CAPACITY DEGRADATION OF FINNED-TUBE HEAT EXCHANGERS DUE TO TUBE-TO-TUBE HEAT TRANSFER

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Abstract. *A simplified mathematical model allows the simulation of the heat transfer process of air flowing in between two parallel fins of a finned-tube heat exchanger. The model is based on the assumption of fully developed flow, being conservative in the sense that it does not account for the increased heat transfer coefficient achieved by the developing flow within the entrance length. The heat transfer from the air to the fin surfaces is modeled via a volumetric heat generation term dependent on the heat transfer coefficient between air and fin surface, and on the air-to-fin local temperature difference. Numerical results from simulating the heat transfer process of a general 4×3 finned-tube, air-water heat exchanger, indicate the existence of strong tube-to-tube heat transfer along the fin surface. This effect accounts for 20 % of the heat exchanger capacity and it is stronger among neighboring tubes presenting very different temperatures. Thin adiabatic layers placed along the fin surface are shown to be very efficient in interrupting the tube-to-tube heat transfer. A preliminary simplified analysis to estimate the tube-to-tube heat transfer effect, suggested as a design tool, is satisfactorily tested against the numerical results.*

Keywords: *finned-tube heat exchanger, capacity degradation, air-cooling/dehumidifying, zeotropic mixture*

1. INTRODUCTION

Finned-tube heat exchangers are used extensively in air conditioning and refrigeration. Among the factors affecting the thermal performance of a heat exchanger are geometry, material, air and coolant flow rates, and thermodynamic conditions of the air and of the coolant.

Physically, the difference between the incoming air temperature and the fin surface temperature is the primary (driving) parameter for the heat transfer process. The heat transfer process takes place along the fin to the coolant that runs in tubes perpendicular to the fin. When the coolant is a simple substance changing phase along the tubes, the coolant temperature is in general assumed uniform and constant (the coolant temperature in fact varies slightly as the coolant flows through a tube because of the pressure drop caused by the viscous friction).

There are, however, several heat exchanger designs in which the coolant temperature varies appreciably along the tubes. This happens, for instance, when the coolant is a simple substance not changing phase (e.g., liquid water) or when the coolant is a zeotropic mixture (zeotropic mixtures do not share the isothermal phase change characteristic of simple substances). In this case, tubes crossing the same fin can be carrying coolant at different temperatures. This difference in temperature might drive a reasonable amount of energy from one tube to another through the fin surface, bypassing the air. Our objective is to develop an efficient model for simulating the heat transfer process taking place in between two fins, and, with this model, investigate the occurrence of tube-to-tube heat transfer and the effect of this heat transfer on the global capacity of the heat exchanger.

2. FINNED-TUBE HEAT EXCHANGER: MATHEMATICAL MODEL

Consider a section between two parallel fins of a finned-tube heat exchanger as shown in Fig. 1. The closely spaced fins, H units high, L units deep and s units apart, resemble a narrow flow passage between two parallel plates. We consider the cooling and possible condensation process along the fins assuming a uniform coolant temperature within each tube (the tubes are perpendicular to the fin surface). Observe that the coolant temperature can vary significantly from tube to tube as in the case of sub-cooled liquid water – or any simple substance not changing phase – or a zeotropic refrigerant mixture being used as the coolant. We assume the top ($y = H$) and bottom ($y = 0$) of the heat exchanger covered by adiabatic solid surfaces forming a rectangular channel.

The continuity and momentum conservation equations for the airflow, considering the air as a Newtonian fluid with constant and uniform properties, are:

$$\nabla \mathbf{v}' = 0 \tag{1}$$

$$\rho(\mathbf{v}' \cdot \nabla)\mathbf{v}' = -\nabla p + \mu \nabla^2 \mathbf{v}' \tag{2}$$

where $\mathbf{v}' = u'\,\mathbf{i} + v'\,\mathbf{j} + w'\,\mathbf{k}$, ρ is the air density and μ is the air dynamic viscosity. Some geometric parameters, the Cartesian coordinates and the corresponding velocity components are shown in Fig. 1.

Assuming fully developed velocity profile in the z-direction (perpendicular to the fins), we can write: $u' = u\,c(z)$, $v' = v\,c(z)$, $w' = \partial p/\partial z = 0$. These approximations are generally valid for closely spaced plates ($s << (L,H)$) when the estimated entrance length x_e is much smaller than (L,H). With the fully developed assumption, Eq. (1) and (2) can be integrated from $z = 0$ to $z = s$. The resulting expressions, with $\mathbf{v} = u\,\mathbf{i} + v\,\mathbf{j}$ and $\nu = \mu/\rho$, are:

$$\nabla \mathbf{v} = 0 \tag{3}$$

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{s}{\rho I_2}\nabla p + \frac{\nu I_1}{I_2}\nabla^2 \mathbf{v} + \frac{I_3}{I_2}\nu \mathbf{v} \tag{4}$$

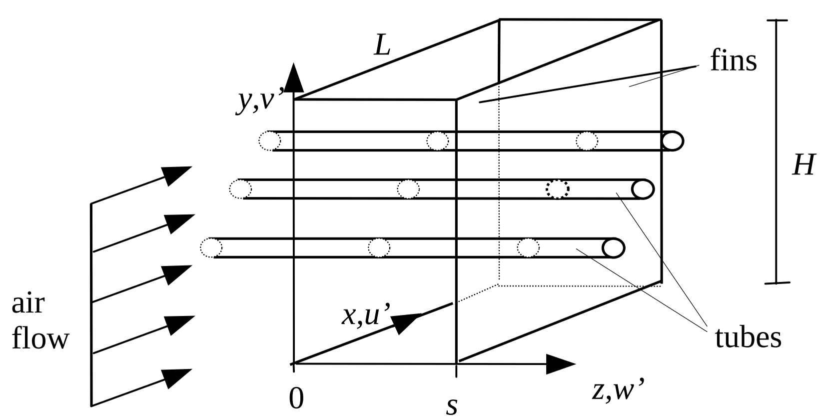

Figure 1 - Air flow channel between two parallel fins with perpendicular tubes.

Notice that a pseudo two-dimensional model is obtained. The three-dimensional effects are accounted for in the model by the last term of Eq. (4) representing the lumped viscous diffusion effect along both fin surfaces. The task now is to determine the new quantities:

$$I_1 = \int_0^s c(z)dz \quad , \; I_2 = \int_0^s c^2(z)dz \quad , \text{ and } \quad I_3 = \int_0^s \frac{d^2c(z)}{dz^2}dz \tag{5}$$

by finding a suitable expression for $c(z)$. In a fully developed laminar flow $c(z)$ is obtained from the Poiseuille solution, leading to

$$c(z) = 6\left[\frac{z}{s} - \left(\frac{z}{s}\right)^2\right] \tag{6}$$

There follows that: $I_1 = s$, $I_2 = 6s/5$, and $I_3 = -12/s$. Observe that for laminar flow with uniform inlet velocity u_{in}, the entrance length can be estimated from $x_e = 0.026\ s^2 u_{in}/\nu$ (Sparrow, 1955) and compared with the dimension L of the heat exchanger.

The momentum formulation Eq. (3)-(6), with a certain inlet velocity profile, $u(0,y)$ and $v(0,y)$ known, no-slip condition at the solid top and bottom surfaces and zero diffusion at the outlet ($\partial u/\partial x = \partial v/\partial x = 0$), is then complete.

After replacing the laminar-flow integral values in Eq. (3) we obtain

$$(\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{5}{6\rho}\nabla p + \frac{5\nu}{6}\nabla^2\mathbf{v} + \frac{10}{s^2}\nu\mathbf{v} \tag{7}$$

Our approach in modeling the heat transfer process starts by writing the energy equation for the air flowing in between the two parallel fins,

$$(\mathbf{v}'\cdot\nabla)T = \alpha\nabla^2 T - \frac{q'''}{\rho c_p} \tag{8}$$

where T is the air temperature, α is the air thermal diffusivity ($\alpha = k/\rho c_p$), and q''' represents the heat-flux extracted from the air stream by the fin surfaces (along x and y) per unit of fin-distance s. Assuming negligible the air temperature variation along the z-direction (perpendicular to the fin surface), and invoking the fully developed velocity profile, Eq. (8) can be integrating from 0 to s taking into consideration that q''' and T are not function of z,

$$(\mathbf{v}\cdot\nabla)T = \alpha\nabla^2 T - \frac{q'''}{\rho c_p} \tag{9}$$

The sink energy term q''' depends on the heat flux across the air-fin interface. This heat flux can occur along dry-fin regions or along wet-fin regions (in the case of condensation). In the first case q''' is represented by

$$sq''' = h_s(T - T_w) \tag{10}$$

where h_s is the local heat transfer coefficient for a dry-fin region (sensible heat) and T_w is the local fin-surface temperature.

In general the heat transfer coefficient between fluid and surface decreases as the flow develops (see for instance the variation in heat transfer coefficient at the entrance of a pipe with slug flow in Bejan, 1995, p.124). Recalling that the thermal entrance length for air, with $Pr \sim 1$, is of the same order as the momentum entrance length, and that the momentum entrance length is generally small in the case of finned-tube heat exchangers, we can be conservative and neglect the entrance effect considering the thermally fully developed case only. In laminar fully developed flow between parallel plates ($Re < 2 \times 10^3$, where $Re = u_{in}D/\nu$, and the equivalent hydraulic diameter $D = 4A/p$, where A is the cross section area and p is the wetted perimeter), the heat transfer coefficient is simply 7.54 k/D for isothermal plates and 8.24 k/D for isoflux plates. In our case, the plates (fins) are neither isothermal nor under an isoflux condition. Conservatively, we use $h_s = 7.54\ k/D$.

For wet-fin regions, we have

$$sq''' = h_c\left(T - T_w\right) \tag{11}$$

where h_c is the local heat transfer coefficient for a wet fin (Rose, 1989),

$$h_c = 0.654\frac{k_L}{\left(\nu_L x\right)^{1/2}}\left(u^2 + v^2\right)^{1/4}\left[\frac{1.508\,Pr_L^{3/2}}{\left(Pr_L + Ja\right)^{3/2}} + \frac{Pr_L}{Ja}\left(\frac{\rho\nu}{\rho_L\nu_L}\right)^{1/2}\right]^{1/3} \tag{12}$$

with the subscript L indicating properties at the liquid-phase, x the distance measured from the location where condensation starts, Ja the Jakob number $c_L(T - T_w)/h_{fg}$ with h_{fg} as the latent heat of condensation, and Pr the Prandtl number ν/α.

From Eq. (10) and (11) we observe that the heat sink term depends on the fin (wall) temperature T_w. Therefore, an extra equation must be solved for the heat transfer along the fin (solid). Considering the fin as very thin (thickness equal to t) so that we can neglect temperature variations along the z-direction,

$$0 = k_w \nabla^2 T_w + \frac{s}{t} q''' \tag{13}$$

where k_w is the fin thermal conductivity. The energy equations for air and fin are coupled in the heat source/sink term q''' because this term depends on the local temperature difference between the air and the fin.

The solution procedure starts by solving the momentum equation for the air flow. Once the velocities are found, the energy Eq. (9) for the air can be solved using a guessed fin temperature distribution. With the air temperature distribution, Eq. (13) can be solved and the results used to update the air temperature, keeping iterating between the two energy equations until converged to the final solution. Observe that the air speed is uncoupled from the energy equations (no need to iterate). Another detail: the heat transfer coefficient to be used in the air equation (sink term) depends on the cooling process taking place (sensible or latent), i.e., it depends on the wall temperature as compared to the dew-point of the air.

The heat exchanger capacity is estimated from the energy exchanged between air and fins (equal to the energy absorbed by the coolant flowing through the tubes) calculated from

$$\dot{q} = s\int_A q''' dA \tag{14}$$

where the integral is over the entire heat transfer surface area, i.e., the fin surface area.

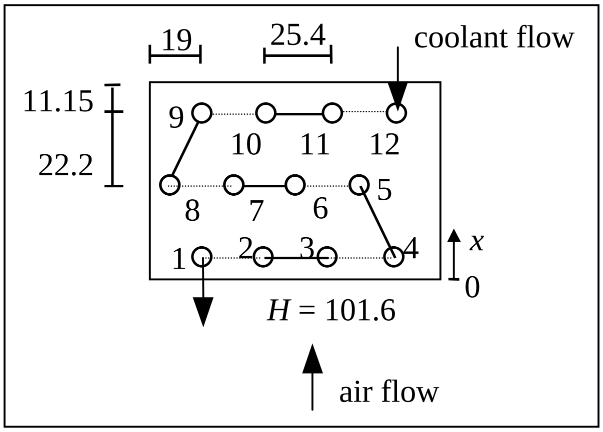

Figure 2 - 4×3 finned-tube, water-air, heat exchanger. (all dimensions are in millimeter)

3. HEAT EXCHANGER CONFIGURATION

Numerical simulations using the mathematical model previously described were performed to provide qualitative information on the impact of tube-to-tube heat transfer on the heat exchanger capacity of a 3×4 air-to-water finned-tube heat exchanger (Fig. 2) studied previously by the Thermal Machinery Group of the Building Environment Division of the NIST. The numerical method is that of control volumes, with details provided elsewhere (Lage, 1997). The numerical results presented here are grid-independent to within 3 %.

The tubes, with external diameter equal to 10 mm, are numbered sequentially from the lower left corner as indicated in the figure, and the air flow is vertical from bottom to top.

The coolant (water) flows through the tubes as shown in Fig. 2 with continuous and dashed lines. The tube temperature values for our simulations, obtained at the NIST, are (in °C), from tube 1 through 12: 19.7, 17.2, 13.7, 11.9, 10.4, 10, 9.8, 10, 8.6, 8.3, 8.1, 8.0.

The inlet air has average flow velocity equal to 1.5 m/s (assumed uniform at the inlet), 27 °C dry bulb temperature and 50 percent relative humidity (air properties, assumed constant, are: k = 0.025 W/mK, $\alpha = 2.08 \times 10^{-5}$ m^2/s, ρ = 1.219 kg/m^3, c_p = 997.5 J/kgK, and Pr = 0.72). The fin spacing is s = 1.8 mm (hydraulic diameter D = 3.5 mm), fin thickness t = 0.20 mm, and fin thermal conductivity k_w = 221.6 W/mK. Observe that the equivalent Reynolds number in this case is 350, so the air flow is laminar and the entrance length is approximately 16 mm, smaller than the depth L of the heat exchanger. The heat transfer coefficient for the dry-fin portion is h_s = 53.86 W/m^{2}K.

4. RESULTS

The air-temperature distribution, obtained numerically, is presented in Fig. 3. We observe that the air temperature distribution is slanted, as a consequence of the temperature decrease as the air flows along the fins (the air inlet temperature is higher than the temperature of the coolant flowing through the tubes). The air saturation temperature (T_{dew} = 15.5 °C) is never achieved in this case (no condensation occurs). The average air temperature at the outlet is 17.309 °C. Adding the energy transferred from the air to the fin throughout the entire domain we get the total heat transferred $\dot{q}$ from air to fin equal to 3.20 W in the present case. This power can be computed independently using the air mass flow rate and the inlet-outlet average air temperature difference, resulting in 3.24 W. The difference between these two values is within the numerical accuracy of our results.