Reynolds number and high shear parameter, and the critical Reynolds number increases approximately linearly with the shear intensity. In our work, the vortex shedding suppression has not been confirmed, even for low Reynolds number and high shear parameter, as can be seen in Fig. 4. This figure shows the vorticity field for Reynolds number $Re = 60$ and $Re = 200$ and shear parameter up to 0.20. The results agree quite well with the numerical studies of Lei et al., 1993 and Jordan and Fromm, 1972. Two other interesting observations can be made from this figure. The first is the shift of the stagnation point towards the high velocity side, which is more evident at high shear parameter. The second is the deformation of the vortex formed in the low velocity side of the cylinder, which creates a sheet of positive vorticity. The formation of a sheet of vorticity is more evident for $Re = 200$ and $\beta = 0.20$.

In Figure 5, the periodic behaviour of the time history of the fluctuating hydrodynamic coefficients confirmed the existence of the vortex shedding.

Strouhal Number

The Strouhal number values obtained in the present simulation, are shown in Fig. 6. From the figure, it can be seen, that the shear parameter influence over the Strouhal number is almost negligible. Figure $6a$ shows that for any shear parameter value, the Strouhal number increases with the Reynolds number in the same way as uniform flow. A very small decrease of this parameter with the shear can be observed in Fig. $6b$, the maximum reduction of the Strouhal value being less than 2.5%. These results agree quite well with previous numerical studies (Lei et al., 1993 and Ayukawa et al., 1993), which have obtained no dependence between shear parameter and the Strouhal number. This behaviour has been also observed recently in a experimental study (Sumner and Akosile, 2003), where a small reduction of Strouhal number with shear parameter has been measured. This behaviour was not confirmed in others previous experimental works, where a increase of Strouhal number with the shear parameter has been found (Kiya et al., 1980 and Kwon et al., 1992).

These results confirm the divergences described in section 1 between numerical results and some of the experimental studies. This may be a consequence of the low aspect ratio and the high blockage found in some experimental studies (Sumner and Akosile, 2003). On the other hand, two dimensional numerical models are unable to reproduce any three dimensional effect, as occurs for Reynolds number greater then 180 for the uniform flow case (Williamson, 1996).

Table 2: Principal results obtained for uniform shear flow.

References	Method	Re	β	It	L_y/D	D/L_z	$Re_c = f(\beta)$	Vortex Shedding Suppression	$St = f(\beta)$	$Cd = f(\beta)$	$\lvert C_L \rvert = f(\beta)$
Kiya et al., 1980	LC	$35 - 1500$	$0.05 - 0.25$	3.0%	$2 - 12.5$	$2.7 - 17\%$	increases	occurs	increases	N.M.	N.M.
Kwon et al., 1992	wT	$600 - 1600$	$0.05 - 0.25$	1.5%	$5.2 - 13$	$6.7 - 16\%$	N.C.	N.O.	increases	decreases	N.M.
Sumner and Akosile, 2003	WT	$4.0 - 9.0 \times 10^4$	$0.02 - 0.07$	3%	$12.3 - 18.4$	$1.8 - 2.7\%$	N.C.	N.O.	small decrease	decreases	increases
Jordan and Fromm, 1972	$2D$ FD	400	0.05	N.C.	94	N.A.	N.C.	N.O.	N.C.	N.C.	increases
El-Refaee and El-Taher, 1985	$2D$ ID	40;80	0.1;0.2	N.C.	26	N.A.	N.C.	N.O.	N.C.	decreases	increases
Lei et al., 1993	$2D$ FD	$80 - 1000$	$0.0 - 0.25$	N.C.	8	N.A.	N.C.	N.O.	small decrease	decreases	increases
current study	$2D$ FD	$60 - 200$	$0.0 - 0.20$	N.C.	12	N.A.	N.O.	dont occur	small decrease	decreases	increases

LC - liquid channel; wT - water tunnel; WT - wind tunnel; FD - finite difference; ID - integral difference.; It - intensity of turbulence; N.C. - no comments; N.A. - not applicable; N.O. - not observed; N.M. - not measured

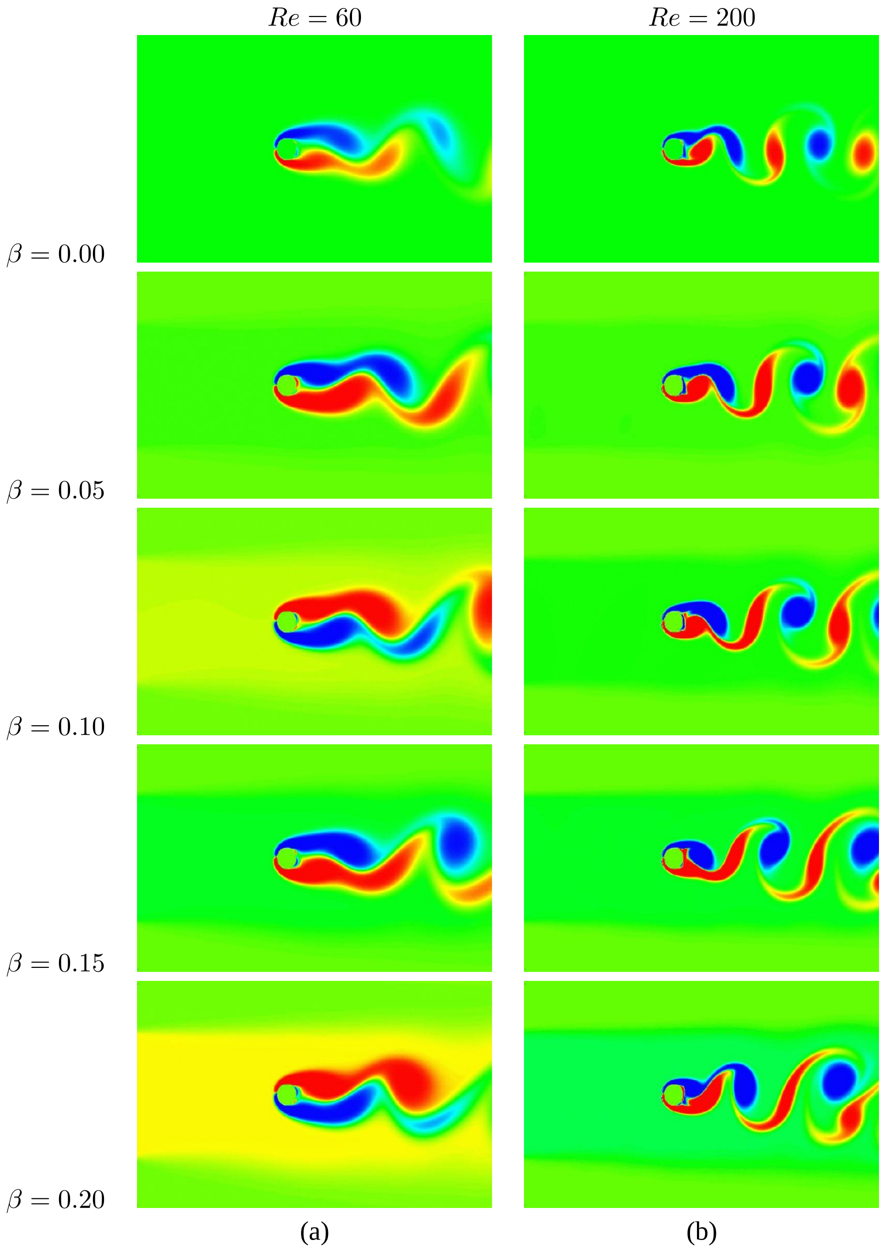


Figure 4: Vorticity field at dimensional time $T = 210$: a) $Re = 60$ and b) $Re = 200$ and $\beta = 0.05 - 0.20$

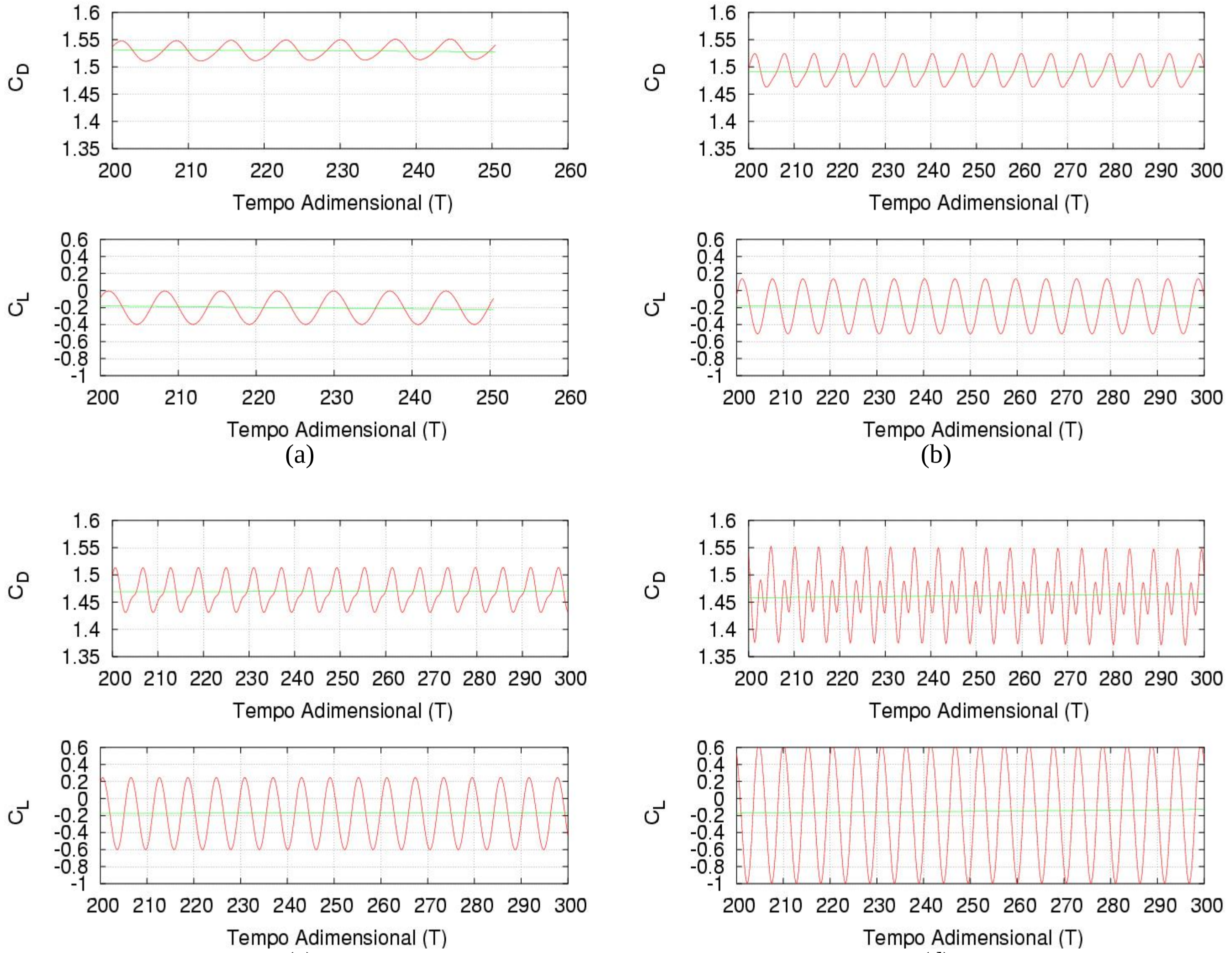
Aerodynamic Forces

The drag coefficient has the same behaviour as in the uniform flow; it decreases when Reynolds number increases (Fig. $7a$). For a given Reynolds number, the shear seems to reduce the mean drag coefficient when compared with the uniform flow case, the maximum reduction is observed for the lower Reynolds number ($R_e = 60$ - Fig. $7b$), and it is less than 3%. This reduction may be a consequence of the stagnation point shift to the high velocity side, which affects the rate of flow passing around the two sides of the cylinder, changing the pressure distribution. These results are consistent with previous works (Kwon et al., 1992, Ayukawa et al., 1993, Lei et al., 1993 and Sumner and Akosile, 2003).

The mean lift coefficient seems to decrease when the Reynolds number increases (Fig. $8a$), while for a given Reynolds number, it seems to increase with the shear parameter (Fig. $8b$). The negative value of the lift coefficient indicates the existence of a force from the high-velocity side towards the low-velocity side. The asymmetry of the flow around the circular cylinder is the origin of this force. These results are consistent with the previous results of Jordan and Fromm, 1972; Lei et al., 1993 and Sumner and Akosile, 2003.

The rms value of the lift force increases with the Reynolds number, both in the shear flow and the uniform one. A small increase of this value is also observed (Fig. 9).

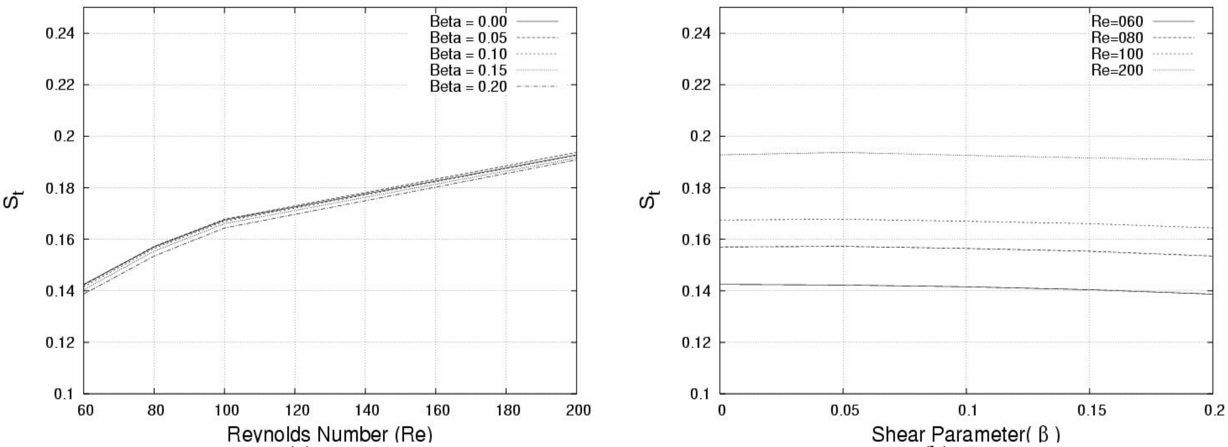
The results presented also indicate that the time series of drag coefficient is more affected than the time series of lift



(a) (b)

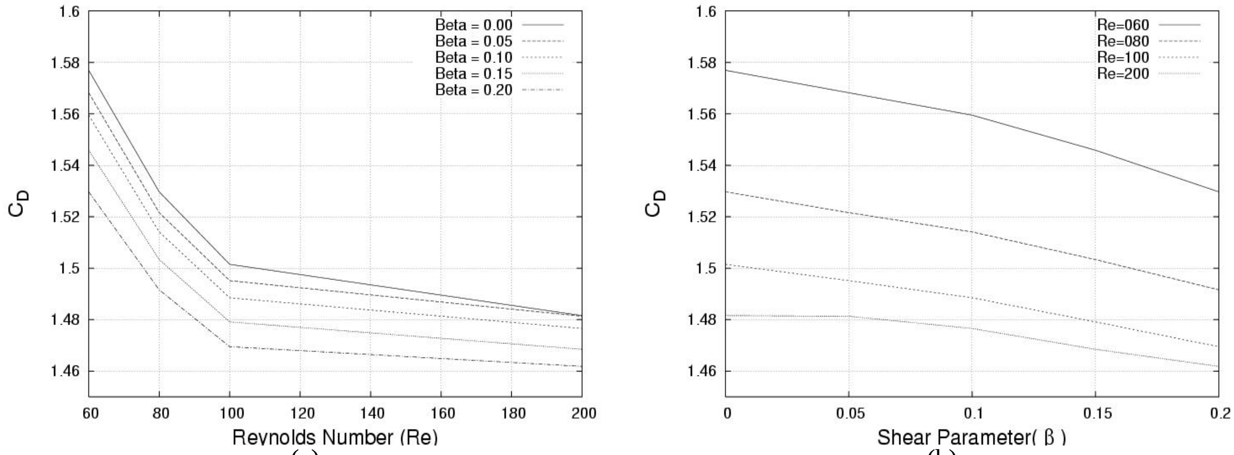
(c) (d)

Figure 5: Time series of the aerodynamics coefficients for shear parameter $\beta = 0.20$ at $Re = 60(a)$, $Re = 80(b)$, $R_e = 100(c)$ and $R_e = 200(d)$



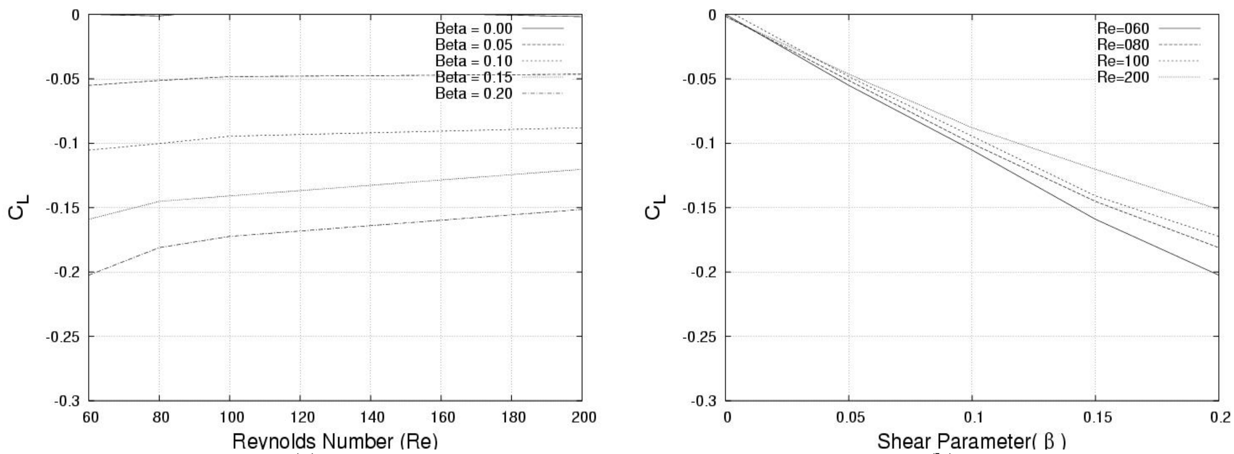
(a) (b)

Figure 6: Strouhal number variation with shear parameter: a) $S_t \times Re$, b) $S_t \times \beta$



(a) (b)

Figure 7: Mean drag coefficient for a circular cylinder in a uniform planar flow : a) $C_D \times Re$, b) $C_D \times \beta$



(a) (b)

Figure 8: Mean lift coefficient for a circular cylinder in a uniform planar flow : a) $C_L \times Re$, b) $C_L \times \beta$

force, as can be seen in Fig. 5, where a second peak can be identified, being more evident for $Re = 200$ and $\beta = 0.20$.

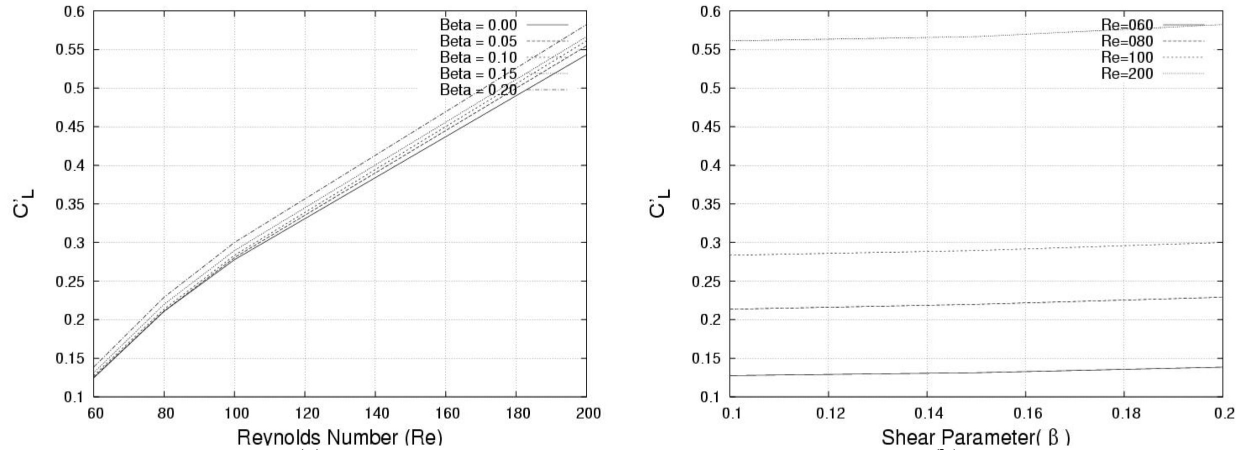
Statistics

The wake velocity profile for a circular cylinder in shear flow at Reynolds number $Re = 200$ in uniform and shear flow are showed in Fig. 10. For the horizontal velocity a small shift of the minimum velocity profile to the low velocity side is observed, which is consistent with the asymmetry of the flow. For the vertical velocity the zero value is shift to the high velocity side. In this figure the Reynolds stress tensor is also shown, where a asymmetry for the $< u'u' >$ and $< u'u' >$ is observed, with both peaks shifted to the high velocity side.

5. Conclusions

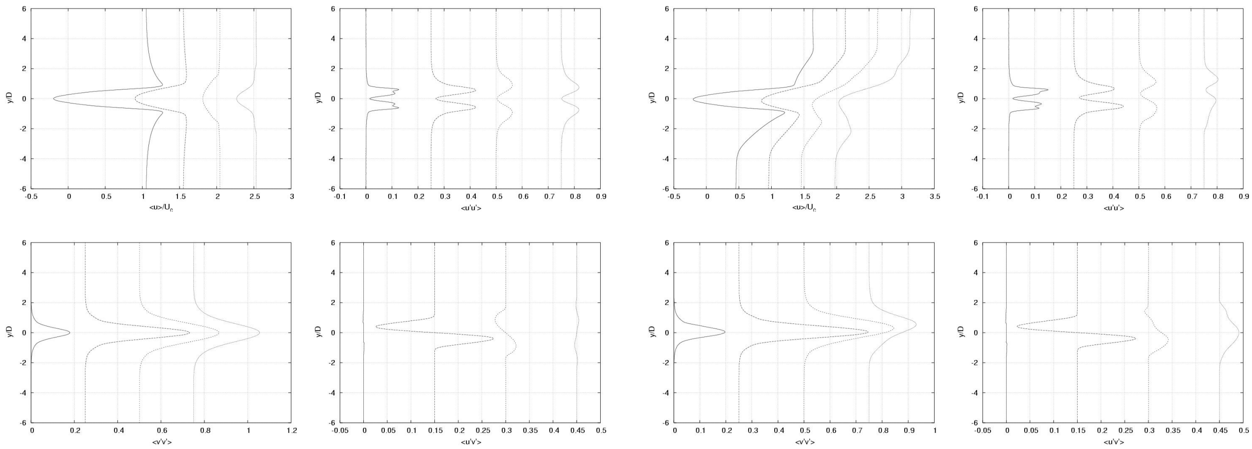
Preliminary numerical results of a study of a uniform shear flow around a circular cylinder at low Reynolds number ($Re = 60 - 200$) and at shear parameter up to 0.20, have been presented.

A small influence of the shear parameter on the Strouhal number has been found, it decreases as the shear parameter increases. No vortex shedding suppression has been observed, even for low Reynolds number ($Re = 60$) and high shear parameter($\beta = 0.20$). Some hypothesis for these disagreements could be made; the increase of the Strouhal number with the shear parameter could be consequence of the low aspect ratio and high blockage used in some of the previous experimental results. On the other hand, two-dimensional numerical simulation are unable to develop any three-dimensional phenomena that could be developed at low Reynolds number and high shear parameter value. Is the authors opinion, that these divergences would be clarified just after a three dimensional numerical study have been done. Others



(a) (b)

Figure 9: Fluctuations rms of the lift coefficient for a circular cylinder in a uniform planar flow : a) $C_D \times Re$, b) $C_D \times \beta$



(a) (b)

Figure 10: Statistics for uniform and shear flow for $Re = 200$. (a) uniform flow ($\beta = 0$) and (b) Uniform shear flow ($\beta = 0.20$).

results are the decrease of the mean drag coefficient and the existence of a small lift force towards the low-velocity side. These results agree quite well with previous studies.

6. Acknowledgements

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