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OPTIMIZATION OF VEHICLE SUSPENSION USING ROBUST ENGINEERING METHOD AND RESPONSE SURFACE METHODOLOGY

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Abstract. *Traditional suspension tuning for ride comfort takes use of a series of physical prototype evaluations by skilled drivers, who analyze the vehicle performance in subjective terms. In this approach, the suspension components (springs, shock absorbers, bumpers, etc) are usually optimized one at a time, regardless of the consequences of the interactions among them in the global suspension behavior. Besides, the costs and construction lead times can not be afforded in the current tight development cycles, and due to its subjective nature, even with extremely skilled drivers, it is not possible to assure that their evaluation is exempt enough. This paper presents an objective approach, based on computer tools. It takes use of a vehicle dynamics simulation tool, used to analyze the vehicle behavior in different road conditions, and optimization tools to select the components. Two optimization tools are studied, Robust Engineering Method and Response Surface Methodology (RSM). Both methods show that, under current simulation conditions, some of the suspension components do not have high influence on ride comfort, so they can be selected by other criteria, such as handling criteria or cost. RSM, unlike Robust Engineering Method, also produce an empirical model that can be used to study parameter interactions.*

Keywords*: vehicle dynamics, suspension, optimization, ride comfort, design of experiments.*

1. Introduction

The use of simulation tools in the development cycle of vehicles is becoming more and more common in the automotive industry, since they are capable of reducing the lead-time for a new model launch and also helping to reduce the development costs. In the case of ride comfort, much has been made in the last years to make better and more reliable vehicle dynamics simulation tools. However, this progress in the simulation tools capacity and precision has not always been followed by an increase use of these tools in the suspension tuning, specially because of the subjectivity nature of the ride comfort evaluation, which makes difficult the development of criteria to correlate the dynamic variables (accelerations, forces, velocities, etc) that can be obtained by the simulation packages and the ride comfort evaluation grades. This problem has already been treated at General Motors do Brasil (GMB), by Vilela, Franceschini and Mesquita (2002), and the next point that deserves special attention is the optimization technique to be adopted. The simulation software, described in Vilela (2003) calculates dynamic variables of the vehicle riding a prescribed road surface, and they are used to calculate the evaluation grade of the simulated suspension, and several suspension configurations can be evaluated in this way. The optimization problem is then finding the best configuration. In this paper two approaches are studied, robust engineering (Phadke, 1989; Ross, 1988) and response surface engineering (Myers and Montgomery, 2002; Myers et al., 2004).

2. Description of the optimization problem

The modeled vehicle is a passenger vehicle with a Mc Pherson front suspension and semi-independent rear suspension with twist beam. Details of the model can be seen in Vilela (2003) and in Vilela and Gueler (2003), . In the present work, the following suspension parameters can be adjusted:

- Stiffness of the stabilizing bar, front suspension.
- Stiffness curve of the front spring.
- Stiffness curve of the rear spring.
- Front shock absorber curve.
- Rear shock absorber curve.
- Stiffness curve of the front bumper.
- Stiffness curve of the rear bumper.
- Rear tire pressure.

Usually the ride comfort is studied for one specific load condition, and, after adjusting the suspension parameters reaching a satisfactory result, this suspension configuration is checked in other load conditions (up to full payload) to see if the results are adequate. This procedure does not lead to an optimal configuration concerning all the load range, and as it is not possible to forecast how the customer will load the vehicle. So the optimization problem is adjusting the suspension parameter that leads to optimum comfort considering that the vehicle load changes within certain limits. The comfort is evaluated by the grade as calculated by the method described in by Vilela, Franceschini and Mesquita (2002), and higher grade means better comfort. In the two optimization approaches studied in this work, robust engineering and response surface method, each simulation is viewed as an "experiment run", and a proper design of experiment needed to be provided in order to accomplish the optimization task.

3. Robust Engineering Method

The robust engineering method deals with a parameter optimization considering that certain conditions can not be adjusted by the engineer. These conditions are denominated as "noise variables", and in the present study the noise variable is the vehicle load. The optimization that can be performed through this methodology is a discrete one, i.e. some discrete values for each parameter must be chosen and the robust engineering will point which one of these is the best one, taken the defined noise into consideration. The robust engineering method also deals well with "qualitative factors", for instance, instead of selecting a stiffness value of a spring, these method can be used to select a "kind" of spring. The robust engineering method takes use of balanced experiment arrangements (an orthogonal array) that can indicate the parameters values that give the best results for the evaluation functions in questions (here they are the ride comfort parameters values) without evaluating all the possible combinations of design parameters (factorial array). The literature presents several orthogonal arrays that can be used, many of then empirically developed. In this study the stiffness of the stabilizing bar (front suspension) can be selected between two values, all the other seven parameters can be adjusted by selecting among three values. The adopted orthogonal array is a L18 array, which results in 18 simulations ("experimental runs") for each of the two adopted "noise" level. The noise levels are the curb (vehicle with full fuel tank and no cargo load) plus driver load and the GVW (gross vehicle weight) load. Table 1 shows the L18 array. In this table (-1) means lower parameter value, (0) means intermediate parameter value and (+1) means higher parameter value.

Table 1. Orthogonal array L18.

Experimental run	Levels of parameters							
	Stiffness of the stabilizing bar	Stiffness curve of the front spring	Stiffness curve of the rear spring	Front shock absorber curve	Rear shock absorber curve	Stiffness curve of the front bumper	Stiffness curve of the rear bumper	Rear tire pressure
1	-1	-1	-1	-1	-1	-1	-1	-1
2	-1	-1	0	0	0	0	0	0
3	-1	-1	+1	+1	+1	+1	+1	+1
4	-1	0	-1	-1	0	0	+1	+1
5	-1	0	0	0	+1	+1	-1	-1
6	-1	0	+1	+1	-1	-1	0	0
7	-1	+1	-1	0	-1	+1	0	+1
8	-1	+1	0	+1	0	-1	+1	-1
9	-1	+1	+1	-1	+1	0	-1	0
10	+1	-1	-1	+1	+1	0	0	-1
11	+1	-1	0	-1	-1	+1	+1	0
12	+1	-1	+1	0	0	-1	-1	+1
13	+1	0	-1	0	+1	-1	+1	0
14	+1	0	0	+1	-1	0	-1	+1
15	+1	0	+1	-1	0	+1	0	-1
16	+1	+1	-1	+1	0	+1	-1	0
17	+1	+1	0	-1	+1	-1	0	+1
18	+1	+1	+1	0	-1	0	+1	-1

From the suspension configuration grades one can calculate the signal-to-noise ratio using Eq. (1).

$$\frac{S}{N} = -10.\log\left[\frac{1}{2}.\left(\frac{1}{L_1^2}+\frac{1}{L_2^2}\right)\right] \tag{1}$$

In Eq. (1), $\frac{S}{N}$ is the signal-to-noise ratio, L_1 is the grade at the curb (vehicle with full fuel tank and no cargo load) plus driver load, and L_2 is the grade at the GVW (gross vehicle weight) load. A higher signal-to noise ratio means a better suspension configuration. Table 2 shows the simulation results.

Table 2: Simulation results.

Experimental run	Grade at "curb + driver" load L_1	Grade at GVW load L_2	Signal-to-noise ratio $\frac{S}{N}$
1	5.9	3.7	12.93
2	5.0	3.5	12.16
3	4.7	3.4	11.81
4	7.0	5.1	15.31
5	6.5	4.9	14.86
6	6.4	4.9	14.81
7	6.9	5.3	15.48
8	6.6	5.3	15.33
9	7.5	5.6	16.05
10	5.5	3.5	12.42
11	5.1	3.6	12.38
12	5.6	3.5	12.46
13	6.7	5.0	15.07
14	6.2	4.8	14.60
15	6.9	4.4	14.40
16	6.5	5.1	15.08
17	7.5	5.6	16.05
18	7.0	5.4	15.63

A signal-to-noise ratio can also be calculated for each parameter level:

$$\frac{S}{N}_{ij} = \frac{1}{n}\sum \frac{S}{N}_{j} \tag{2}$$

The signal-to-noise ratio $\frac{S}{N}_{ij}$ of parameter i at level j is the mean value of the signal-to-noise ratios $\frac{S}{N}_{j}$ of all experimental runs where the value of the i^{th} parameter is at level j.

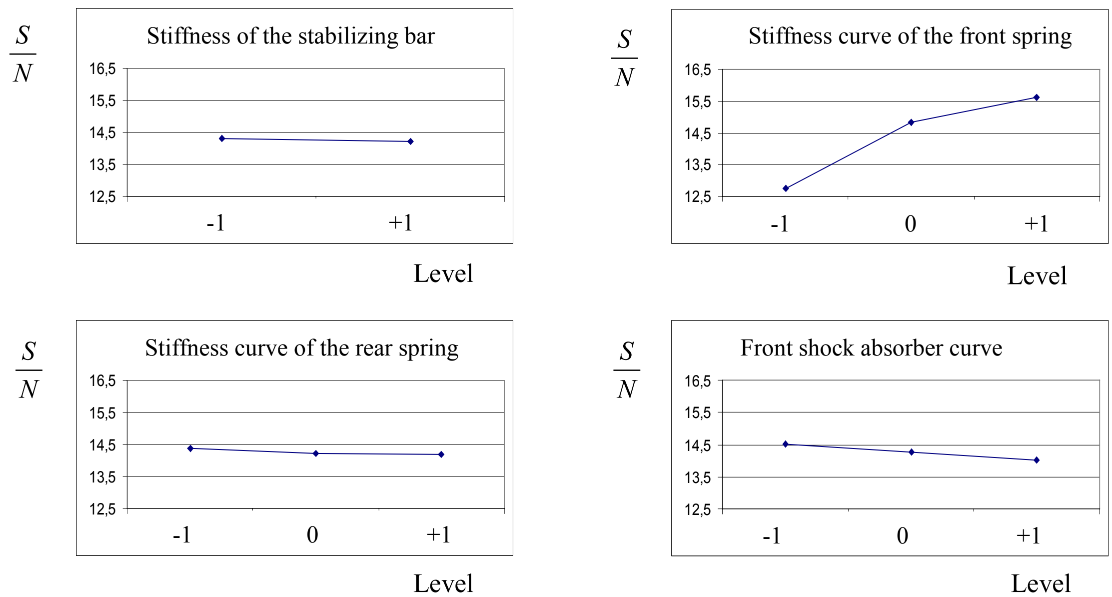

Figure 1. Signal-to-noise ratio for the suspension parameters.

Figures (1) and (2) show the signal-to-noise ratio for the suspension parameters. The optimal suspension configuration is then set up from the best parameter levels.

From Fig. (1) the best levels are –1 for the stiffness of the stabilizing bar, +1 for the stiffness curve of the front spring, –1 for the stiffness curve of the rear spring, and –1 for the front shock absorber curve.

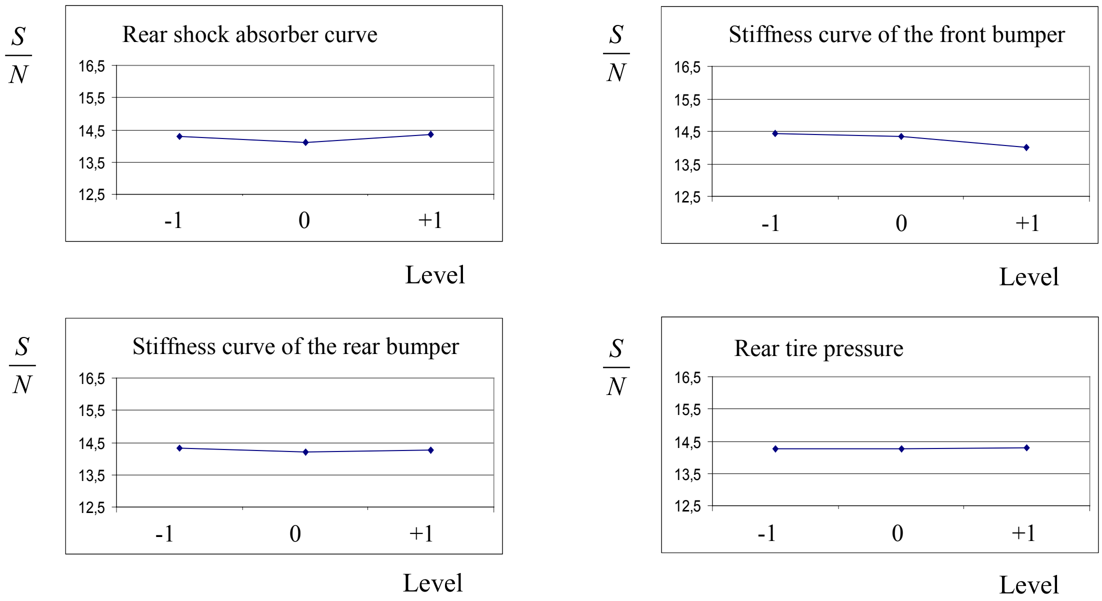

Figure 2. Signal-to-noise ratio for the suspension parameters.

From Fig. (2) the best levels are +1 for the rear shock absorber curve, –1 for the stiffness curve of the front bumper, –1 for the stiffness curve of the rear bumper, and +1 for the rear tire pressure.

An simulation is then made with the selected parameter values. The results are:

Grade at the curb (vehicle with full fuel tank and no cargo load) plus driver load: 7.5.

Grade at the GVW (gross vehicle weight) load: 5.7.

Signal-to-noise ratio: 16.15.

4. Response Surface Methodology

The response surface methodology also works with an array of "experimental runs". These arrays may have different structures, being the central composite design one of the most known (Myers and Montgomery, 2002). The objective of these arrays is to perform an adequate experiment in order to develop an empirical model of the process. If the process output is dependent of two inputs (controllable variables or adjustable parameters, also known as factors), the resulting model can be grafically represented by a surface, and this fact has led to the term "response surface methodology". The selection or development of the array of experimental runs depends on the empirical model that will be adjusted to the experimental data. In this paper the empirical model is supposed to be of first order plus two-factor interactions. In this case only two levels of each parameter needed to be used in the simulations, and a fractional factorial design can be used. The factors (adjustable parameters) are the same of the robust engineering approach:

x_1 – Stiffness of the stabilizing bar, front suspension.

x_2 – Stiffness curve of the front spring.

x_3 – Stiffness curve of the rear spring.

x_4 – Front shock absorber curve.

x_5 – Rear shock absorber curve.

x_6 – Stiffness curve of the front bumper.

x_7 – Stiffness curve of the rear bumper.

x_8 – Rear tire pressure.

The "noise" (vehicle load) also may be considered a factor, if a combined array (Myers and Montgomery, 2002) is used:

z – Vehicle load.

Then the model of the suspension configuration grade can be represented by the following scalar function:

$$y(\mathbf{x}, z) = b_0 + \mathbf{b}^{\mathrm{T}} \cdot \mathbf{x} + \mathbf{x}^{\mathrm{T}} \cdot \mathbf{B} \cdot \mathbf{x} + \mathbf{x}^{\mathrm{T}} \cdot \mathbf{h} \cdot z \tag{3}$$

In Eq. (3), y is the suspension configuration grade, $\mathbf{x}$ is the vector of the parameter values (its elements are x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8), b_0 is the value of the function y at the center point ($z = 0$ and $\mathbf{x} = \mathbf{0}$), and is also known as the intercept value, $\mathbf{b}$ is a column vector of partial regression coefficients, $\mathbf{B}$ is a matrix that establishes the two-factor interactions (also composed of partial regression coefficients), and $\mathbf{h}$ is a column vector that establishes the interaction between the parameters values and the noise (also composed of partial regression coefficients). Using least squares method, the following approximation is attained:

$$\hat{y}(\mathbf{x}, z) = \hat{b}_0 + \hat{\mathbf{b}}^{\mathbf{T}} \cdot \mathbf{x} + \mathbf{x}^{\mathbf{T}} \cdot \hat{\mathbf{B}} \cdot \mathbf{x} + \mathbf{x}^{\mathbf{T}} \cdot \hat{\mathbf{h}} \cdot z + \varepsilon \tag{4}$$

As the specific selected model does not have quadratic terms (as x_1^2 or x_5^2), the diagonal terms of the matrix $\mathbf{B}$ are all null. The total number of the coefficients to be estimated is 46, as there are 8 adjustable parameters plus 1 noise variable. To estimate these partial regression coefficients a fractional factorial design is used. As there are 9 factors with 2 levels each, a full factorial design, which is the simulations of all possible combinations of parameter levels, will demand 2^9 simulations ($2^9 = 512$). Considering the model shown in Eq. (3), a 2^{9-2} fractional factorial design is adequate to estimate the coefficients, which results in $2^7 = 128$ simulations. The 2^{9-2} fractional factorial design is composed of a full factorial design of 7 coefficients, which results in an array of 7 columns (A,B,C,D,E, F and G) and 128 lines, plus 2 additional columns which are formed by the element by element multiplication of columns ACDFG and BCEFG. As these columns is composed by the factor levels, they all have only elements –1 and +1.

For each combination of the factor levels, that is a line in the matrix $\mathbf{X}$ in Eq. (5), there is a suspension configuration grade y_i. So, for 128 simulations, the vector $\mathbf{y}$ of grades has 128 elements and can be represented by:

$$\mathbf{y} = \mathbf{X} \cdot \mathbf{b} + \boldsymbol{\varepsilon} \tag{5}$$

Considering that the experimental runs are, in this work, simulations of a deterministic model, the vector of random errors $\boldsymbol{\varepsilon}$ is null. The matrix $\mathbf{X}$, which is 128×46, is composed by a first column composed only by ones (the intercept column I), all the 9 columns of the fractional factorial array, plus the 36 columns regarding the two factor interations:

$$\begin{array}{c} \begin{array}{ccccccccccccccc} \text{I} & \text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} & \text{G} & \text{H} & \text{J} & \text{AB} & \text{AC} & \text{AD} & & \text{HJ} \end{array} \\ \mathbf{X} = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & \cdots & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \tag{6}$$

$$\text{H} = \text{ACDFG} \tag{7}$$

$$\text{J} = \text{BCEFG} \tag{8}$$

He least square estimator of $\mathbf{b}$ is:

$$\hat{\mathbf{b}} = \left(\mathbf{X}^{\mathbf{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{y} \tag{9}$$

The fitted regression model is:

$$\hat{\mathbf{y}} = \mathbf{X} \cdot \hat{\mathbf{b}} \tag{10}$$

The vector of residues is:

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} \tag{11}$$

In order to test the significance of the regression the test statistic F_0 is used:

$$F_0 = \frac{S_{SR}/k}{S_{SE}/(n-k-1)} \quad (12)$$

In Eq. (12) S_{SE} is the sum of squares of the residuals, S_{SR} is the sum of squares due to the regression model, k is the number of estimated coefficients (except for the intercept) and n is the number of simulations, and in Eq. (15) S_{ST} is the total sum of squares.

$$S_{SE} = \mathbf{e}^{\mathrm{T}}\mathbf{e} \quad (13)$$

$$S_{SR} = S_{ST} - S_{SE} \quad (14)$$

$$S_{ST} = \mathbf{y}^{\mathrm{T}}\mathbf{y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} \quad (15)$$

The regression results show that many of the factors influence are not statistically significant. Using the backward elimination method (Myers and Montgomery, 2002) to select the significant factors results in the following list:

Table 3: Significant factors.

Factor	Regression coefficients
Intercept	+ 5.324
z – Vehicle load.	– 0.838
x_2 – Stiffness curve of the front spring.	+ 0.872
x_4 – Front shock absorber curve.	– 0.232
x_6 – Stiffness curve of the front bumper.	– 0.141
x_8 – Rear tire pressure.	– 0.030
$z.\ x_2$	+ 0.025
$z.\ x_4$	+ 0.104
$z.\ x_6$	+ 0.066
$x_2.\ x_4$	– 0.128
$x_2.\ x_6$	+ 0.074

The F_0 value is 1,442.5, which is far higher than the $F_{0.99,\ 45,\ 128} \cong 1.7$, which means that the hypothesis of null regressors can be rejected.

The empirical model of the suspension grade is then:

$$\hat{y}(\mathbf{x}, z) = +5.324 - 0.833z + 0.872x_2 - 0.232x_4 - 0.141x_6 - 0.030x_8 + 0.025zx_2 + 0.104zx_4 + 0.066zx_6 - 0.128x_2x_4 + 0.074x_2x_6 \quad (16)$$

The maximization of the grade is attained with the following configuration:

$x_2 = +1$
$x_4 = -1$
$x_6 = -1$
$x_8 = -1$

These values agree with the robust engineering selections.

One important characteristic of the response surface method is its sequential nature. In the simulations with factor $x_2 = +1$, the signal of the coefficient of factor zx_6 do not agree with the estimated one. Then, using the same set of already done simulations, a new model is adjusted, with the value of the factor x_2 fixed in +1.

The resulting model, which is valid only for $x_2 = +1$, is attained using the forward inclusion method (Myers and Montgomery, 2002):

$$\hat{y}(\mathbf{x}, z) = +6.196 - 0.813z - 0.362x_4 - 0.021x_5 - 0.067x_6 - 0.042x_8 + 0.131zx_4 \quad (17)$$

The F_0 value is 10,087.7, which is far higher than the F_0 value of the previous model.

Since the "noise" is the vehicle load (factor z), which can be considered a variable with a null mean value with variance σ_z^2, the model in Eq. (17) can be divided in:

$$\bar{y}(\mathbf{x}) = +6.196 - 0.362x_4 - 0.021x_5 - 0.067x_6 - 0.042x_8 \quad (18)$$

$$g(\mathbf{x}, z) = -0.813z + 0.131zx_4 \quad (19)$$

The value of the grade with intermediate vehicle load is then $\bar{y}$, and the variability V_{ar} of this value due to the vehicle load can be expressed by:

$$V_{ar}[y(\mathbf{x}, z)] = \sigma_z^2 \left[\frac{\partial g(\mathbf{x}, z)}{\partial z}\right]^2 + \hat{\sigma}^2 \quad (20)$$

In Eq. (20) $\hat{\sigma}^2$ is the mean squared error of the fitted model.

Using Eq. (18) and Eq. (19), and a linear quadratic programming solver, is then possible to find a configuration that comply with following conditions:

$$\min_{\mathbf{x}} V_{ar}[y(\mathbf{x}, z)] \text{ subject to } \bar{y}(\mathbf{x}) > y_{\min}$$

and $-1 < x_i < +1$.

Selecting a very low value for $\bar{y}(\mathbf{x})$ we find the conditions for minimum variability, or maximum grade (recalling that $x_2 = +1$). Considering an intermediate adjustment, allowing a lower comfort grade in exchange of a lower variability, the optimum configuration is then selected:

Table 4: Suspension configurations.

Factor	Minimum variability	Maximum grade	Optimum configuration
	Grade = 5.7 V_{ar} = 0.56	Grade = 6.7 V_{ar} = 0.78	Grade = 6.4 V_{ar} = 0.68
x_2 – Stiffness curve of the front spring.	+ 1	+ 1	+ 1
x_4 – Front shock absorber curve.	+ 1	– 1	– 0.2
x_5 – Rear shock absorber curve.	+ 1	– 1	– 1
x_6 – Stiffness curve of the front bumper.	+ 1	– 1	– 1
x_8 – Rear tire pressure.	+ 1	– 1	– 1

The convenience of an analytical model allows the use of several optimization algorithms. In particular, the numerical algorithms used in this work is from Scilab, a free software package for scientific purposes (Scilab Consortium, 2004).

5. Conclusions

Two optimization procedures are studied for the vehicle ride comfort optimization problem, the robust engineering, and the response surface methodology. Both are capable of dealing with the problem, but the response surface methodology provides a higher understanding of the influence of the parameter variation in the suspension performance, but at the cost of a much greater number of simulations (128 against 36). Although the simulation costs are decreasing with the development of computer hardware and software, the number of simulations may be important, because ride comfort alone is not a good criterion for suspension tuning, and more complex models of the vehicle dynamics may be necessary for simultaneous ride comfort and handling optimization.

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