# CURVATURE IDENTIFICATION IN A PRISMATIC JOINT OF A ROBOT USING GYROSCOPES BASED DUAL QUATERNIONS PROCESSING 

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Abstract. This paper shows a solution to measure the curvature of a rail-guide in which a robot manipulator moves. Such robot, denominated Roboturb, is destined to welding tasks in rotors of high power hydraulic turbines damaged by cavitation effect. This robot has as mainly characteristics the fact of presenting small load (total mass of 20kg), high mobility level due to its seven degrees of freedom and, it has one of the joints in the form of a curvilinear rail with undefined geometry. The influence that this rail curvature impose on paths executed by the robot is analyzed. Tthis is decisive to Roboturb operate because without rail coordinates knowledge is impossible any precision movement. The principles of solution includes a way to obtain 1-axis angular displacement from angular rate gyroscope information; a formulation to derive complete 3-axis angular orientation from two gyroscopes only; and a algorithm to represent the rail profile using dual quaternions. At the end some practical results are show. The implementation was developed in Labview software, and has used two technological kinds of gyroscopes: FOG (fiber optics gyros) and crystal micromachined gyroscopes. With a laser pointer installed on the robot hand and fixed in a target, the robot runs along rail. So, with the profile information (acquired with the measurement system proposed) was possible minimize the orientation error to 40 arc-seconds maximum value.

Keywords: robotic, quaternion, gyroscope, curvature, rail

## 1. Introduction

Hydraulic turbines of high power present erosion due to the cavitation phenomenon and need to be repaired through material deposition by welding process. The manual repair operation is troublesome because there is not enough working space for the welder. Thereby, it has been developed, through a partnership project among Electric Power Company of Paraná (COPEL) and the Federal University of Santa Catarina (UFSC), a robot manipulator capable to realize welder task. This project, named Roboturb, already has a prototype shown in Fig. 1b. With this robot the deposition process will be optimised. A laser sensor will be added in the system to measure and restore the geometric form of the blades.


Figure 1. Roboturb: (a) installed over Francis turbine and (b) a close view

Due to the reduced space between adjacent blades in the turbine's rotor (Fig. 1a), it was necessary to develop a small load robot with high mobility, especially to reach points of difficult access. An articulated robotic manipulator with 7

DOF seems to be the most appropriate solution for this case (Guenther, Simas and Pieri, 2000), showed in Fig. 1b. At the same time, because of the great planar extension of a turbine blade, the robot needs to travel in all cavited areas. To achieve that, the robot is installed on a flexible rail (as guided for the base carriage of Roboturb), fixed to the turbines blade by suckers. That allow the rail molding to the complex curvature of the blade (Kapp, Martin and Dos Santos, 2002). This means that the rail profile will change at each new installation, and it is necessary to measure the rail profile in a fast and highly precise way (Janschek and Dos Santos, 2003). Some measurement requirements and condition others important condictions contours are shown in Table 1 in relationship with Fig. 2.

Table 1. Measurements requirements and conditions contours

| End-effector positioning uncertainty | $<1 \mathrm{~mm}$ |
| :--- | ---: |
| Total length of robot arm | $<1,0 \mathrm{~m}$ |
| Angular uncertainty (on the rail) | $<0,06^{\circ}$ |
| Max. velocity (along the rail) | $0,1 \mathrm{~m} / \mathrm{s}$ |
| Max. acceleration | $1 \mathrm{~m} / \mathrm{s}^{2}$ |
| Length of rail | $\sim 1,0 \mathrm{~m}$ |
| Interval between measurements (in rail) | $0,4 \mathrm{~mm}$ |
| Payload | 2 kg |



Figure 2. Top view of robot arm

A previous analysis to choose the most appropriate sensor (which has included shape-tape, tilt-sensor, rotary encoder and gyroscope) has proved that the gyroscope is the best choice for this measurement system (Dos Santos, 2003). Such gyroscopes will be installed in the carriage of the robot, and has to have at least, the capacity to sense angular rates in $+/-10 \%$ s with an uncertainty not greater than $+/-0,026 \%$. In this paper we will make a first approach of the real complete solution considering the base robot carriage guided by means of four wheels disposed in two axes, like shown in Fig. 3b below.


Figure 3. Robot carriage: (a) real case with six wheels and (b) approximation for initial analyze

In the real case robot carriage have six wheels disposed in three axes (Fig. 3a previous), therefore some additional herein not treated considerations, would be necessary. This approximation makes it possible to analyze any rail situation in which their curvature is restricted to have constant behavior (for example like shown in Fig. 10 ahead).

## 2 Angular displacement from an angular rate gyroscope

The first principle of solution consists in deriving the 1 -axis angular displacement from the angular rate gyro information. Ideally a gyro is a sensor which returns the angular velocity around the axis sensor. But, in practice, the outputs have some bias and noise added to the signal. Figure 4a shows an ideal gyroscope, while the yellow block of Figure 4 b shows a real gyroscope.
(a)

(b)


Figure 4. Gyroscopes models: (a) Ideal gyroscope; (b) Real gyroscope with an output integrator
In order to get the mechanical angular displacement around the sensing axis, an integrator has to be added to the gyro output (see Figure 4b). Unfortunately, bias and noise are integrated together with the real mechanical displacement as well and, leading for an error in the estimation of this angle occurs. Another uncertainty factor, that will occur when using gyroscopes, is the unknowledge of the Earth angular rate vector. In this case the Earth angular rate will be embeded in the output gyroscope signal.

In the Matlab-Simulink diagram at Fig. 5, a simulation of this principle is showed. In such diagram the bias and the Earth angular rate are treated as uncertainty which will be added to the generated noise. So, assuming a null rate in the input (gyroscope motionless), is possible to evaluate the angular measure uncertainty will be added. In the showed case, the numerical values are specific to a piezoeletric crystal gyroscope. For this gyroscope the total uncertainty added at the end of 12 seconds measurement period was $0,6^{\circ}$, accordingly illustrated in Fig. 6 below.


Figura 5. MATLAB-Simulink diagram for procedure simulation


Figura 6. MATLAB-Simulink graphs obtained by simulation: (a) noise, (b) angle after integration

## 3. Formulation to get the orientation using two gyroscopes

A formulation to get a complete 3-axis angular orientation, using only two gyros, is possible if some mechanical constraints are introduced (Janschek and Dos Santos, 2003). Due the fact that the rail has flexibility for only twist and bend angles (respectively $\varphi$. and $\theta$ angles), the orthonormal angle $\psi$ is a consequence of the previous two angles, according to the following approximations referenced to the Figures 7 and 8.


Figure 7. Angles in rail: (a) bend and (b) twist


Figure 8. Infinitesimal approximation of the rail profile
Firstly for small angles the tangent value coincides with the angle (in radians), then :

$$
\begin{equation*}
\tan \Delta \varphi=\Delta \varphi=\frac{\Delta y}{\Delta x} \tag{1}
\end{equation*}
$$

where $\Delta y=\Delta z \cdot \sin (\Delta \psi)$; but, because is further possible to consider $\Delta z \cong \Delta s$, and that for small angles the sine value coincides with the angle (in radians), this equation becomes:
$\Delta y=-\Delta s . \Delta \psi$

And, in a similar way it is possible to compute $\Delta x$ as follows:
$\Delta x=\Delta s . \Delta \theta$

Then, with (3) and (2) in (1):
$\Delta \psi=-\Delta \varphi \cdot \Delta \theta$

Means that $\psi$ depends only upon $\varphi$ and $\theta$.

## 4. Dual quaternion computation

To understand the computation applied, is necessary in first some dual quaternion review. Quaternions is a method to describe spatial rotations firstly introduced by Hamilton (1853). They are generalized numbers using four units, the ordinary real number and three additional units $i, j$ and $k$ that satisfy the next relations:

$$
\begin{align*}
& \mathbf{i}^{2}=\mathbf{j}^{2}=\mathbf{k}^{2}=-1  \tag{5}\\
& \mathbf{i . j}=-\mathbf{j} \mathbf{i}=\mathbf{k}  \tag{6}\\
& \mathbf{j} \cdot \mathbf{k}=-\mathbf{k} \cdot \mathbf{j}=\mathbf{i}  \tag{7}\\
& \mathbf{k} . \mathbf{i}=-\mathbf{i} . \mathbf{k}=\mathbf{j} \tag{8}
\end{align*}
$$

A generic quaternion has the form

$$
\begin{equation*}
\mathbf{q}=q_{1}+q_{2} \cdot \mathbf{i}+q_{3} \cdot \mathbf{j}+q_{4} \cdot \mathbf{k} \tag{9}
\end{equation*}
$$

where $q_{1}$ is the real part, and $q_{2}, q_{3}$ and $q_{4}$ form a three dimensional spatial complex part. In order to represent a 3D rotation, a quaternion will be formed by the following vector:

$$
\mathbf{q}=\left[\begin{array}{c}
\cos (\Phi / 2)  \tag{10}\\
e_{1} \cdot \sin (\Phi / 2) \\
e_{2} \cdot \sin (\Phi / 2) \\
e_{3} \cdot \sin (\Phi / 2)
\end{array}\right]
$$

where $\mathbf{e}\left(e_{1}, e_{2}, e_{3}\right)$ is the unit vector pointing to main axis of rotation (in a Euler formulation), and $\Phi$ is rotation angle around the same axis. This is the best algorithm way to rotation modeling (Dos Santos, Langer and Souza, 2004).

Additionaly, the dual numbers was first proposed by Clifford (1873), and after improved by Study (1891). A dual number is mathematically expressed as:

$$
\begin{equation*}
\stackrel{v}{z}=a+\varepsilon . b, \tag{11}
\end{equation*}
$$

where $a$ and $b$ are real numbers $(a, b \in \mathfrak{R}), \varepsilon$ is the dual operator. This operator is similar to the $\boldsymbol{i}$ operator in complex numbers, but instead of $\varepsilon$ is subject to the following rule:

$$
\begin{equation*}
\varepsilon^{2}=0 \tag{12}
\end{equation*}
$$

And, where must be in mind that:

$$
\begin{align*}
& \varepsilon \neq 0  \tag{13}\\
& 0 . \varepsilon=\varepsilon .0=0 \text { and }  \tag{14}\\
& 1 . \varepsilon=\varepsilon .1=1 \tag{15}
\end{align*}
$$

A important property of dual numbers is associated to the function application in which arguments are dual numbers. Due the fact that all potencies of $\varepsilon$ greater than unit become null, the Taylor series expansions makes equal to:

$$
\begin{equation*}
f(a+\varepsilon . b)=f(a)+\varepsilon . b . f^{\prime}(a) \tag{16}
\end{equation*}
$$

Dual quaternions are a generalized class of dual numbers where quaternions are used instead of real numbers in order to represent space lines with Plücker coordinates (Shevlin, 1989). The general form of a dual quaternion is:

$$
\begin{equation*}
\stackrel{\vee}{\mathbf{q}}=\mathbf{q}_{a}+\varepsilon \mathbf{q}_{b} \tag{17}
\end{equation*}
$$

where $\varepsilon$ is the dual operator, $\mathbf{q}_{a}$ is the quaternionic expression for a pure rotation presented in the spatial displacement, while $\mathbf{q}_{b}$ is the dual part quaternion expressing the translation portion of the same displacement considering the main axis of rotation (Daniilidis, 1999).

By using dual quaternions is it easy to represent spatial displacements (Perez and McCarthy, 2002). Besides, there is still possible to define transformations rules as simple as that used to represent rotations (Goddard and Abidi, 1998). Through this relations is possible to realize a dual quaternion computation for position and orientation for a rail based robot using two gyros. The following block-diagram in Fig. 9 shows this algorithm:


Figure 9. Block-diagram to compute position and orientation
Where in this block-diagram $\Delta \mathrm{q}$ is the quaternion for small angles, $\Delta t_{r}$ is the current incremental displacement computed; $\Delta \stackrel{\vee}{\mathbf{q}}$ is the dual quaternion for incremental displacements (position and orientation); $\stackrel{\vee}{q}_{n}$ and $\stackrel{\vee}{\mathbf{q}}_{n-1}$ are respectively actual and previous dual quaternion computations; and $\otimes$ is the rotational product (a particular kind of quaternion multiplication operator) defined by Dos Santos (2003).

## 5. Results from algorithm application

The effectiveness of the curvature identification measurement system proposed was then, realized by practical tests and statistical analysis applied to the measured data get from the two gyroscopes while the robot runs over a constant curvature surface (Dos Santos, 2003). The applied tests approach included the following items:
a. Roboturb rail was installed on a circular surface in a way to promote twist and bend angles;
b. A repetition of tests using various gyroscope technologies both to measure twist and bend angles;
c. Variations in the sampling methods by:
i. Acquisition with motion in two rail directions,
ii. Acquisition with motion in three different velocity levels and
iii. Acquisition using 12 and 16 bit resolution.

Thus, a curved surface was conformed by means of a steel plate, in a semi-circular shape. Over this surface the Roboturb rail-arm set was then installed as showed in the next picture.


Figure 10. Picture of Roboturb installed on a curved surface
The particular mode in which the rail was mounted over the plate (axially tilted), originated both twist and bend angles. To prevent undesirable influence factors such as different gravitational forces, the arm was always hold in the same posture during all the time of data acquisition.

Three gyroscope models were used in the tests. One gyroscope of the fiber optic technology type model E-Core RA2030, from KVH Inc, and two other gyroscopes using piezoelectric crystal technology models GyrochipII and Horizon, from Bei Systron Donner make.

The signals obtained from the gyroscopes were filtered with a 20 Hz low-pass RC anti-alias filter. The sampling rate used was 50 Hz while the robot carriage was running along the rail in three different velocities: 10,50 and $100 \mathrm{~mm} / \mathrm{s}$.

### 5.1 Graphic visualization

After a statistical treatment on the collected data (a lot of 2160 total points), and considering a probability density fusion between applied gyroscopes, was possible to evaluate the robot carriage orientation along the whole trajectory executed with a uncertainty less than 0,6 arc-min.; this value was computed by normalized metrologic evaluation methods. This result was also computed by considering all the measurement set collected when in a motion of $50 \mathrm{~mm} / \mathrm{s}$, independently of direction, bits of resolution, and gyroscope model. After that, for this analyzed profile, the graphical curvature among the 1000 mm rail length, after a treatment in a dedicated computer graphical routine, was presented in the next illustrated views, shown in Fig. 11.


Figure 11.3D view of curved rail acquired with the measurement system

### 5.2 Orientation characteristic for a course near to the rail axis

In order to get a performance evaluating of the applied measurement system using gyroscopes, there was been developed an inverse kinematics algorithm for the Roboturb (also based in dual quaternion method instead of traditional matricial form), for the compensate rail curvature (Dos Santos, 2003). With this algorithm is possible to realize a test looking at orientation characteristic curve on to the robot end-effector, for a motion with parallel course to the rail axis, more specifically for a motion with course parallel to the Z axis.

This kind of trajectory tends to maintain constant X and Y target coordinates, and has as peculiarity the fact that the movements in the rotations joints are exclusively generated by the rail compensation algorithm part. The graphic in Fig. 13 allows to visualize the angular deviations of pursued target, by a test realized with a laser emission pointer installed at the robot end-effector (like show in Fig. 12) at same position showed in Fig. 10.


Figure 12. Test to obtain orientation characteristic using laser pointer looking at a target


Figure 13. Angular target deviation among all rail course

In 0.9 meter Z-axis trajectory the maximum angular orientation error was 38 arcmin. That first result encourage to proceed with using gyroscopes to measure rail curvature. It's important to observe that in the conditions when this test was executed a precision robot calibration was not maked yet. Of course a lot of anothers errors source like belt elasticity or control resolution for example will be influence in the errors level too.

Although this result was obtained considering a carriage simplification to four wheels (like discussed around Fig. 3), the complete solution for six wheels follows the same principles however some additional geometric algorithms make it yet necessary. On the other hand the complete solution is perfectly applicable in similars navigations problems where a vehicle runs over irregular tracks.

## 6. Conclusions

The developed measurement system for curvature identification in a prismatic joint of Roboturb using two gyroscope based dual quaternion processing have shown good results. By using a infinitesimal analysis it was possible to obtain a formulation to achieve 3D orientation using only two gyroscopes instead. The gyroscopes measure angular rate, however they present noise that increase the uncertainty in the orientation estimation. In this sense the bias instability is the major source of errors, although the repeatability in the measured angular rate was not possible be better than the Earth angular rate if this angular axis vector is ignored (case of Roboturb). For the proposed measurement system was possible to compensate robot end-effector trajectory deviations along rail parallel axis course with accuracy better then 40 arc-min.

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