Proceedings of the XII International Symposium on Dynamic Problems of Mechanics (DINAME 2007),
P.S.Varoto and M.A.Trindade (editors), ABCM, Ilhabela, SP, Brazil, February 26 - March 2, 2007

Effect of Boundary Conditions in the Sound Transmission through Double Panel with Poroelastic Core

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Abstract: Porous materials are very important in noise control applications. They are able to dissipate sound energy reducing reverberant noise and improving sound insulation. There are several models available which can be used to predict acoustic performance of porous materials. More simplified models consider the porous medium as an equivalent fluid with effective properties. However, in some porous materials, such as foams, frame motion plays an important role in the sound propagation and they are called poroelastic materials. Often, these kinds of materials are modeled using more complex model obtained from Biot's theory. In this case the three waves which can propagate in the porous medium are considered and their intensity will be directly related to the boundary conditions. In this paper prediction of sound transmission through a double panel with a poroelastic core is presented. The porous material formulation is based on the Allard-Johnson's model which takes into account five fluid-acoustic properties. The results of equivalent fluid model and Biot's model are compared for different boundary conditions. It is shown that these models can be equivalent in some cases and a simpler equivalent fluid model becomes advantageous since no information regarding elastic properties of the frame is required.

Keywords: porous material, sound transmission, boundary conditions

INTRODUCTION

Dynamic behavior of porous materials is quite complex and a theoretical approach is necessary in order to design efficient multilayered systems for sound insulation. A comprehensive porous material model was developed by Biot (1956 and 1962). This model assumes that three waves may propagate in the porous medium. Although there are many situations where one can model porous medium approximately as an equivalent fluid (Allard, 1993), Biot's model has been adopted as a standard model for porous materials in general. It has been observed that it gives better results for flexible structures lined with porous materials (Lauriks et al., 1992, Bolton and Green, 1993, Bolton et al., 1996) and it is specially appropriate for foams. For these materials, the elastic properties of the frame are very important and they are usually called poroelastic materials.

In this paper the two porous material models, poroelastic and equivalent fluid (rigid frame), are used to model a porous material layer between two elastic panels. The transfer matrix method is adopted to compute the sound transmission loss of the multilayered system with distinct layers. This is a general method and it can be used to model large multilayered systems (Allard et al., 1986 and 1987, Lauriks et al., 1992, Brouard et al., 1995). The porous layer is submitted to different boundary conditions and the results are presented for different elastic properties of the porous frame.

POROELASTIC MODEL

Biot's theory yields the following wave equations (Bolton and Green, 1993, Bolton et al., 1996)

$$N\nabla^2\mathbf{u} + \nabla[(A+N)\theta^s + Q\theta^f] = -\omega^2(\widetilde{\rho}_{11}\mathbf{u} + \widetilde{\rho}_{12}\mathbf{U}) \tag{1}$$

$$\nabla[Q\theta^s + R\theta^f] = -\omega^2(\widetilde{\rho}_{12}\mathbf{u} + \widetilde{\rho}_{22}\mathbf{U}) \tag{2}$$

where $\mathbf{u}$ is the vector solid displacement field and $\mathbf{U}$ is the vector fluid displacement field. $\theta^s = \nabla.\mathbf{u}$ and $\theta^f = \nabla.\mathbf{U}$ are volumetric deformations in the phases. $N = E_1/[2(1+\nu)]$ is the shear modulus with E_1 being the *in vacuo* Young's modulus of the bulk solid phase and ν the Poisson ratio; $A = \nu E_1/[(1+\nu)(1-2\nu)]$ is the first Lamé constant; $Q = K_e(\omega)(1-\phi)$ is positive and represents the coupling between the volume change of the solid and that of the fuid with ϕ being the porous material porosity. The frequency-dependent property K_e is the bulk modulus of elasticity of the fuid in the pores that will be presented later. R relates fuid stress and strain and is assumed to equal $\phi K_e(\omega)$.

The parameters ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients that account for the effects of non-uniform relative fluid flow through pores. These coefficients depend on the fluid and solid masses and inertial coupling. Allowance for viscous energy dissipation resulting from the relative motion between the solid and the fuid phases of the porous material results in complex mass coefficients as follows

$$\widetilde{\rho}_{11} = (1-\phi)\rho_s + \rho_a - i\sigma\phi^2 G(\omega)/\omega \qquad \widetilde{\rho}_{22} = \phi\rho_f + \rho_a - i\sigma\phi^2 G(\omega)/\omega \qquad \widetilde{\rho}_{12} = -\rho_a + i\sigma\phi^2 G(\omega)/\omega \tag{3}$$

where ρ_s and ρ_f are the densities of the solid and fluid phases. Frequently, porous materials are characterized in terms of a bulk density $\rho_1 = (1-\phi)\rho_s$. The additional mass ρ_a resulting from intertial coupling is related to the tortuosity α_∞ as follows

$$\rho_a = \phi\rho_f(\alpha_\infty - 1) \tag{4}$$

The term related to viscous dissipation includes the flow resistivity σ and the frequency-dependent function $G(\omega)$ that will be presented later.

Equations (1) and (2) can be rearranged in order to present the following equation (Bolton et al., 1996)

$$\nabla^4\theta^s + A_1\nabla^2\theta^s + A_2\theta^s = 0 \tag{5}$$

Considering harmonic propagation $e^{i\omega t - ikr}$ in Eq. (5), two solutions are possible and their wavenumbers are given by

$$\delta_1^2 = \frac{A_1 - \sqrt{A_1^2 - 4A_2}}{2} \qquad \delta_2^2 = \frac{A_1 + \sqrt{A_1^2 - 4A_2}}{2} \tag{6}$$

where A_1 and A_2 are obtained from

$$A_1 = \frac{\omega^2\left(\widetilde{\rho}_{11}R - 2\widetilde{\rho}_{12}Q + \widetilde{\rho}_{22}P\right)}{PR - Q^2} \qquad A_2 = \frac{\omega^4\left(\widetilde{\rho}_{11}\widetilde{\rho}_{22} - \widetilde{\rho}_{12}^2\right)}{PR - Q^2} \tag{7}$$

where $P = 2N + A$. Thus, two longitudinal waves may propagate in an elastic porous material with distinct wavenumbers. The ratios between the fluid and solid phase displacements μ are given by (Allard, 1993)

$$\mu_i = \frac{U_i}{u_i} = \frac{P\delta_i^2 - \omega^2\widetilde{\rho}_{11}}{\omega^2\widetilde{\rho}_{12} - Q\delta_i^2}, \quad i = 1,2 \tag{8}$$

The wave equations can be also rearranged in another manner yielding (Bolton et al., 1996)

$$\nabla^2\Psi + \delta_3^2\Psi = 0 \tag{9}$$

where Ψ is rotational strain of the solid phase. Equation (9) governs the propagation of the shear wave in the solid phase and its wavenumber is given by

$$\delta_3^2 = \frac{\omega^2}{N}\left(\widetilde{\rho}_{11} - \frac{\widetilde{\rho}_{12}^2}{\widetilde{\rho}_{22}}\right) \tag{10}$$

A quantity μ_3 which represents the ratio of fluid and solid phase displacements can be written as

$$\mu_3 = -\frac{\widetilde{\rho}_{12}}{\widetilde{\rho}_{22}} \tag{11}$$

EQUIVALENT FLUID MODEL

Considering porous material as a fluid with effective properties may be of interest in some situations and for some kinds of porous materials (Bolton and Kang, 1997). Since the porous medium is considered as an equivalent fluid, Helmholtz equation becomes the governing equation. Thus, for an equivalent fluid with effective properties, one can write

$$\nabla^2 p + \omega^2\frac{\rho_e}{K_e}p = 0 \tag{12}$$

where ρ_e and K_e are the effective properties of an equivalent fluid. Equation (12) represents the propagation of a single compressional wave through the porous medium. The wavenumber can be directly related to the effective density ρ_e and the effective fluid bulk modulus K_e by (Allard, 1993)

$$k_e = \omega\sqrt{\frac{\rho_e}{K_e}} \tag{13}$$

The fluid effective density ρ_e in the pores is frequency-dependent and also depends on four porous material macroscopic propreties ϕ, σ, α_∞ and Λ (viscous characteristic length). The Biot-Allard model gives the following expression (Allard, 1993)

$$\rho_e(\omega) = \alpha_\infty\rho_0\left[1+\frac{\sigma\phi}{i\omega\rho_0\alpha_\infty}G(\omega)\right] \tag{14}$$

where,

$$G(\omega) = \sqrt{1+\frac{4i\alpha_\infty^2\mu_0\rho_0\omega}{\sigma^2\Lambda^2\phi^2}} \tag{15}$$

Note that an equivalent fluid model is a particular case of Biot's theory since

$$\widetilde{\rho}_{22} = \phi\rho_e \tag{16}$$

The effective fluid bulk modulus K_e is a frequency-dependent property and also depends on the thermal characteristic length Λ'. The Biot-Allard model presents the following expression

$$K_e(\omega) = \frac{\gamma P_0}{\gamma-(\gamma-1)\left[1+\left[(8\mu_0)/(i\Lambda' N_{pr}\omega\rho_0)\right]G'(\omega)\right]^{-1}}, \tag{17}$$

where,

$$G'(\omega) = \sqrt{1+i\frac{\rho_0\omega N_{pr}\Lambda'^2}{16\mu_0}}, \tag{18}$$

where P_0 is the atmospheric pressure, γ the specific heat ratio, N_{pr} the Prandtl number and μ_0 dynamic viscosity of the fluid in the pores.

The equivalent fluid model can be advantageous in relation to the poroelastic model (Biot's model) since it does not require frame elastic properties. However, this alternative model is only applicable to some cases (Bolton and Kang, 1997). It is also important to mention that the equivalent fluid model presented here is applicable to porous materials with rigid frame. Limp porous materials should include a correction in the effective density ρ_e (Lai et al., 1997, Lai and Bolton, 1998).

TRANSFER MATRIX APPROACH

Figure 1 shows a plane acoustic wave impinging upon a material of thickness h, at an incidence angle θ. Various types of waves can propagate in the material, according to its nature. The geometry of the problem is bidimensional, in the (x,z) plane. A dependency of $e^{-ik_x x}$ is used in the transverse coordinate direction.

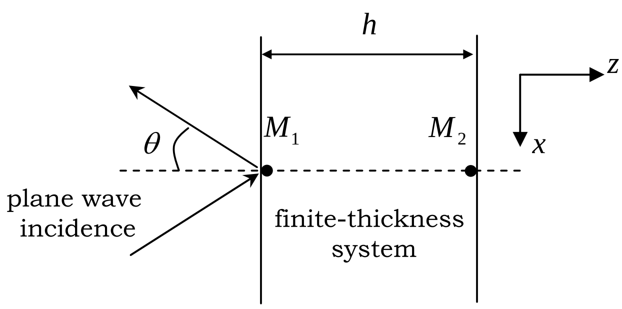


Figure 1 – Plane wave impinging upon finite-thickness system

The x component of the wavenumber for each wave in the finite medium is equal to the x component of the incident wavenumber in the fluid on the incident side of the finite medium. The x component of the wavenumber is given by

$$k_x = \frac{\omega}{c_0} \sin\theta \tag{19}$$

where ω is the angular frequency of the wave, and c_0 is the speed of sound in the fluid. Sound propagation in the layer is represented by a transfer matrix $[T]$ such that

$$V(M_1) = [T]V(M_2) \tag{20}$$

where M_1 and M_2 are two points set close to the forward and the backward face of the layer, respectively, and where the components of the vector $V(M)$ are the variables which describe the acoustic field at a point M of the medium. The matrix $[T]$ depends on the thickness h and the physical properties of each medium.

Fluid layer

This type of layer is primarily used to model air gaps between material layers. The acoustic field is completely defined at any point M by the vector

$$V^f(M) = [p(M) \;\; v_z(M)]^T \tag{21}$$

where p and v_z are the acoustic pressure and the z component of the fluid velocity, respectively.

The transfer matrix of a fluid layer $[T^f]$ with thickness h is given by

$$[T^f] = \begin{bmatrix} \cos(k_z h) & i(\omega\rho_0)/(k_z)\sin(k_z h) \\ i(k_z)/(\omega\rho_0)\sin(k_z h) & \cos(k_z h) \end{bmatrix} \tag{22}$$

This type of layer can also be used to model an equivalent fluid layer. In this case, the fluid density ρ_0 becomes an effective density ρ_e and the wavenumber k becomes an effective fluid wavenumber k_e. Thus, the z component of the wavenumber becomes $k_z = \sqrt{k_e^2 - k_x^2}$.

Solid layer

This type of layer is used to model solid materials in general, i.e., it is not limited to thin solid layers and is applicable to thick layers. In this case, one compressional wave and a shear wave may propagate through the solid medium. The two wavenumbers can be obtained as

$$\delta_1^2 = \frac{\omega^2\rho_s}{\lambda_s + 2\mu_s} \qquad \delta_3^2 = \frac{\omega^2\rho_s}{\mu_s} \tag{23}$$

where ρ_s is the solid density. The first Lamé constant λ_s and the shear modulus and μ_s are given by

$$\lambda_s = \frac{E_s(1+i\eta_s)\nu_s}{(1+\nu_s)(1-2\nu_s)} \qquad \mu_s = \frac{E_s(1+i\eta_s)}{2(1+\nu_s)} \tag{24}$$

where E_s is the Young's modulus, η_s the loss factor and ν_s the Poisson ratio.

The vector of the field variables which describes the acoustic field at any point M of the solid layer is given by

$$V^s(M) = [v_x(M) \;\; v_z(M) \;\; \sigma_z(M) \;\; \tau_{xz}(M)]^T \tag{25}$$

where v_x and v_z are the x and z components of the velocity, respectively, σ_z the normal stress and τ_{xz} shear stress at the point M.

The transfer matrix of a solid layer $[T^s]$ with thickness h can be obtained as follows

$$[T^s] = [\Gamma^s(-h)][\Gamma^s(0)]^{-1} \tag{26}$$

where,

$$[\Gamma^s(z)] = \begin{bmatrix} \omega k_x \cos(k_{1z}z) & -i\omega k_x \sin(k_{1z}z) & i\omega k_{3z}\sin(k_{3z}z) & -\omega k_{3z}\cos(k_{3z}z) \\ -i\omega k_{1z}\sin(k_{1z}z) & \omega k_{1z}\cos(k_{1z}z) & \omega k_x \cos(k_{3z}z) & -i\omega k_x \sin(k_{3z}z) \\ -(\lambda_s\delta_1^2 + 2\mu_s k_{1z}^2)\cos(k_{1z}z) & i(\lambda_s\delta_1^2 + 2\mu_s k_{1z}^2)\sin(k_{1z}z) & 2i\mu_s k_x k_{3z}\sin(k_{3z}z) & -2\mu_s k_x k_{3z}\cos(k_{3z}z) \\ 2i\mu_s k_x k_{1z}\sin(k_{1z}z) & -2\mu_s k_x k_{1z}\cos(k_{1z}z) & -\mu_s(k_x^2 - k_{3z}^2)\cos(k_{3z}z) & i\mu_s(k_x^2 - k_{3z}^2)\sin(k_{3z}z) \end{bmatrix}$$

This 4x4 matrix relates wave amplitudes and field variables in any point of the solid layer along the z direction (Brouard et al., 1995).

Poroelastic layer

This type of layer can be used to take into account all possible waves that may propagate in a porous medium. According to Biot's theory, 3 waves may propagate in a porous elastic medium. The three z components of the wavenumbers can be obtained as

$$k_{jz} = \sqrt{\delta_j^2 - k_x^2} \quad j = 1,2 \text{ and } 3 \tag{27}$$

where the wavenumbers δ_j are given by Eqs. (6) and (10).

The acoustic field at any point M of a poroelastic layer can be described by a vector V^p as follows

$$V^p(M) = [v_x^s(M) \;\; v_z^s(M) \;\; v_z^f(M) \;\; \sigma_z^s(M) \;\; \tau_{xz}(M) \;\; \sigma^f(M)]^T \tag{28}$$

which contains six independent acoustic quantities: the x and z components (v_x^s) and (v_z^s) of the frame velocity, the z component (v_z^f) of the fluid velocity, two components (σ_z^s) and (τ_{xz}) of the frame stress tensor, and the component (σ^f) of the fluid stress tensor.

The transfer matrix of a poroelastic layer $[T^p]$ with thickness h can be obtained as follows

$$[T^p] = [\Gamma^p(-h)][\Gamma^p(0)]^{-1} \tag{29}$$

where,

$$[\Gamma^p(z)] = \left[\begin{matrix} \omega k_x \cos(k_{1z}z) & -i\omega k_x \sin(k_{1z}z) & \omega k_x \cos(k_{2z}z) \\ -i\omega k_{1z}\sin(k_{1z}z) & \omega k_{1z}\cos(k_{1z}z) & -i\omega k_{2z}\sin(k_{2z}z) \\ -i\omega\mu_1 k_{1z}\sin(k_{1z}z) & \omega\mu_1 k_{1z}\cos(k_{1z}z) & -i\omega\mu_2 k_{2z}\sin(k_{2z}z) \\ -[(P+\mu_1 Q)\delta_1^2 - 2Nk_x^2]\cos(k_{1z}z) & i[(P+\mu_1 Q)\delta_1^2 - 2Nk_x^2]\sin(k_{1z}z) & -[(P+\mu_2 Q)\delta_2^2 - 2Nk_x^2]\cos(k_{2z}z) \\ 2iNk_x k_{1z}\sin(k_{1z}z) & -2Nk_x k_{1z}\cos(k_{1z}z) & 2iNk_x k_{2z}\sin(k_{2z}z) \\ -(\mu_1 R + Q)\delta_1^2\cos(k_{1z}z) & i(\mu_1 R + Q)\delta_1^2\sin(k_{1z}z) & -(\mu_2 R + Q)\delta_2^2\cos(k_{2z}z) \end{matrix} \right.$$

$$\left. \begin{matrix} -i\omega k_x \sin(k_{2z}z) & i\omega k_{3z}\sin(k_{3z}z) & -\omega k_{3z}\cos(k_{3z}z) \\ \omega k_{2z}\cos(k_{2z}z) & \omega k_x \cos(k_{3z}z) & -i\omega k_x \sin(k_{3z}z) \\ \omega\mu_2 k_{2z}\cos(k_{2z}z) & \omega\mu_3 k_x \cos(k_{3z}z) & -i\omega\mu_3 k_x \sin(k_{3z}z) \\ i[(P+\mu_2 Q)\delta_2^2 - 2Nk_x^2]\sin(k_{2z}z) & 2iNk_x k_{3z}\sin(k_{3z}z) & -2Nk_x k_{3z}\cos(k_{3z}z) \\ -2Nk_x k_{2z}\cos(k_{2z}z) & -N(k_x^2 - k_{3z}^2)\cos(k_{3z}z) & iN(k_x^2 - k_{3z}^2)\sin(k_{3z}z) \\ i(\mu_2 R + Q)\delta_2^2\sin(k_{2z}z) & 0 & 0 \end{matrix} \right]$$

This 6x6 matrix relates wave amplitudes and field variables in any point of the poroelastic layer along the z direction (Brouard et al., 1995).

GLOBAL TRANSFER MATRIX

A global transfer matrix can be obtained by combining the transfer matrices of each layer and interface matrices between layers. Consider a sound transmission problem as illustrated in Fig. (2).

As an example, the acoustic field at point A can be related to the acoustic field at the point M_2 by the following equation

$$[I_{f1}]V^f(A) + [J_{f1}][T^{(1)}]V^{(1)}(M_2) = 0 \tag{30}$$

where $[I_{f1}]$ and $[J_{f1}]$ are the interface matrices that depend on the nature of the layers. These matrices will be presented later.

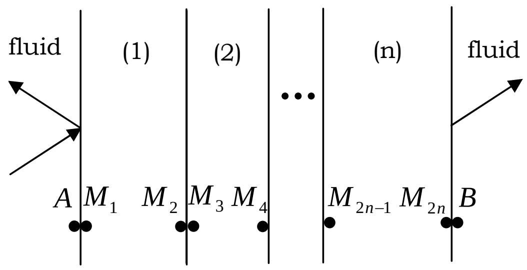


Figure 2 – Sound transmission through multilayered system

This procedure can be extended to various layers. For $k = 1, 2, \dots, n-1$ one can write

$$[I_{(k)(k+1)}]V^{k}(M_{2k}) + [J_{(k)(k+1)}][T^{(k+1)}]V^{(k+1)}(M_{2(k+1)}) = 0 \tag{31}$$

This set of equations can be written in a form

$$[D_0]V_0 = 0 \tag{32}$$

where $V_0 = [V^f(A) \;\; V^{(1)}(M_2) \;\; \cdots \;\; V^{(n-1)}(M_{2n-2}) \;\; V^{(n)}(M_{2n})]^T$.

Another equation can be used to relate a point near the free surface in the layer n to point B in the transmission side. Thus, the vector $V^f(B)$ and the vector $V^{(n)}(2n)$ of the last layer are related by

$$[I_{(n)f}]V^{(n)}(M_{2n}) + [J_{(n)f}]V^f(B) = 0 \tag{33}$$

where $V^f(B) = [p(B) \;\; v_z^f(B)]^T$.

Also, the impedance Z_B of the fluid at the point B is related to the characteristic impedance of the fluid Z_c as follows

$$Z_B = p(B)/v_z^f(B) = Z_c/\cos\theta \tag{34}$$

or,

$$[-1 \;\; Z_B]V^f(B) = 0 \tag{35}$$

A new set of equations can be obtained inserting these equations into Eq. (32) yielding

$$[D]V = 0 \tag{36}$$

where,

$$[D] = \begin{bmatrix} & [D_0] & [0] \\ [0] & [I_{(n)f}] & [J_{(n)f}] \\ 0 & \dots \;\; 0 & [-1 \;\; Z_B] \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} V_0 \\ V^f(B) \end{bmatrix} = \begin{bmatrix} V^f(A) \\ V^{(1)}(M_2) \\ V^{(2)}(M_4) \\ \vdots \\ V^{(n-1)}(M_{2n-2}) \\ V^{(n)}(M_{2n}) \\ V^f(B) \end{bmatrix} \tag{37}$$

Sound transmission loss

A relationship between the reflection factor r and the transmission factor t is given by

$$\frac{p(A)}{1+r} - \frac{p(B)}{t} = 0 \tag{38}$$

The addition of this new equation to Eq. (36) yields

$$\begin{bmatrix} t \;\; 0 \;\; \dots \;\; -(1+r) \;\; 0 \\ [D] \end{bmatrix} V = 0 \tag{39}$$

The determinant of this matrix is null and, thus, the surface impedance Z_A at the point A is given by

$$Z_A = -\frac{\det[D_1]}{\det[D_2]} \tag{40}$$

where $\det[D_1]$ is the determinant obtained when the first column of the matrix $[D]$ is eliminated and $\det[D_2]$ the determinant when the second column is eliminated. As a result, the reflection factor r can be computed as

$$r = \frac{Z_A \cos\theta - Z_0}{Z_A \cos\theta + Z_0} \tag{41}$$

The transmission factor t is calculated as

$$t = -(1+r)\frac{\det[D_{N+1}]}{\det[D_1]} \tag{42}$$

where $\det[D_{N+1}]$ is the determinant obtained when the $(N+1)th$ column of the matrix $[D]$ is eliminated. Finally, the sound transmission loss TL is given by

$$TL = -10\log(\tau) = -10\log(|t|^2) \tag{43}$$

Note that τ represents the sound transmission coefficient.

INTERFACE MATRICES

In the transfer matrix method the boundary conditions are imposed by interface matrices. These matrices are therefore presented below for solid, fluid, equivalent fluid and poroelastic layers interfaces.

Solid-fluid interface

$$[I_{sf}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [J_{sf}] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad [I_{fs}] = [J_{sf}] \text{ and } [J_{fs}] = [I_{sf}]$$

Solid-equivalent fluid interface

$$[I_{sef}] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [J_{sef}] = \begin{bmatrix} 0 & -1 \\ 1/\phi & 0 \\ 0 & 0 \end{bmatrix}, \quad [I_{efs}] = [J_{sef}] \text{ and } [J_{efs}] = [I_{sef}]$$

Solid-poroelastic interface

$$[I_{sp}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad [J_{sp}] = -\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad [I_{ps}] = [J_{sp}] \text{ and } [J_{ps}] = [I_{sp}]$$

Poroelastic-fluid interface

$$[I_{pf}] = \begin{bmatrix} 0 & 1-\phi & \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad [J_{pf}] = \begin{bmatrix} 0 & -1 \\ 1-\phi & 0 \\ 0 & 0 \\ \phi & 0 \end{bmatrix}, \quad [I_{fp}] = [J_{pf}] \text{ and } [J_{fp}] = [I_{pf}]$$

CONFIGURATIONS AND PROPERTIES

Three different lining configurations were used in order to investigate the relevance of the elastic properties of a porous layer in a double wall system. Figure (3) illustrates these configurations which impose different boundary conditions to the porous layer. In the configuration 1, the two $1mm$ aluminium panels are bonded to a porous material layer which is $20mm$ thick. For configuration 2, one face of the porous layer is lightly uncoupled from the panel and a very thin air gap will appear ($0.1mm$ fluid layer). In the configuration 3, the two porous faces are uncoupled from the two panels and two air gaps are formed.

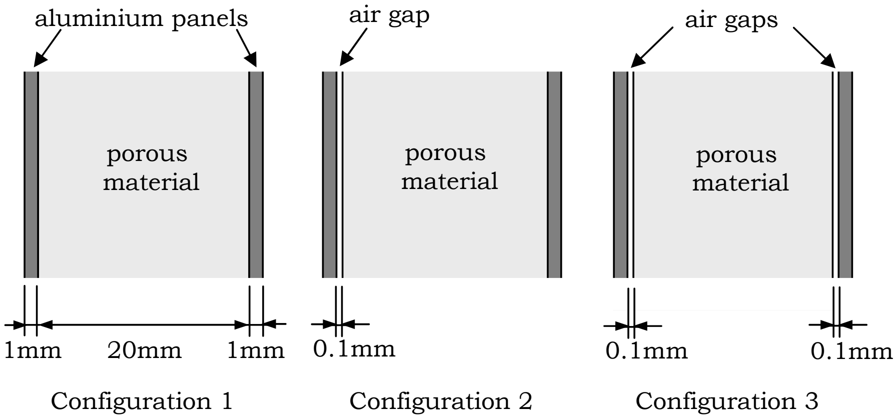


Figure 3 – Configurations for double panel lining

The properties of porous materials used for sound insulation can vary over a wide range. However, in this paper, the elastic properties of the porous frame are the focus of analysis. These properties are the Young's bulk modulus E_1, the Poisson ratio ν and the loss factor η. In this study only Young's bulk modulus E_1 and Poisson ratio ν of the porous frame are allowed to vary and other porous material properties remain the same. The effect of the loss factor of the porous material is not shown. However, it is known that an increase in the damping tends to smooth the transmission loss curve since frame waves are more attenuated.

A typical foam used for noise control purposes may present bulk density $\rho_1 = 40kg/m^3$, loss factor $\eta = 0.2$, porosity $\phi = 0.95$, flow resistivity $\sigma = 20\ 10^3 rayls/m$, tortuosity $\alpha_\infty = 2$ and characteristic lengths $\Lambda = \Lambda' = 100\mu m$. These properties were adopted for the porous layer in all analyses.

The fluid is air with density $\rho_0 = 1.21kg/m^3$, sound speed $c_0 = 342m/s$, dynamic viscosity $\mu_0 = 1.85\ 10^{-5}Ns/m^2$, specific heat ratio $\gamma = 1.4$, Prandtl number $N_{pr} = 0.73$ and atmospheric pressure $P_0 = 1.0132\ 10^5 N/m^2$. The solid panels are made of aluminium with $E_s = 70GPa$, $\eta_s = 0.01$, $\rho_s = 2700kg/m^3$ and $\nu_s = 0.33$.

RESULTS

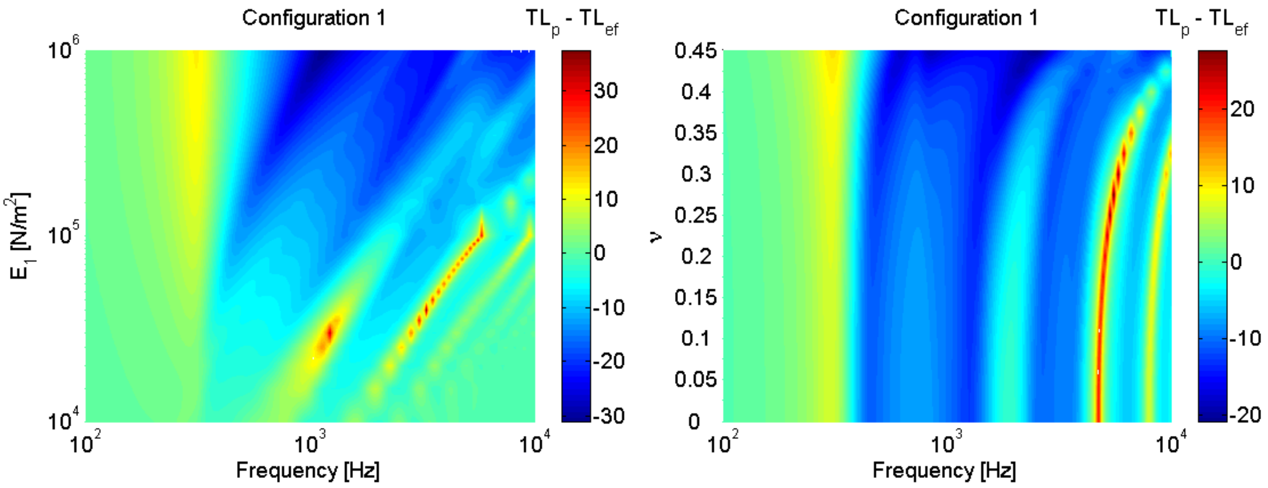
Only the poroelastic model accounts for the elastic properties of the porous frame. Therefore, a direct comparison to an equivalent fluid model for different elastic properties provides important information regarding the frame waves relevance. Figures (4), (5) and (6) show the results for the configurations 1, 2 and 3, respectively. Results are the difference between the transmission loss obtained with a poroelastic model for the porous layer (TL_p) and the transmission loss obtained with an equivalent fluid model for the porous layer (TL_{ef}). The transmission loss is calculated for oblique incidence ($\theta = 60^0$) in order to also excite shear waves in the porous frame. All calculations were carried out from 10^2Hz to 10^4Hz.

Subfigures (a) shown in the Figs. (4), (5) and (6) consider the variation of E_1 for $\nu = 0.3$ and subfigures (b) present the effect of the variation of ν for $E_1 = 10^5 N/m^2$. The limits of variation which were adopted for E_1 are $10^4 N/m^2$ and $10^6 N/m^2$ since most of porous materials will be inside this range. Following the same idea, the Poisson ratio was allowed to vary up to 0.45.

It can be verified by comparing Figs. (4), (5) and (6) that the range of variation of $TL_p - TL_{ef}$ is large for configuration 1 and small for configuration 3. For example, the difference between models is lower than 1.1dB for configuration 3. This clearly shows that frame waves are more important when the porous frame is directly excited by a solid interface.

For all configurations there is a low frequency region where models present very similar results. This region ends around 300Hz where it occurs the mass-stiffness-mass resonance. This frequency mainly depends on the panels masses and the stiffness of the air between the two panels. For configuration 1, where frame is directly bonded to the two panels, frame stiffness is also important and an increase in E_1 shifts this resonance to higher frequencies.

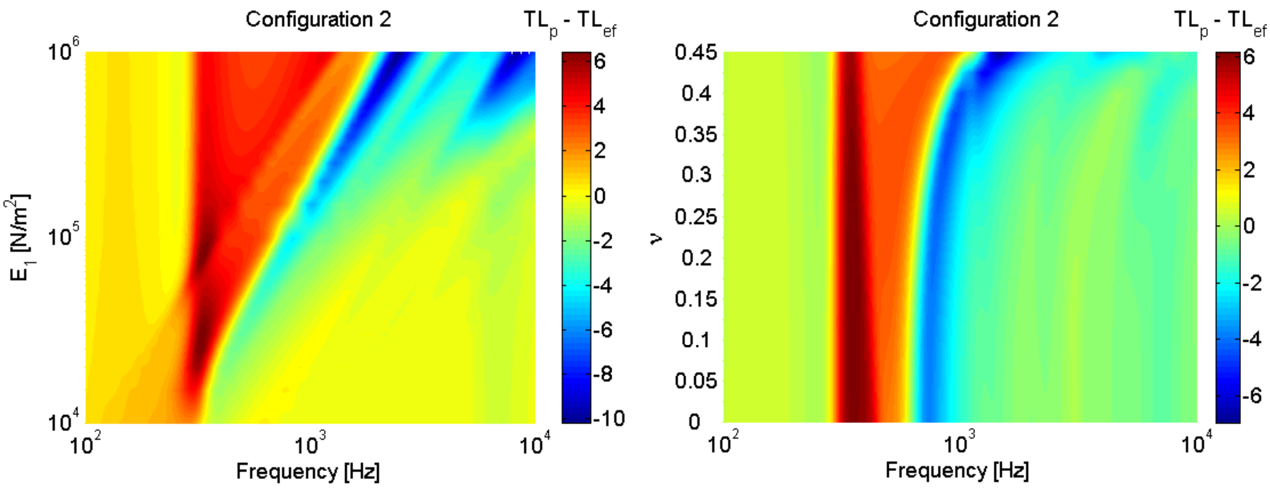
From a global point of view, it can also be observed in Figs. (4.a), (5.a) and (6.a) that models lead to closer results as



(a) Young's bulk modulus effect for $\nu = 0.3$

(b) Poisson ratio effect for $E_1 = 10^5 N/m^2$

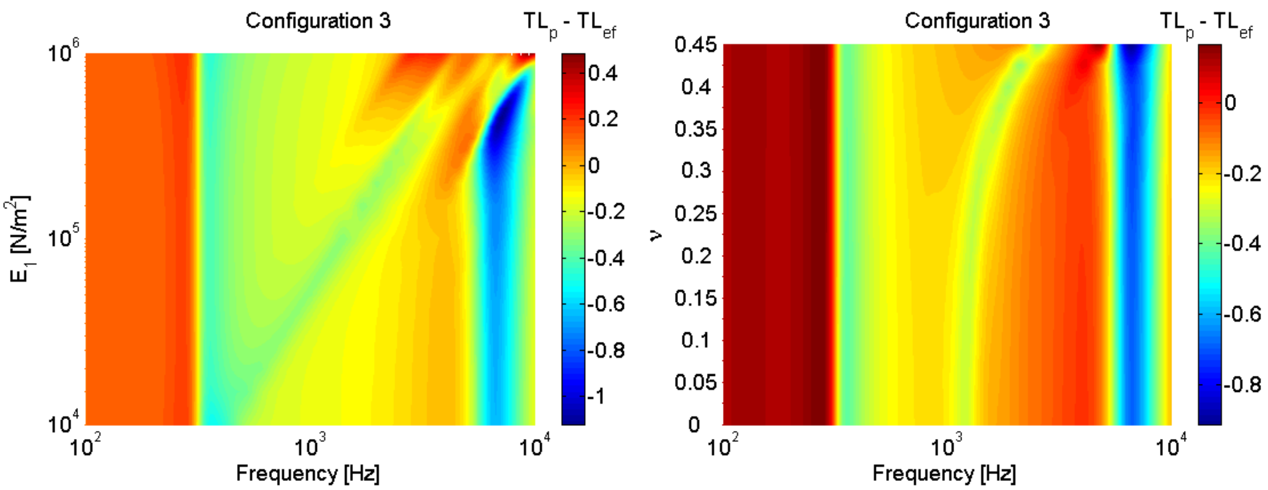
Figure 4 – Difference in the TL of the models for configuration 1



(a) Young's bulk modulus effect for $\nu = 0.3$

(b) Poisson ratio effect for $E_1 = 10^5 N/m^2$

Figure 5 – Difference in the TL of the models for configuration 2



(a) Young's bulk modulus effect for $\nu = 0.3$

(b) Poisson ratio effect for $E_1 = 10^5 N/m^2$

Figure 6 – Difference in the TL of the models for configuration 3

Young's bulk modulus is reduced. The only exception is found for configuration 3 as shown in Fig. (6.a). In this case a very rigid frame ($E_1 \cong 10^6 N/m^2$) provides smaller difference (there is no dark blue region) between models.

The effect of the Poisson ratio ν is similar for all configurations. It can be observed that variations in relation to the TL_{ef} caused by frame waves are shifted to higher frequencies as ν is increased. This fact is clearly seen for configuration 1 where elastic properties are more relevant as shown in the red region in Fig. (4.b).

CONCLUSIONS

Poroelastic and equivalent fluid models were described in detail and adopted to model a porous layer between two solid panels. Different boundary conditions for the porous layer were imposed by interface matrices and the relevance of the elastic properties of the frame was investigated for each case. The transfer matrix method was also described and applied to obtain the sound transmission loss through the multilayered systems. It is shown that the results of the two models are similar when the porous material layer is not directly connected to a solid interface (configuration 3). This means that, in sound transmission problems, porous layers can be modeled as an equivalent fluid when they are not bonded to vibrating solid surfaces. Besides, for very limp porous materials, the influence of boundary conditions becomes lower and models also present similar results. This indicates that acoustic performance of porous materials tends to be independent of boundary conditions for low stiffness frames.

It is interesting to highlight that acoustic performance of porous materials is dependent of several properties. In this paper only elastic properties of the frame were considered. However, inertial and viscous coupling between phases can also be important. Light porous materials with high flow resistivity and tortuosity present strong coupling and the fluid phase may excite the frame with high intensity. As a result, frame motion becomes more evident and its elastic properties will influence more strongly acoustic performance even when frame is not directly excited by a solid interface. Future work will address this topic.

ACKNOWLEDGMENTS

The authors wish to acknowledge the financial support of the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil.

REFERENCES

Allard, J. F., 1993, Propagation of Sound in Porous Media - Modelling sound absorbing materials, Elsevier Applied Science, New York.

Allard, J. F., Bourdier, R. and Depollier, C., 1986, "Biot waves in layered media", Journal of Applied Physics, Vol. 60(6), pp. 1926-1929.

Allard, J. F., Champoux, Y. and Depollier, C., 1987, "Modelization of layered sound absorbing materials with transfer matrices", Journal of the Acoustical Society of America, Vol. 82(5), pp. 1792-1796.

Biot, M. A., 1962, "Generalized theory of acoustic propagation in porous dissipative media", Journal of the Acoustical Society of America, Vol. 34(5), pp. 1254-1264.

Biot, M. A., 1956, "Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid: I. Low-Frequency Range II. High-Frequency Range", Journal of the Acoustical Society of America, Vol. 28, pp. 168-191.

Bolton, J. S. and Kang, Y.J., 1997, "Porous materials for sound absorption and transmission control", SAE Congress, 971878.

Bolton, J. S. and Green, E. R., 1993, "Normal Incidence Sound Transmission through Double-Panel Systems Lined with Relatively Stiff, Partially Reticulated Polyurethane Foam", Applied Acoustics, Vol. 39, pp. 23-51.

Bolton, J. S., Shiau, N. M. and Kang, Y. J., 1996, "Sound transmission through multi-panel structures lined with elastic porous materials", Journal of Sound and Vibration, Vol. 191, pp. 317-347.

Brouard, B. , Lafarge, D. and Allard, J.F., 1995, "A general method of modeling sound propagation in layered media", Journal of Sound and Vibration, Vol. 183(1), pp. 129-142.

Lai, H. Y., Katragadda, S., Bolton, J.S. and Alexander, J. H., 1997, "Layered fibrous treatments for sound absorbing and sound transmission", Proceeding of SAE Congress, pp. 1553-1560.

Lai, H., and Bolton, J.S., 1998, "Optimization of acoustical systems incorporating fibrous elements for sound absorption", Noise-Con, pp. 265-270.

Lauriks, W., Mees, P. and Allard, J.F., 1992, "The acoustic transmission through layered systems", Journal of Sound and Vibration, Vol. 155(1), pp. 125-132.

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