HEAT TRANSFER IN THE RE-ATTACHMENT REGION OF A BACKWARD-FACING STEP FLOW

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Abstract. *The characteristics of a turbulent boundary layer near the reattachment point behind a backward-facing step flow are studied here. The emphasis is on providing experimental data that can be used to validate a previous formulation of the problem advanced by Cruz and Silva Freire(IJHMT, 41, 2097–2111, 1998). Two different flow visualization techniques are used to find the location of the separation point. Data for the mean velocity and mean temperature are then presented for the non-recirculation flow region. A data reduction technique based on the inverse problem method is used to furnish predictions of the local skin friction coefficient and Stanton number. All results are also compared with the data of Vogel and Eaton(J. Heat Transfer, vol. 107, 922–929, 1985).*

Key words: *Turbulence, backward-facing step flow, re-attachment, heat transfer*

1. INTRODUCTION

Boundary layer separation from a solid surface is certainly one of the most important and difficult problems of fluid mechanics. Most of the theoretical attempts at describing separated flows have resorted to asymptotic methods. Despite the fair success these methods have achieved, most real significant advances on the problem have come from experimental studies. The complexities associated with the short scales near points of separation or reattachment are difficult to overcome by any means so that the physical behaviour of such flows has been by and large constructed through phenomenologycal models.

In recent years, the present authors have dedicated a good parcel of their time to the development of asymptotic theories for the description of turbulent boundary layers near a seperation point. In a series of articles, both the problems of a velocity boundary layer and of a temperature boundary layer heading for separation have been studied. The works have appealed to the matched asymptotic theory and to Kaplun limits to propose a new changeable asymptotic structure to the flow near a separation point. Basically, the asymptotic structure of the flow is supposed to undergo some modifications reducing from the classical two-deck structure to an one deck structure.

All of our results in the past have been validated through the data of two authors: Vogel and Eaton(1985) and Driver and Siggmiller(1987). These authors have studied the backward-facing step flow geometry. The emphasis was on characterizing simultaneously well the velocity and temperature fields. These authors realized that there was a lack in measurements for the thermal fields and carried out works were the chief concern was describing well all the relevant fluid-dynamic and heat-transfer quantities. In many phenomena, reattaching boundary layers can cause an increase in local heat transfer rate and this effect can certainly not be captured by simple turbulent models based on the turbulent Prandtl number concept. This fact has created a great interest on the physical modelling of the reattaching flows.

The purpose of the present work is twofold: i) to add further experimental evidence to the boundary layer behaviour near to a reattachment point, and ii) to extend the metodology developed in Cruz and Silva Freire(1999) to a theoretical evaluation of the local skin-friction coefficient and Stanton number in the recirculation, reattachment and downstream of reattachment regions.

In the latter part, the wall solutions of Cruz and Silva Freire(1999) are used according to their original formulation. However, the data are reduced using an inverse problem solution procedure.

2. SHORT LITERATURE REVIEW

The difficulties associated with the asymptotic description of turbulent separated flows result in a failure of the classical two-layered structure to represent the flow near a separation point. Basically, the vanishing of the shear stresses at a separation point implies that alternative scaling parameters have to be sought. Important works that have tried to overcome thmany difficulties are the works of Melnik(1989), of Durbin and Belcher(1992) and of Gersten(1987). These works, however, only deal with the velocity field.

In Cruz and Silva Freire(1998) a new asymptotic structure for the flow near a separation point, for both, the velocity and the temperature fields was proposed. In this work, a new scaling procedure was developed which resulted in a changeable asymptotic structure for the boundary layer, different from those of other authors, but consistent with the experimental data. The theory, in particular, led to a new expression for the velocity law of the wall and to a skin-friction equation that were supposed to hold up to the separation point and in the reverse flow region. Also, new expressions were proposed for the temperature law of the wall and for the Stanton number equation.

The same authors, Cruz and Silva Freire(1999) improved the perviously developed theory by makink it more ammenble for a numerical treatment. In relation to the original paper, the following modifications were introduced: 1) a new formulation for the reference velocity, u_R, 2) a new expression for the velocity law of the wall, and 3) a new expression for the temperature law of the wall. The new reference velocity was specified through the total shear stress, as opposed to the previous one which had to be evaluated from an algebraic transcendental equation. The single expression advanced for the velocity law of the wall replaced the three law of the wall expressions (Eqs. 25, 26 and 27) of Cruz and Silva Freire(1998); this expression is supposed to hold in the whole fluid region. The temperature law of the wall is written with the help of reasonably sophisticated expressions for its angular and linear coefficients; these are a function of the turbulent

Prandlt number, the pressure gradient, the reference velocity and the shear stress at the wall. The main consequence of all these modifications was that much better results are found for the prediction of Stanton number near the separation point. This was a particular concern of the present authors when this work started.

3. THEORY

3.1. Velocity law of the wall

Here, the main theoretical developments will be shortly presented. For a complete description of the asymptotic solution of the problem, the reader is referred to Cruz and Silva Freire(1998).

Following the procedure of Cruz and Silva Freire(1999), the law of the wall for a separating flow can be written as

$$u = \frac{\tau_w}{|\tau_w|}\frac{2}{\varkappa}\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dP_w}{dx}y} + \frac{\tau_w}{|\tau_w|}\frac{u_\tau}{\varkappa}\ln\left(\frac{y}{L_c}\right). \tag{1}$$

where

$$L_c = \frac{\sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\nu\frac{dP_w}{dx}u_R} - \frac{\tau_w}{\rho}}{\frac{1}{\rho}\frac{dP_w}{dx}} \tag{2}$$

and all symbols have their classical meaning; $\varkappa$ is the von Kármán constant (=0.4), u_τ is the friction velocity, and u_R ($= \sqrt{\tau}$, τ = total shear stress) is a reference velocity.

The above equation is a generalisation of the classical law of the wall and replaces the three expressions advanced in Cruz and Silva Freire(1998). Far away from the separation point, where the shear stress is positive and $dP_w/dx << \tau_w$, equation (1) reduces to

$$u = \frac{2}{\varkappa}u_\tau + \frac{u_\tau}{\varkappa}\ln\left(\frac{y}{L_c}\right), \quad L_c = \nu/u_\tau, \tag{3}$$

that is, to the classical law of the wall.

Close to the separation point where τ_w=0, the equation reduces to

$$u = \frac{2}{\varkappa}\sqrt{\frac{y}{\rho}\frac{dP_w}{dx}}, \tag{4}$$

Stratford's equation (see Stratford(1959)).

In the reverse flow region where $dP_w/dx >> \tau_w$, equation (1) can be written as

$$u = -\frac{2}{\varkappa}u_\tau - \frac{u_\tau}{\varkappa}\ln\left(\frac{y}{L_c}\right), \quad L_c = 2\left|\frac{\tau_w}{dP_w/dx}\right|. \tag{5}$$

We will now describe how the wall shear stress was evaluated from the above equations for application in a κ-ϵ turbulence model.

The total shear stress can be evaluated from

$$\tau_p = C_\mu^{1/2} \kappa_p + \nu \frac{\partial u}{\partial y}\Big|_p \tag{6}$$

where the subscript p denotes the first grid point.

This equation can be used as a first estimate for the wall shear stress if we consider

$$\tau_{wo} = \frac{u_p C_\nu^{1/4} {\tau_p}^{1/2} \rho \varkappa}{\ln\left(E y \frac{{\tau_p}^{1/2}}{\nu}\right)}. \tag{7}$$

and E=9.8.

The pressure gradient term can be evaluated from

$$\frac{1}{\rho}\frac{dP_w}{dx} = \frac{\tau_p - \tau_w}{y}. \tag{8}$$

Next, the characteristic length can be calculated from

$$L_c = \frac{\sqrt{\left(\frac{\tau_{wo}}{\rho}\right)^2 + 2\nu \frac{dP_w}{dx} u_R} - \frac{\tau_{wo}}{\rho}}{\frac{1}{\rho}\frac{dP_w}{dx}}. \tag{9}$$

Finally, the wall shear stress is calculated from

$$\tau_w = \frac{u_p {\tau_p}^{1/2} \rho \varkappa}{2\sqrt{\left|\frac{\tau_p}{\tau_{wo}}\right|} + \ln\left(\frac{y}{L_c}\right)}. \tag{10}$$

3.2. Temperature law of the wall

An asymptotic theory for the thermal boundary layer near a separation point is also described in some detail in Cruz and Silva Freire(1998). Here, we write the temperature law of the wall as

$$\frac{T_w - T}{Q_w} = \frac{P_{rt}}{\varkappa_t \rho c_p u_\tau} \ln \frac{\sqrt{\tau_w/\rho + \frac{1}{\rho}\frac{dP_w}{dx} y} - \sqrt{\tau_w/\rho}}{\sqrt{\tau_w/\rho + \frac{1}{\rho}\frac{dP_w}{dx} y} + \sqrt{\tau_w/\rho}} + C_q, \tag{11}$$

where

$$C_q = \frac{P_{rt}}{\varkappa_t \rho c_p u_R} \ln \frac{4 E u_R^3}{\nu \left|\frac{dP_w}{dx}\right|} + AJ, \tag{12}$$

$$AJ = 1.11 P_{rt} \sqrt{\frac{A}{\varkappa}} \left(\frac{P_r}{P_{rt}} - 1\right) \left(\frac{P_r}{P_{rt}}\right)^{0.25}, \tag{13}$$

$$A = 26\frac{\tau_w^{1/2}}{u_R}, \quad P_{rt} = 0.9, \tag{14}$$

and all symbols have their classical meaning.

Equation (13) was first proposed by Launder and Spalding(1974). Equation (14) has been modified from the original formulation ($A = 26$) in order to perform better in the separated flow region. In Cruz and Silva Freire(1998) the predicted values of S_t were well below the experimental values so that Eqs. 12 to 14 had to be introduced to rectify that. Please, note that far away from the separation point the standard law of the wall is recovered.

4. EXPERIMENTS

The experiments were performed in the high turbulence wind tunnel located at the Laboratory of Turbulence Mechanics of the Mechanical Engineering Program of COPPE/UFRJ; for the considered test conditions, the free stream level of turbulence was about 2%. The tunnel is an open circuit tunnel with a test section of dimensions 670x670x4000 cm; the test section has an adjustable inclination roof to assure the flows to have a zero pressure gradient.

Measurements were performed for values of the free-stream velocity of 3.12 m/s. Mean velocity profiles and turbulence intensity levels were obtained using a constant temperature hot-wire anemometer. The boundary layer probe was of the type 55P15. A Pitot tube, an electronic manometer, and a computer controlled traverse gear were also used. In getting the data, 10,000 samples were considered which yielded a precision of 0.6% in the mean velocity data.

To obtain accurate measurements, the mean and fluctuating components of the analogic signal given by the anemometer were treated separately. Two output channels of the anemometer were used. The mean velocity profiles were calculated directly from the untreated signal of channel one. The signal given by channel two was 1 Hz high-pass filtered leaving, therefore, only the fluctuating velocity. The latter signal was then amplified with a gain controlled between 1 and 500 and shifted by an offset so as to adjust the amplitude of the signal to the range of the A/N converter.

The mean temperature profiles were obtained through a chromel-constantan microtermocouple mounted on the same traverse gear system used for the hot-wire probe.

A schematic view of the step is shown in Figure 1. The step is 50 mm high and is placed 2.000 mm dowstream of the last of four small-mesh screens. The boundary layer thickness measured 1.800 mm from the screen was 3 cm thick.

The flow visualization was made by two methods: smoke and surface oil. The results are shown in Figures 2 and 3. The position of the reattachment point was evaluated from these figures.

The velocity and temperature mean profiles are shown in Figures 4 and 5 in physical dimensional coordinates. The measurement stations were taken from the step edge. The existence of a logarithmic region is clear from the data. Only the profile taken at $x = 300$ mm does not obey the logarithmic law.

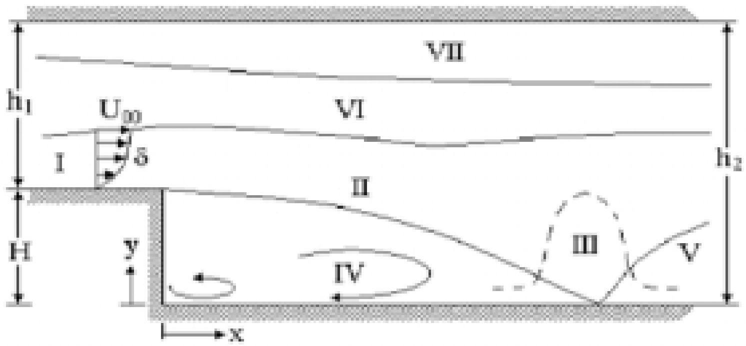

Figure 1: Step geometry.

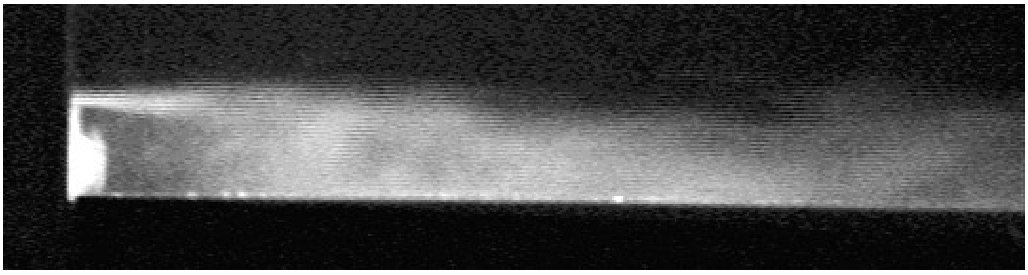

Figure 2: Flow visualization with smoke.

5. Results

For an evaluation of the local skin-friction coefficient and Stanton number, the expressions developed in Cruz and Silva Freire(1999) are used. The three unknowns C_f, S_t and dP_w/dx are estimated through an Inverse Problem Solution Method. While in the direct problem the cause is given and the effect is determined, in an inverse problem an estimation of the cause is obtained through the knowledge of the effect. The solution of the inverse problem used here is obtained throught the Levenberg-Marquardt Method; that is, an iterative method for solving nonlinear least squares problems of parameter estimation (see, e.g., Özisik and Orlande(2000)). The inverse problem is solved by a minimization of the remainder of the least-square norm which can be expressed as

$$S(P) = \sum_{i=1}^{I} (Y_i - K_i(P))^2 \tag{15}$$

where $P = (P_1, P_2, ..., P_N)$ = vector with parameters to be estimated, $K(P) = K(P, y_i)$ = quantity estimated at position y_i, $Y_i = Y(x_i)$ = measured velocity or temperature at position x_i, N = total number of parameters to be estimated, I = total number of measurements. The estimated quantities $K(P)$ are obtained through solution of the direct problem by using a current estimation for the unknown parameters $P_j, j = 1...N$.

The above procedure was validated for the evaluation of C_f through the data of Vogel and Eaton(1985). The flow conditions of Vogel and Eaton are shown in Table 1. The inverse problem technique was applied to equation 1 for the evaluation of C_f.

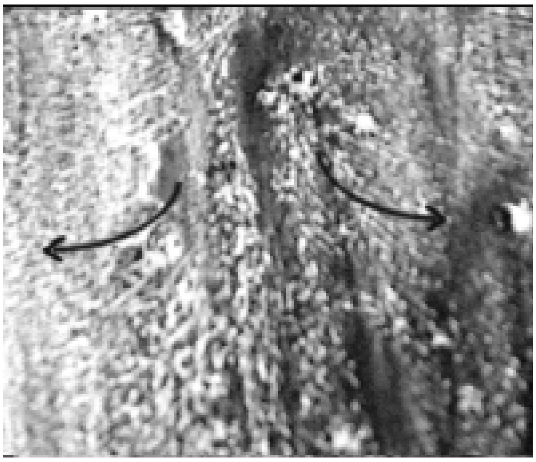

Figure 3: Flow visualization with surface oil.

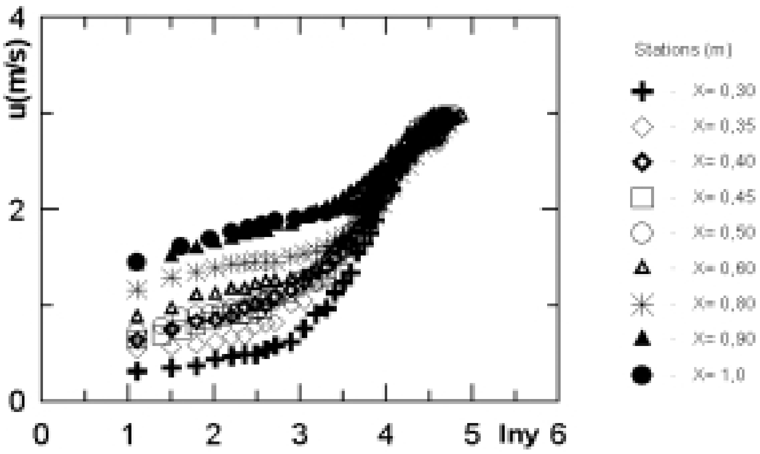

Figure 4: Velocity profile in dimensional coordinates.

Next, the inverse problem technique was applied directly to our data. This time the technique was applied to equations 1 and 11. The results are shown in Figures 6 and 7 and in Table 2

The overall behaviour of C_f and of S_t follows the expected trend. What is known from literature is that the heat transfer rate is low in the recirculation region; approachin the reattachment point it then rises sharply attaining greater values than those typical of a ordinary boundary layer. Downstream, the heat transfer rate relaxes back to the flat plate solution.

Values of C_f and of S_t were also calculated through the integral momentum and energy equations. However, the number of measuring stations proved to be inadequate for the obtaining of reliable results.

Normally, it is expected that the boundary layer relaxes back to a equilibrium state of development several hundreds of step height downstream of the step edge. In our case this would mean at least 5.000 mm. The asymptotic behaviour of our solution does assure that this will occur for the Reynolds analogy is recovered for large values of x.

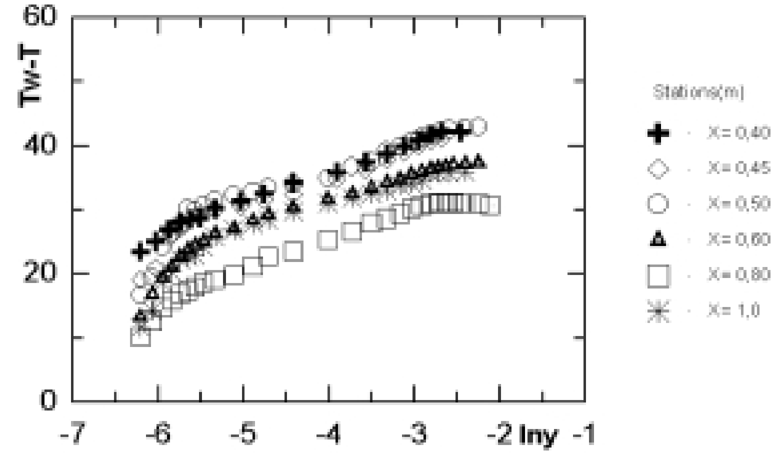

Figure 5: Temperature profile in dimensional coordinates.

Table 1: The experimental data of Vogel and Eaton.

Author	$U[m/s]$	R	$Q_w[W/m^2]$
Vogel and Eaton(1985)	11.3	28000	270
Present	3.12	28000	270

CONCLUSION

The present work had a very distinct goal at its beginning, to provide a robust method for the calculation of flows subjected to separation. Specially, we wanted to improve the calculation methods for skin-friction coefficient and Stanton number which were developed in the past to use the law of the wall.

Apparently, the goal has been achieved with the specification of expressions (1) and (11). These expressions were shown to stand very well against the data of Vogel and Eaton(1985), giving very good results for the velocity and temperature fields and the skin-friction coefficient and the Stanton number.

Presently, the author are subjecting those expressions to further scrutiny. This will be reported in another occasion.

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Table 2: Estimated values of C_f and of S_t.

$Station$	$C_f x10^3$	$S_t x10^3$	dP_w/dx
30	0.068	8.87	0.41
40	0.34	5.63	0.88
45	0.30	5.72	1.07
50	0.50	4.54	0.21
60	0.70	3.52	0.06
80	1.05	3.98	0.01
90	1.43	3.14	0.01
100	1.482	4.72	0.02

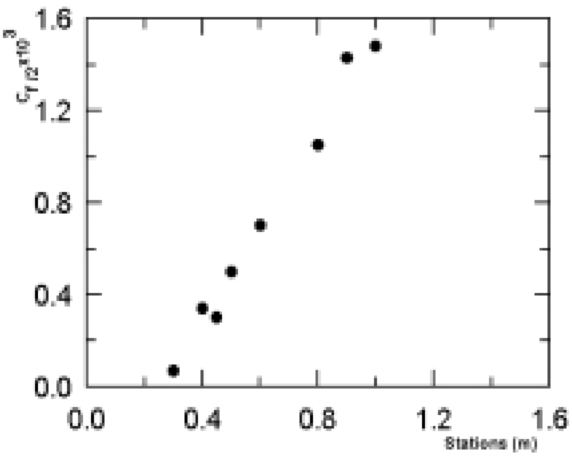

Figure 6: Skin friction coefficient results.

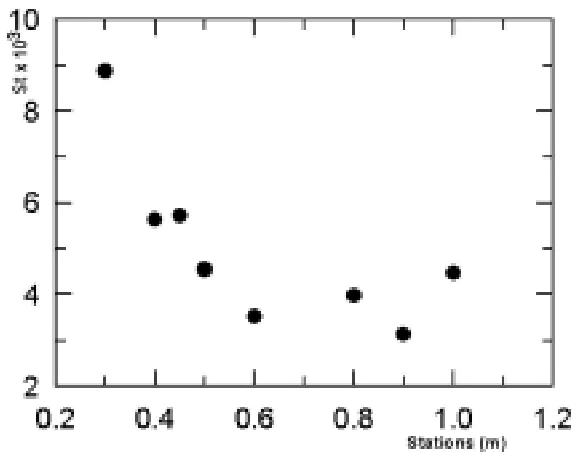

Figure 7: Stanton number results.

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