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A NEW *k*-ε MODEL FOR VISCOELASTIC DRAG REDUCING PIPE FLOW

Pedro R. Resende
Centro de Estudos de Fenómenos de Transporte, DEMEGI, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal
resende@fe.up.pt

Daniel Onofre Cruz
Departamento de Engenharia Mecânica, Universidade Federal do Pará- UFPa, Campus Universitário do Guamá, 66075-900 Belém, Pará, Brazil
doac@ufpa.br

Fernando Tavares de Pinho
Centro de Estudos de Fenómenos de Transporte, Escola de Engenharia, Universidade do Minho, Campus de Azurém, 4800-058 Guimarães, Portugal
fpinho@dem.uminho.pt

Abstract. *The new stress term in the time-average momentum equation of the modified generalised Newtonian fluid model of Cruz and Pinho (2003) is modeled for improved predictions of turbulent viscoelastic flows. The modified generalised Newtonian model was introduced by Pinho (2003) to mimic viscoelastic effects of fluids exhibiting drag reduction in turbulent pipe flow. The new stress quantifies the cross-correlation between the fluctuating viscosity and the fluctuating rate of strain and had been neglected in the* k-ε *low Reynolds number model originally developed by Cruz and Pinho (2003). The inclusion of this model for the new stress improves the predictions of turbulent kinetic energy (* k^+ *) in drag reducing pipe flow at the cost of a negligible deterioration in the prediction quality of other quantities for some of other fluids tested and for which the literature provides data. The original model of Cruz and Pinho (2003) has also been modified to correct for a mistake and this was shown to improve the predictions for all fluids except for the solution of 0.125%PAA used to calibrate the model.s.*

Keywords*. Viscoelastic, drag reduction, turbulence model, extensional viscosity,* k-ε *model*

1. Introduction

The development of turbulence models for drag reducing fluids remains a challenge that is currently motivating a wealth of new research, especially aimed at understanding its relationship with polymer dynamics and fluid rheology. On this front most of the research tends to be based on direct numerical simulation (DNS), as reviewed by Pinho (2003). Turbulent flows of viscoelastic fluids are relevant in various industrial applications, such as in drilling oil and gas wells (here it is essential to be able to predict accurately the pressure drop, amongst other things) or during maintenance of pumping stations in pipelines (polymers are added to reduce friction and compensate for the shutdown of pumps).

The most recent DNS research continues to represent the rheology of dilute polymer solutions with the FENE-P model (for instance Dimitropoulos et al, 1998; De Angelis et al, 2002; Ptasinski et al, 2003), but Zhou and Akhavan (2003) have shown that it incurs large errors in predicting extensional flow properties under transient flow conditions (a consistent finding also made by Ptasinski et al, 2003), and recommended multimode models or different closures of the basic FENE model. In contrast, Stone and Graham (2003) defend the use of FENE-P because of its simplicity and ability to capture relevant features.

However, a useful simple single-point turbulence closure based on these fundamental investigations has yet to appear, not least because the link with experimental data has yet to be reinforced. In the meantime, it is more advantageous to rely on a model derived from a top-bottom approach: based on one of the widely accepted theoretical concepts, relating drag reduction with strain-hardening of the Trouton ratio initially formulated by Lumley (1969), Pinho (2003) has modified a Generalized Newtonian constitutive equation and derived the corresponding framework required for single-point turbulence closures. Then, Cruz and Pinho (2003) developed a low Reynolds number *k*-ε model and compared its predictions with data of Escudier et al (1999) and Presti (2000) which includes both detailed rheological properties and flow characteristics. The advantage of this turbulence model relative to previous single-point closures for viscoelastic fluids, reviewed in Cruz and Pinho (2003), is that only parameters related to the shear-viscosity and the Trouton ratio need to be used as input data, together with the bulk velocity. Another important characteristic of the formulation proposed by Pinho (2003) and Cruz and Pinho (2003), and probably the main reason for the above-mentioned advantage, is that the model was not developed by *ad-hoc* modifications of some of the parameters of a previous selected Newtonian model, to take into account the non-Newtonian effects, but most of the parameters and functions appearing in the model were specifically deduced considering important features of turbulent non-Newtonian flows. However, in their model, Cruz and Pinho (2003) neglected a new stress term in the momentum equation and its corresponding effect on the transport of turbulent kinetic energy.

The neglected stress term accounts for the coupling between viscosity fluctuations and rate of strain fluctuations and its role on the transport equation for turbulent kinetic energy is akin to that of any other stress: it can be either a production or a dissipation term. Pinho (2003) demonstrated these terms to be smaller than the Reynolds stress terms and negligible for constant viscosity or weakly shear-thinning fluids. However, for strongly shear-thinning fluids, for fluids with a strain-hardening extensional viscosity and in the near vicinity of walls the term may not be so small and helps to improve the predictions especially of the turbulent kinetic energy.

Here, a closed model is proposed to account for this new stress term in the context of low Reynolds number k-ε model of Cruz and Pinho (2003) and this is shown to improve predictions for several viscoelastic fluids. As discussed later, the improvement is also due to the correction of a mistake in the model of Cruz and Pinho (2003), in the absence of this new stress.

In the next section all the conservation equations, including those of the turbulence model, are presented and the terms requiring modelling are identified. Section 3 proposes a model for the new stress appearing in the momentum and turbulent kinetic energy equations, and the performance of the model is shown in Section 4 via comparisons with experimental data. The paper ends with a summary of the main conclusions.

2. Governing equations

The basic equations to be solved are the Reynolds-averaged mass and momentum equations and those associated with the adopted k-ε turbulence model for fully-developed duct flow. The mass equation is not affected by the fluid rheology hence it is not modified. Following Pinho (2003), the momentum equation is

$$-\frac{\partial \overline{p}}{\partial x_i}+\frac{\partial\left(2\overline{\mu}S_{ik}-\rho\overline{u_i u_k}+2\overline{\mu' s_{ik}}\right)}{\partial x_k}=0 \tag{1}$$

The average molecular viscosity in Eq. (1) is given by the combination in Eq. (2) of a pure viscometric shear viscosity contribution (η_v in Eq. 3) and the high Reynolds number time-average molecular viscosity contribution ($\overline{\mu}_h$) of Eq. (4).

$$\overline{\mu}=f_v\overline{\mu}_h+(1-f_v)\eta_v \tag{2}$$

$$\eta_v=K_v\left[\dot{\gamma}^2\right]^{\frac{n-1}{2}} \tag{3}$$

$$\overline{\mu}_h=\left(C_\mu\rho\right)^{\frac{3m(m-1)A_2}{8+3m(m-1)A_2}}2^{\frac{4m(m-1)A_2}{8+3m(m-1)A_2}}k^{\frac{6m(m-1)A_2}{8+3m(m-1)A_2}}\varepsilon^{\frac{[8-3(m-1)A_2]m}{8+3m(m-1)A_2}}B^{\frac{8}{8+3m(m-1)A_2}} \tag{4}$$

where K_v and n are the power law parameters fittd to the shear viscosity and f_v is a damping function. This viscosity model is based on the modified constituve equation $\sigma_{ij}=2\mu S_{ij}$ where the variable viscosity is given by Eq. (5)

$$\mu=\eta_v K_e\left[\dot{\varepsilon}^2\right]^{\frac{p-1}{2}}=\frac{1}{3}\frac{\eta_e(\dot{\varepsilon})}{\eta_v(\dot{\gamma})} \tag{5}$$

The viscosity includes the shear contribution (η_v) and was modified by inclusion of a term accounting fo strain-thickening of the extensional viscosity (η_e) via the Trouton ratio. Note that $\dot{\gamma}=\sqrt{3}\dot{\varepsilon}$, with more details presented in Cruz and Pinho (2003).

The Reynolds stresses are expressed by the turbulent viscosity hypothesis

$$-\overline{u_i u_j}=2\nu_T S_{ij}-\frac{2}{3}k\delta_{ij}=2C_\mu f_\mu\frac{k^2}{\tilde{\varepsilon}}S_{ij}-\frac{2}{3}k\delta_{ij} \tag{6}$$

with the turbulent viscosity given as a function of k and ε, as for other low Reynolds number models. $\tilde{\varepsilon}$ stands for the modified rate of dissipation ($\tilde{\varepsilon}=\varepsilon-D$) and here the model of Nagano and Hishida (1987) is adopted for term D ($D=2\overline{\nu}\left(\partial\sqrt{k}/\partial r\right)^2$).

Hence, the transport equations for k and ε, presented below, must also be solved, and to close the momentum equation it is necessary to determine the new stress $2\overline{\mu' s_{ij}}$ in the momentum equation. The development of a model for this new stress is the main contribution of this work and is addressed in the next section.

The modeled transport equation for k in fully-developed duct flow is

$$0 = \frac{\partial}{\partial x_j}\left[\left(\frac{\bar{\mu}}{\rho} + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \frac{2}{\rho}\overline{\mu' s_{ij}} S_{ij} - \overline{u_i u_j} S_{ij} - \varepsilon \tag{7}$$

which differs from that in Cruz and Pinho (2003), because the second term on the right-hand-side is now included.

For fully-developed pipe flow the transport equation for $\tilde{\varepsilon}$ is unmodified relative to that of Cruz and Pinho (2003)

$$0 = \frac{1}{r}\frac{d}{dr}\left[r\left(\bar{\mu} + \rho\frac{\nu_T}{\sigma_\varepsilon}\right)\frac{d\tilde{\varepsilon}}{dr}\right] + \rho f_1 C_{\varepsilon 1}\frac{\tilde{\varepsilon}}{k}P - \rho f_2 C_{\varepsilon 2}\frac{\tilde{\varepsilon}^2}{k} + \rho E + C_{\varepsilon 4}\frac{\nu_T}{\sigma_E \bar{\nu}}\frac{d\tilde{\varepsilon}}{dr}\frac{d\bar{\mu}}{dr} \tag{8}$$

The various terms and damping functions appearing in this set of equations are

$$E = \bar{\nu}\nu_T\left(1 - f_\mu\right)\left(\frac{\partial^2 U}{\partial r^2}\right)^2;\ f_1 = 1.0;\ f_2 = 1 - 0.3\exp\left(-R_T^2\right) \text{ with } R_T = \frac{k^2}{\bar{\nu}\tilde{\varepsilon}};\ f_v = f_\mu\ . \tag{9}$$

The eddy viscosity damping function f_μ was derived by Cruz and Pinho (2003) and is

$$f_\mu = \left\{1 - \left[1 + \left|\frac{1-n}{1+n}\right| y^+\right]^{-\left|\frac{1+n}{1-n}\right| / A^+}\right\}\left\{1 - \left[1 + \left|\frac{p-1}{3-p}\right| y^+ C^{\frac{1-p}{2-p}}\right]^{-\left|\frac{3-p}{p-1}\right| / A^+}\right\} \tag{10}$$

The wall coordinate is calculated with the friction velocity and the average molecular viscosity at the wall $y^+ = u_\tau y / \bar{\nu}_w$. The remaining parameters are listed in Table 1. The Reynolds number used throughout this paper is always based on pipe diameter, bulk flow velocity and wall viscosity.

Table 1- Values of the parameters of the low Reynolds k-ε model.

C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	$C_{\varepsilon 3}$	A_ε	A_2	C
0.09	1.0	1.3	1.45	1.90	1.0	10	0.45	45

In the original work of Cruz and Pinho (2003), a value of C= 18 was determined on the basis of comparisons of flow dynamic data from numerical predictions of their turbulence model and experimental data of Escudier et al (1999), but by mistake the wrong value of parameter K_e was used for the solution of 0.125% PAA. Using the correct value of K_e leads to C= 45, as shown in Section 4.1.

3. Model for the new stress

Closure of the set of equations requires a model for the new stress term $2\overline{\mu' s_{ij}}$ that accounts for the correlation between fluctuations of the viscosity and of the rate of strain tensor. To develop such model, estimates are made separately for the viscosity and the rate of strain fluctuations. The model should include dependence on both n and p so that the stress vanishes in the Newtonian limit (n=1, p= 1).

By definition (see Pinho, 2003), the fluctuating viscosity is proportional to $\mu' \propto K_v K_e (\dot{\varepsilon}')^{p-1}(\dot{\gamma}')^{n-1}$ where the fluctuating invariants $\dot{\gamma}'$ and $\dot{\varepsilon}'$ are given by $\dot{\gamma}' \sim \sqrt{s_{ij}s_{ij}}$ and $\dot{\varepsilon}' \sim \sqrt{s_{ij}s_{ij}} / A_\varepsilon$. Defining $\mathrm{S} \equiv \sqrt{s_{ij}s_{ij}}$ and back-substituting, gives

$$\mu' s_{ij} \propto \frac{K_v K_e}{A_\varepsilon^{p-1}} \mathrm{S}^{p+n-2} s_{ij} \tag{11}$$

Estimates must be made for S and s_{ij} as functions of known quantities in order to arrive at a closed set of equations. These are made in the context of a boundary layer and two assumptions are invoked here: (1) that we are in the equilibrium region, where production of turbulence balances its rate of dissipation and (2) that the new stress does not interfere significantly with this equilibrium. Since, within a boundary layer $\mathrm{S} \sim \partial u'/\partial y$, then

$$P_k = -\rho\overline{uv}\frac{dU}{dy} \approx \rho\varepsilon = 2\overline{\hat{\mu}s_{ij}^2} \sim 2\overline{\mu}\mathrm{S}^2 \tag{12}$$

Invoking the turbulent viscosity hypothesis for the Reynolds shear stress (Eq. 6)

$$\rho C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}\left(\frac{dU}{dy}\right)^2 \sim 2\overline{\mu}\mathrm{S}^2 \tag{13}$$

from which S is estimated to be

$$\mathrm{S} = \sqrt{\frac{\rho C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}\left(\frac{\partial U}{\partial y}\right)^2}{2\overline{\mu}}} \tag{14}$$

Within the boundary layer $s_{ij} \approx \partial u_i/\partial x_j \sim \partial u/\partial y$ and $u_i \sim \sqrt{\overline{u_i u_j}}$. Given the model assumed for the Reynolds shear stress (Eq. 6), the estimate of the velocity fluctuation is

$$u_i \sim \sqrt{C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}}\sqrt{\frac{\partial U}{\partial y}} \tag{15}$$

and the estimate of s_{ij} is $s_{ij} \sim \partial u_i/\partial x_j \sim u_i/L_c$.

L_c is a spatial scale of turbulence that takes into account high Reynolds number flow away from the wall and the damping effect of the approaching wall. Hence, far from the wall it is given by $L_c = k^{3/2}/\varepsilon$ whereas close to the wall L should be $L_c = u_\tau^3/\varepsilon$. To match smoothly these two behaviours, $L_c = u_R^3/\varepsilon$ with

$$u_R^2 = k\Bigg/\left|\exp\left(-\left(\frac{k}{u_\tau^2}\right)^\alpha\right)-1\right|^{\frac{1}{\alpha}} \quad \text{where} \quad \alpha = 4 \tag{16}$$

In conclusion, the final expression for the new stress term is given by Eq. (17), which for pipe flow simplifies because $S_{ij} \equiv \partial U/\partial y$. Parameter $\tilde{C}$ was introduced to account for discrepancies in modelling.

$$2\overline{\mu' s_{ij}} = \tilde{C}\frac{K_v K_e}{A_\varepsilon^{p-1}}\left[\frac{\rho C_\mu f_\mu k^2}{2\overline{\mu}\tilde{\varepsilon}}S_{ij}S_{ij}\right]^{\frac{p+n-2}{2}}\sqrt{C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}}} \times \frac{1}{L_c} \times \frac{S_{ij}}{\sqrt[4]{S_{ij}S_{ij}}} \tag{17}$$

The new stress must vanish in the Newtonian limit because there are no viscosity fluctuations, and to arrive at an adequate expression for $\tilde{C}$ the following argument was developed. By definition, the fluctuating viscosity is

$$\mu' = C_1\left[\hat{S}_{ij}{}^2\right]^{\frac{n+p-2}{2}} - C_1\overline{\left[\hat{S}_{ij}{}^2\right]^{\frac{n+p-2}{2}}} \tag{18}$$

where C_1 is a parameter (see Pinho (2003) for the exact equation). Assuming $\overline{\left(\hat{s}_{ij}^2\right)^a} \approx \left(\overline{s_{ij}^2}\right)^a$, on the basis of high Reynolds number turbulence (Tennekes and Lumley, 1972), then

$$\mu' = C_1\left[\left(S_{ij}+s_{ij}\right)^2\right]^{\frac{n+p-2}{2}} - C_1\left[\overline{s_{ij}^2}\right]^{\frac{n+p-2}{2}} \rightarrow \mu' = C_1\left[\overline{s_{ij}^2}\right]^{\frac{n+p-2}{2}}\left\{\left[\frac{\left(S_{ij}+s_{ij}\right)^2}{\overline{s_{ij}^2}}\right]^{\frac{n+p-2}{2}} - 1\right\} \tag{19}$$

Since the term within the curly brackets must vanish in the Newtonian limit (n=p=1)

$$\left[\frac{\left(S_{ij}+s_{ij}\right)^2}{\overline{s_{ij}^2}}\right]^{\frac{n+p-2}{2}} - 1 = \left[\left(\frac{S_{ij}}{\sqrt{\overline{s_{ij}^2}}} + \frac{s_{ij}}{\sqrt{\overline{s_{ij}^2}}}\right)^2\right]^{\frac{n+p-2}{2}} - 1 \sim \left[\left(C_0+1\right)^2\right]^{\frac{n+p-2}{2}} - 1, \tag{20}$$

we arrive at the form of $\tilde{C}$ that contains the parameter to be quantified, (C_0) $\tilde{C} = \left(1+C_0\right)^{p+n-2} - 1$.

So, the model for the new stress in Eq. (17) can be expressed as the product of the time-average rate of deformation tensor (S_{ij}) by a coefficient akin to a second turbulent viscosity that also depends on the mean and turbulent flow fields.

4. Results and discussion

To assess the performance of the new turbulence model we follow the same philosophy as Cruz and Pinho (2003): using the experimental data of Escudier et al (1999) and Presti (2000) for the aqueous solution of 0.125% PAA, the effect of the new stress is investigated first and the model is optimized through quantification of C_0 and C. Then, simulations and comparisons are carried out for the other fluids in these experiments. However, prior to that, a correction to the original model of Cruz and Pinho (2003) is made in the next section.

The numerical simulations were carried out with the same finite-volume code used in Cruz and Pinho (2003). Non-uniform meshes of 199 cells across the pipe were used, having at least 12 computational cells within each of the viscous sublayers ($y^+ < 5$). This mesh gives mesh independent results for Newtonian and non-Newtonian fluids within 0.1%.

4.1. Behaviour of the model without the new stress

The rheological parameters for the 0.125% PAA solution are $K_v = 0.2491$ Pas$^{0.425}$, n= 0.425, $K_e = 8.25$ and p= 1.4796 as seen in Table 2, which contains also information for the other fluids tested. By mistake, in Cruz and Pinho (2003) a value of $K_e = 1.9394$ was used for the 0.125% PAA solution, instead of the correct value of 8.25, thus underpredicting the extensional viscosity of the fluid by a factor of about 4.

Table 2- Parameters of viscosity law used to fit the viscosity data in Escudier et al (1999).

Fluid	K_v [Pas^n]	n	K_e	p
0.25% CMC	0.2639	0.6174	2.0760	1.2678
0.09% CMC/0.09% XG	0.15178	0.5783	2.1833	1.1638
0.2% XG	0.2701	0.4409	3.8519	1.2592
0.125% PAA	0.2491	0.425	8.25*	1.4796

* Correct value

As a consequence, and since the solution of 0.125% PAA is used to calibrate the turbulence model, the correct value of C (in the damping function f_μ) should be 45 instead of 18. The effect of this difference can be assessed in Figure 1, which compare the performance of the various turbulence models in this work with the corresponding experimental data. It is clear that with the correct value of K_e, the use of C=18 results in underprediction of drag reduction, mean velocity and k^+, both in relation to the experimental data as well as in relation to the more correct prediction for C= 45.

The slope of the predicted *f-Re* curve in Figure 1-a) is lower than that of the experiments although there is a small, but insufficient, improvement for *C*= 45. This value was chosen to match the *f-Re* predictions with experimental data at a Reynolds number of around 42,900, the flow condition for which there are experimental data. The differences in mean velocity at *Re*= 42,900 in Figure 1-b) are consistent with the differences in the friction factor. For the turbulent kinetic energy profiles plotted in Figures 1 (c) and (d), the changes are more dramatic. The turbulence model underpredicts both k/U^2 and k^+, with the effect of *C* being small in terms of the former and quite significant in terms of the former. This is an indication that both models predict not too different values of *k*, but act strongly upon the friction velocity. So, by correcting the mistake of Cruz and Pinho (2003) the true predictions for the flow of 0.125% PAA are of similar quality as those seen in that work for mean flow quantities, but are worse in terms of turbulent flow quantities.

Regarding the other fluids in Table 2, the corresponding predictions in Cruz and Pinho (2003) for *C*= 18 were correct, but since the optimization of the turbulence model resulted in *C*= 45, the use of this value is to be preferred, which has the aditional benefit of improving predictions for all other fluids.

4.2. Effect of new stress

Figure 1-a) includes predictions of friction factor by the new model for two different values of parameter C_0. Since from the outset the sign of the new stress was unknown, both possibilities had to be investigated. What was known was that, under drag reducing conditions, the Reynolds shear stress was strongly dampened and, because the viscous stress is low, there must be a compensating stress usually called polymer stress (Warholic et al, 1999). In the original turbulence model of Cruz and Pinho (2003) it was the modified viscous stress that accounted for the difference between

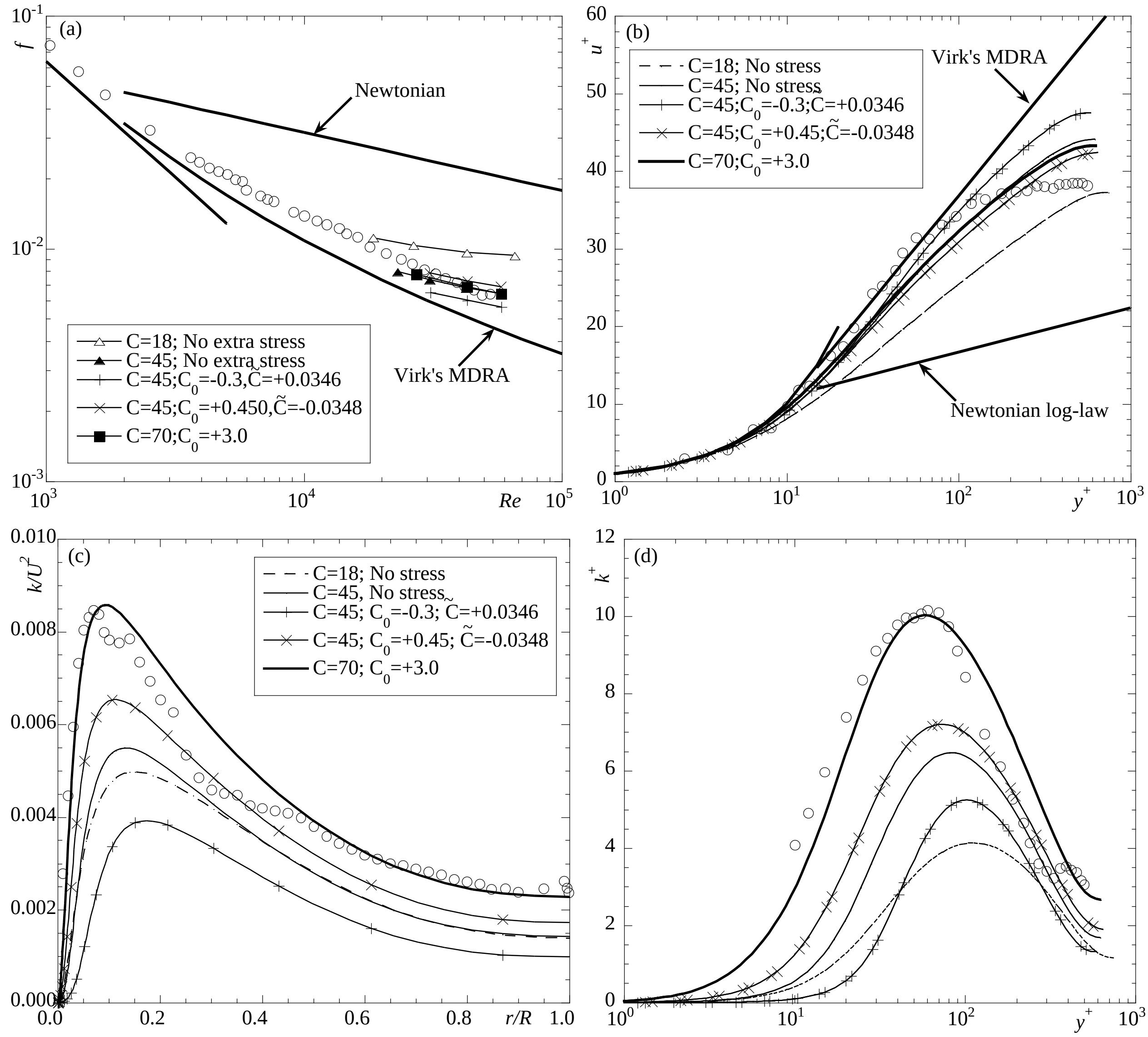

Figure 1- Effect of *C* and of new stress on flow for 0.125% PAA solution. Open circles: experimental data of Escudier et al (1999) (a,b) and Presti (2000) (c,d): (a) Friction factor; (b) Mean velocity at *Re*= 42,900; (c) k/U^2; (d) k^+.

the total stress and the Reynolds shear stress. This may suggest that the new stress is positive but this is not necessarily true: in a real drag reducing fluid the viscous stress is that due exclusively to the time-average shear flow, so the difference between the total and viscous stresses must be positive. Here, in contrast, it is not known how much of this polymer stress is being taken by the modified viscous stress ($2\bar{\mu}S_{ij}$) and so the new stress ($2\overline{\mu' s_{ij}}$) can in effect be negative, but its sum with the modified viscous stress must clearly be a positive quantity.

The predictions with the new stress model in show the effects of the sign and intensity of the new stress. The values of C_0 were such that the values of $\tilde{C}$ are symmetric and the new stress has the sign of $\tilde{C}$. The positive new stress increases drag reduction, whereas a negative stress increases the fiction factor, with the former effect larger than the latter. The predictions of the mean velocity profile in wall coordinates are consistent with the predictions of the friction factor, with a higher shift of the profile towards Virk's asymptote for the positive stress (higher drag reduction) and a lower shift when the stress is negative. Again, the effect of the positive stress is larger than that of the negative stress.

For the turbulent kinetic energy, however, changes are dramatic with the positive new stresses reducing the values of k, whereas negative stresses increase k. One of the shortcomings of the original model of Cruz and Pinho (2003), now is accentuated by the increase of C to 45, is its underprediction of the turbulent kinetic energy. Here, a significant increase in k is obtained when the new stress is negative at the cost of deteriorating the predictions of f and u^+, but this can be easily corrected by increasing the value of C and thus reducing the friction factor while increasing k^+. So, combining a negative new stress with a higher value of C should improve predictions of k/U^2, and especially of k^+, while maintaining the performance of the model in terms of friction factor and mean velocity.

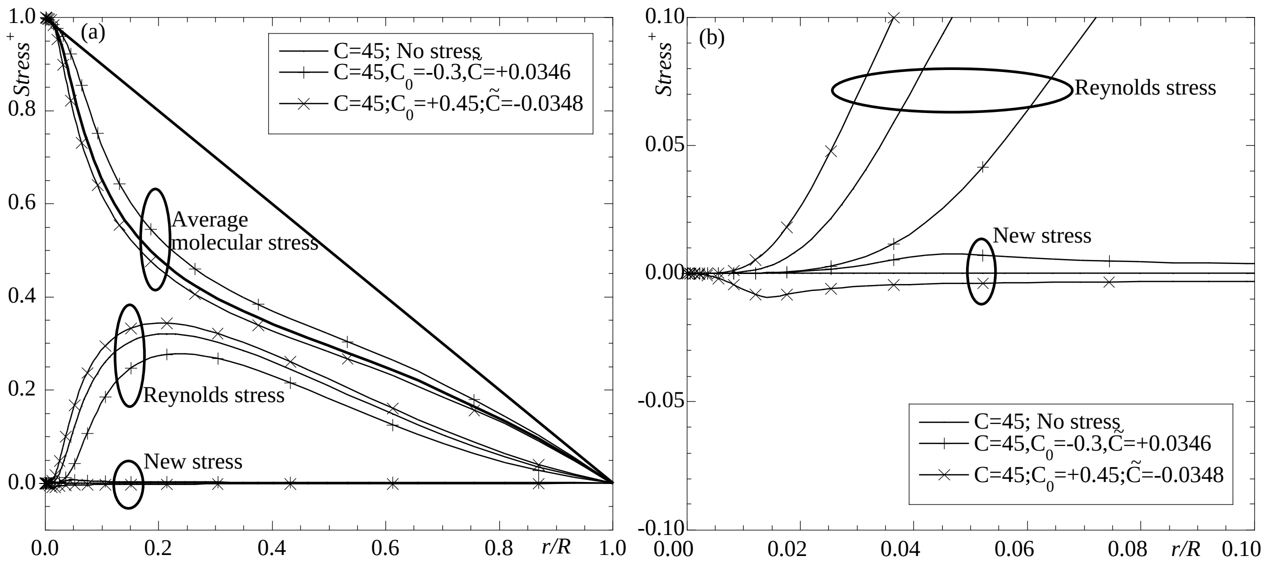

Figure 2- Effect of C and of sign of new stress on radial distribution of various shear stresses across the pipe for flow of 0.125% PAA at Re= 42,900. (a) radial profile; (b) zoom in the near-wall region (for $C_0 > 0$, $-2\overline{\mu' s_{ij}}$ is plotted).

Before proceeding to suggest an adequate combination of C and C_0, it is advantageous to analyse in detail the radial profiles of the various shear stresses, shown in Figure 2. It is clear from Figure 2-a), that in spite of the very low value of the new stress, the Reynolds shear stress and the time-average viscous stress are strongly affected. Figure 2 shows the new stress to be non-zero, but small across the pipe, peaking only very close to the wall. Still, the impact of the new stress is dramatic. Inspection of the transport equation of k (Eq. 7) helps understand this finding: the Reynolds stress acts to produce k, the viscous stress acts to dissipate it and the new stress will dissipate energy if positive but will create k if negative. The other contributions to the balance of k are less important. Very close to the wall, the magnitude of the new stress is similar to that of the Reynolds stress, regardless of its sign, so that when the new stress is positive it annihilates the effect of $-\rho\overline{uv}$ and production of turbulence is delayed until much farther away from the wall, thus leading to lower turbulence. Elsewhere, there is also a decrease in turbulence production since the new stress is non-zero, although small in magnitude. In contrast, when the new stress is negative, production of k can increase significantly close to the wall and is increased elsewhere, helping to raise the overall turbulence across the pipe.

Increasing C to 70 and considering a negative new stress with $C_0 = +3.0$, there is a significant improvement in the prediction of k/U^2 and k^+ while the quality of the remaining predictions (fRe and $u^+ - y^+$) are maintained. The friction factor in Figure 1-a) matches the experiments at a Reynolds number in the vicinity of 42,000 as for the old model with *C*=45. There is also good agreeement for the mean velocity in Figure 1-b), but the main benefit is in predicting the turbulent kinetic energy. As shown by the thick solid lines in Figures 2-a) and 2-b), the predictions and the experimental data for k/U^2 and k^+ match well and it is especially noteworthy that both the peak values and their locations are well predicted.

4.3. Performance of model for other fluids

In general, these improvements are consistent in that they take place with all fluids for which we possess experimental information. In Figures 3 and 4, the performance of the new model is assessed by comparing its predictions with the predictions of previous turbulence models for *C*= 18 and *C*= 45 and with the experimental results of Escudier et al (1999) and Presti (2000) for 0.25% CMC and a blend of 0.09%/0.09% XG/CMC, respectively.

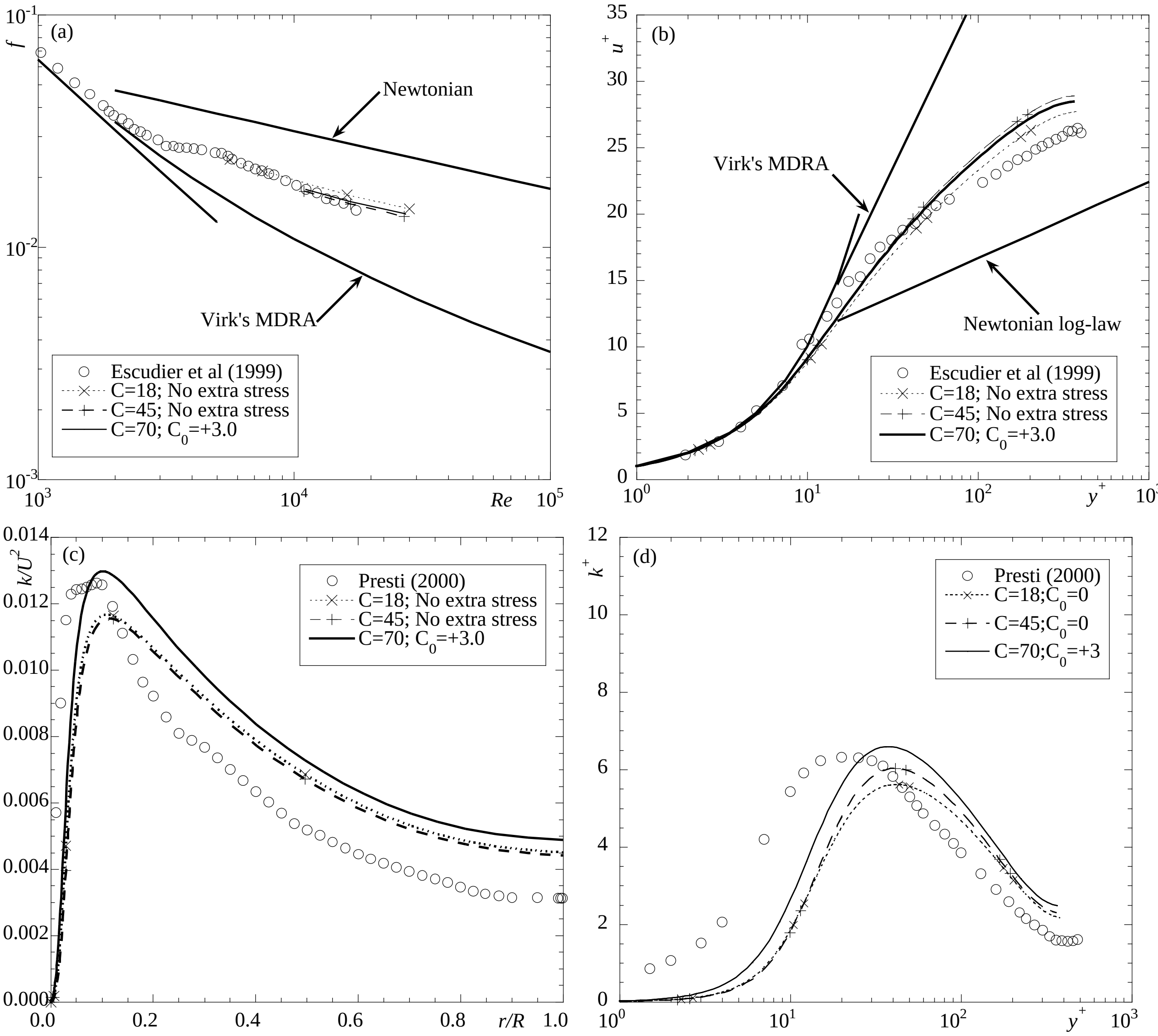

Figure 3- Predictions of flow of 0.25% CMC at *Re*= 16,600 with new stress (*C*= 70 and C_0= +3.0) and comparisons with model of Cruz and Pinho (2003) (old: *C*= 18 and C_0= +0.0), (corrected: *C*= 45 and C_0= +0.0) and experimental data: (a) friction factor; (b) u^+ versus y^+; (c) k/U^2 versus *r/R*; (d) k^+ versus y^+.

In all cases, there are improvements relative to the original model in Cruz and Pinho (2003) (*C*= 18) both with the improved version of the original model (*C*= 45; $C_0 = 0$) as well as by the new model containing the new stress (*C*= 70; $C_0 = +3$). For the modified original model (*C*= 45), these improvements, assessed relative to the experimental data, result in a decrease in the friction factor, an increase in the upward shift of the mean velocity and a slight increase in turbulent kinetic energy. For the new model, however, there is sometimes a small deterioration in the prediction of f and u^+, but these are largely compensated by significant improvements in predicting the turbulent kinetic energy.

For the 0.25% CMC solution, the friction factor in Figure 3-a) increases slightly for the new stress model, and the mean velocity profile is correspondingly downshifted (c.f Figure 3-b), but these variations are very small in comparison to the increase of both turbulent quantities (k^+ and k/U^2) shown in Figures 3-c) and d). Both peak values are well predicted, but are farther from the wall than the experimental data.

For the blend of CMC and xanthan gum, the results in Figure 4 show similar trends, but with less advantage coming from the new stress term. Here, the predictions of *fRe* and $u^+ - y^+$ for the new model are worse than those of the original model without the new stress (C= 18 and C= 45), although the difference is small. However, for the turbulent kinetic energy there is again a clear advantage in using the formulation with the new stress term. The peak in k/U^2 is now overpredicted by an amount similar to the underprediction of the peak by the two models with $C_0 = 0$, although all models overpredict the quantity elsewhere, but for k^+ there are important and larger improvements in terms of peak value, its location and the behaviour at low y^+. Considering the various positive and negative variations, the overall change was an improvement.

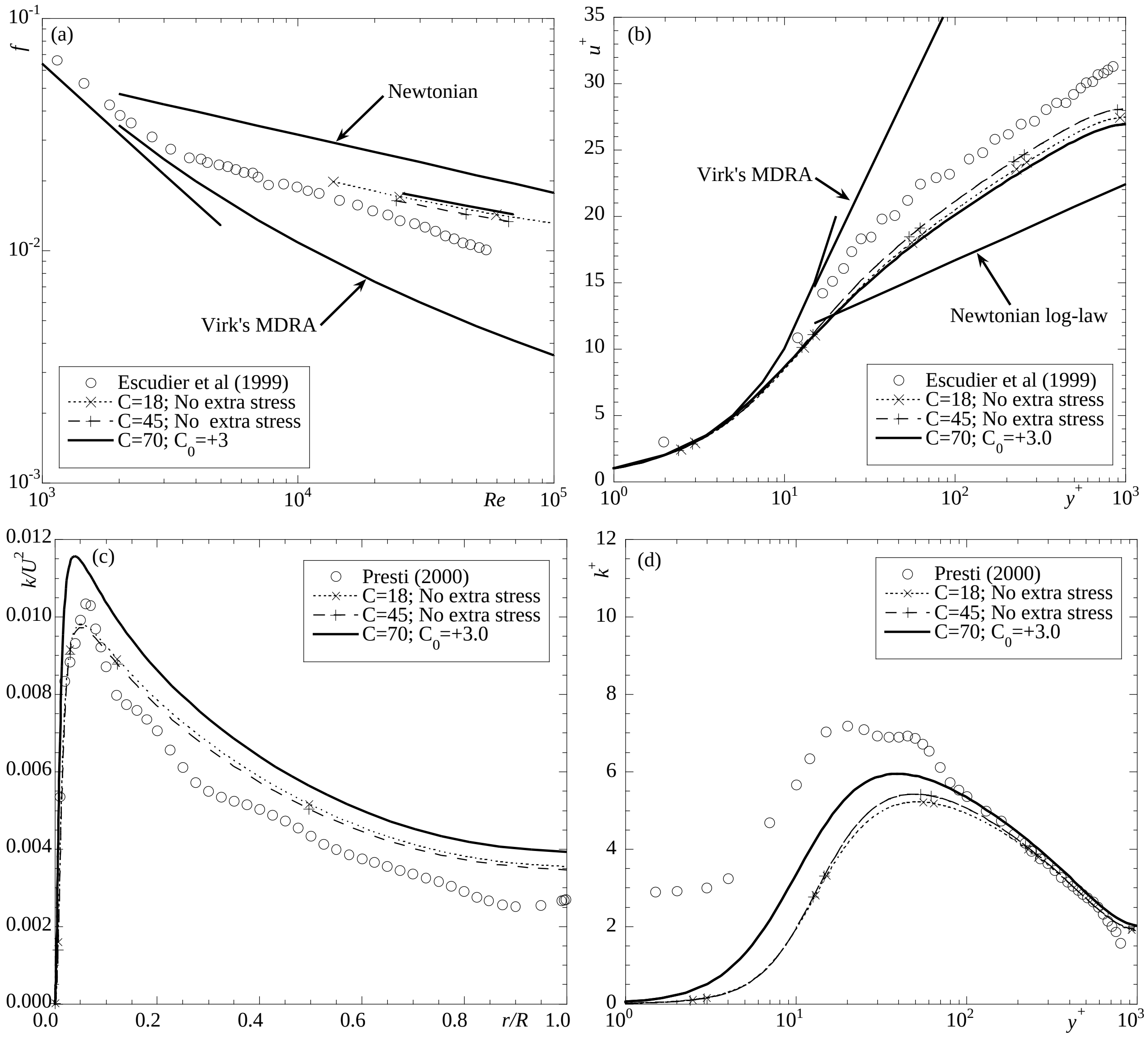

Figure 4- Predictions of flow of 0.09%/0.09% CMC/XG at *Re*= 45,300 with the model with new stress (C= 70 and C_0= +3.0) and comparisons with model of Cruz and Pinho (2003) (old: C= 18 and C_0= +0.0), (corrected: C= 45 and C_0= +0.0) and experimental data: (a) friction factor; (b) u^+ versus y^+; (c) k/U^2 versus *r/R*; (d) k^+ versus y^+.

Finally, for the 0.2%XG (not shown here for space limitations) net improvements are also observed, but the comparison is less favourable than for the other fluids, as already in Cruz and Pinho (2003). Nevertheless, with the new model, predictions for 0.2% XG are now closer to experiments than in Cruz and Pinho (2003).

5. Conclusions

A turbulence model for drag reducing viscolastic fluids was improved by inclusion of a new stress term previously neglected by Cruz and Pinho (2003) in the momentum and turbulent kinetic energy equations. The new stress accounts for the correlation between fluctuations of viscosity and fluctuations of the rate of deformation and a model was developed here for the new stress in order to close the full set of equation. However, prior to testing this new turbulence model, the original value of K_e for 0.125% PAA used by Cruz and Pinho was corrected and this lead to an increase in *C* to 45 for improved predictions.

Calibration of the new turbulence against experimental data for a solution of 0.125% PAA gave *C*= 70 in the damping function f_μ and C_0= +3.0 for the new stress closure. The model was then tested by comparison with experimental data from other fluids. The advantage of this new model is that it has improved the distribution of turbulent kinetic energy, and especially of k^+, while decreasing only slightly the quality of the predictions for the other quantities (*f* and u^+). However, in some case k/U^2 was overpredicted and this requires future modifications. These improvements were found to be general, happening with all fluids tested.

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