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A New Approach of Reliability-based Design Optimization of Complex Structures using Modal Synthesis Methods

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Abstract. The purpose Significant research over the last three decades has focused on the search for the "optimum" structural system. The Reliability-Based Design Optimization (RBDO) seeks to find the best compromise between cost and safety levels. This model often needs a very high computing time and leads to weak convergence stability. A hybrid method has been proposed to overcome these difficulties. However, when applying on real dynamic cases, we have a big number of degrees of freedom. In this paper presents an appraisal of coupling the method of synthesis modal and reliability-based structural system optimization.

Keywords: Reliability-Based Design Optimization, Guyan, Grai Bampton,, Structural Reliability

1. Introduction

In the field of deterministic structural optimization, the designer reduces the structural cost without taking into account uncertainties concerning materials, geometry and loading. This way the resulting optimal design may represent a lower level of reliability and thus a higher risk of failure. Since structural problems are non-deterministic, it is clear that the introduction of the reliability concept plays an important role in the structural optimization field. The integration of reliability analysis into design optimization problem represents the Reliability-Based Design Optimization (RBDO) model. The object of this model is to design structures which should be both economic and reliable. In the RBDO model for robust system design, the mean values of uncertain system variables are usually used as design variables, and the cost is optimized subject to prescribed probabilistic constraints as defined by a mathematical nonlinear programming problem. Therefore, a RBDO solution that reduces the structural weight in uncritical regions does not only provide an improved design but also a higher level of confidence in the design. This approach can be carried out in two separate spaces: the physical space and the normalized space. Since very many repeated searches are needed in the above two spaces, the computational time for such an optimization is a big problem. The solution of the above nested problems leads to very high computational cost, especially for large-scale structures. The major difficulty lies in the structural reliability evaluation, which is carried out by a special optimization procedure. Fortunately, an efficient approach called the hybrid method has been elaborated. This method, which is based on simultaneous solution of the reliability and the optimization problem, has successfully reduced the computational time problem and has been shown to verify the optimality conditions. The solution is achieved in a Hybrid Design Space (HDS) containing the deterministic and the random variables, and a specified reliability level is satisfied. Structures with static linear and/or perfectly plastic responses were generally studied RBDO. But few RBDO studies considering dynamic analyses were reported in the literature. When applying the RBDO procedures on real dynamic cases (for large-scale problems), the computing time becomes much higher. For example, when designing a car model, we need several millions degrees of freedom that leads to a very complex problem. In order to apply the sequential RBDO process, it may be impossible to solve this case because of its weak stability and high computing time. However, the hybrid RBDO can efficiently reduce the computing time and allows us coupling between different models. In order to increase its efficiency, we propose in this paper to implement the simplified model technique. The objective of this technique is to reduce the degrees of freedom (simplify the complex structures) by using a simplified model of the real structure. An application of the hybrid method is carried out using synthesis modal technique.

2. Deterministic Design Optimization (DDO)

2.1 Design optimization problem

In Deterministic Design Optimization (DDO), the system safety may be taken into account by assigning safety factors to certain structural parameters. Using these safety factors, the optimization problem which is carried out in the physical space (figure 1), consists in minimizing an objective function (cost, volume of material …) subject to geometrical, physical or functional constraints in the form

$$\begin{aligned} &\text{min} &&: f(\{x\}) \\ &\text{subject to} &&: g_k(\{x\}) \le 0 \end{aligned} \quad (1)$$

where {x} designates the vector of deterministic design variables. The values of the proposed safety factors principally depend on the engineering experience, but, when designing a new structure, we cannot pre-determine the real critical points, and the choice of these coefficients may therefore be wrong.

Suitable geometry, material properties and loads are assumed, and an analysis is then performed to provide a detailed behavior of the structure. However, changes of the loads, variability of material properties, and uncertainties regarding the analytical models all contribute to the probability that the structure does not perform as intended. To address this concern, analysis methods have been developed to deal with the statistical nature of the input information. As structures are becoming still more complex (e.g. space shuttle engine components, space structures, advanced tactical fighters, etc.) and performance requirements are becoming more ambitious, the need for analyzing the influence of uncertainties and computation of reliability has been growing.

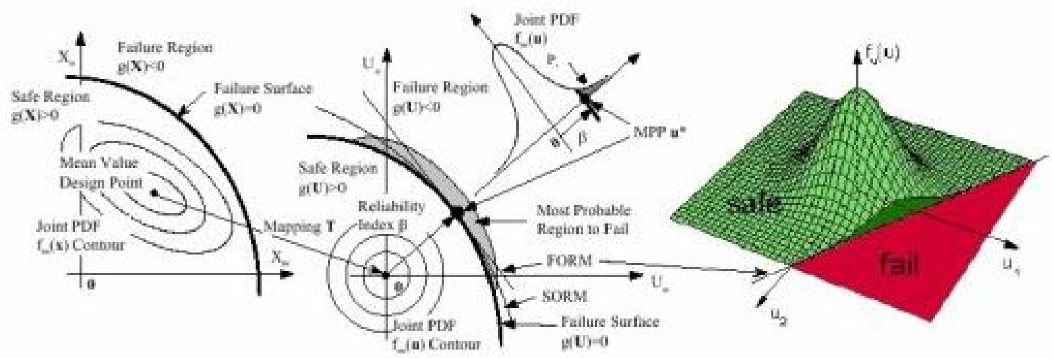

Figure 1: Physical and probabilistic spaces

Over the last ten years there has been an increasing trend in analyzing structures using probabilistic information on loads, geometry, material properties, and boundary conditions. In order to evaluate the structural safety level, a reliability analysis must be carried out without taking into account the safety factor from problem (1).

2.2. Reliability analysis of the optimal solution

For a given failure scenario, the reliability index β is evaluated by solving a constrained minimization problem:

$$\beta = \min d(\{u\}) = \sqrt{\sum_{1}^{m} u_j^2} \quad \text{subject to } H(\{x\},(u)) \le 0 \quad (2)$$

Where {u} is the vector modulus in the normalized space, measured from the origin (see figure 1b). The solution of problem (2) is called the design point P*. The solution is subject to classical difficulties in nonlinear programming: existence of local minima, gradient approximation and computational time. Although problem (2) can be solved with any appropriate optimization method, special techniques have been developed to take advantage of the particular form of the reliability problem. Liu and Der Kiureghian (1991) compared different algorithms on the basis of four criteria: generality, robustness, efficiency and capacity. They recommended three algorithms for structural reliability evaluations: the Sequential Quadratic Programming (SQP), the modified Rackwitz-Fiessler (RF) approach and the gradient projection method. In nonlinear finite element analysis, the latter method is the least efficient. After having followed the Deterministic Design Optimization (DDO) procedure by a reliability analysis, we can distinguish between two cases:

Case 1: High reliability level: when choosing high values of safety factors for certain parameters, the structural cost (or weight) will be significantly increased because the reliability level becomes much higher than the required level for the structure. So, the design is safe but very expensive.

Case 2: Low reliability level: when choosing small values of safety factors or bad distribution of these factors, the structural reliability level may be too low to be admissible. For example, Grandhi and Wang (1998) found that the resulting reliability index of the optimal deterministic design of a gas turbine blade is β =0.0053 under some uncertainties. This result indicated that the reliability at the deterministic optimum is quite low and needs to be improved by the probabilistic design.

For both cases, we can find that there is a strong need to integrate the reliability analysis in the optimization process in order to control the reliability level and to minimize the structural cost or weight in the non-critical regions of the structure. In the next section, we show how this can be performed efficiently. The integration of reliability analysis into engineering design optimization is termed Reliability-Based Design Optimization (RBDO).

3. Reliability-Based Design Optimization (RBDO)

The ultimate goal of design under uncertainty is to reach an optimum in terms of total cost. In principle, an optimum balance between structural system reliability and other conflicting societal goals must be obtained. This is a difficult task. We first present different subjects for RBDO: the objective(s), the design variables, the system response, the limit states, and the solution techniques and next the classical and the hybrid RBDO methods.

3.1. RBDO subjects

● Objective(s) – Many objective functions have been proposed for RBDO. These include cost and utility functions that should be minimized and maximized, respectively. A major obstacle to use these functions is the difficulty in assessing monetary value for human life and injuries. However, when such loss can be avoided, the optimal lifetime cost and optimal lifetime utility can be established (Mori & Ellingwood 1994; Tao & Corotis 1994).

● Design variables – A natural hierarchy exists among the different classes of design variables used in RBDO: (a) cross-sectional variables; (b) configurational variables; (c) topological variables; and (d) material variables. These variables could be continuous or discrete. In RBDO most studies are at the cross-sectional level, but also configurational (Furuta 1980), topological (Murotsu & Shao 1990; Thoft-Christensen 1991), and even material (Shao 1991) design variables have been used. In general, continuous variables are used in RBDO. Sometimes discrete variables are also used in conjunction with catalog dimensions for steel structures, among others. For life-cycle cost and topological optimization situations, integer variables also have to be used (e.g., number of inspections during the lifetime of a structure, number of spans in a bridge) (Lin 1995). The work reported on RBDO is generally limited to one class of design variables. As previously stated, it is generally limited to cross-sectional variables. Much work remains to be done using, perhaps, interactive system optimization combined with expert systems, to achieve a practical RBDO solution for a complex structure in which the material, topology, configuration, and member sizes will be treated simultaneously as design variables.

● Random variables – In addition to the vector of deterministic variables to be used in the engineering design optimization, the uncertainties are modeled by a vector of stochastic physical variables affecting the failure scenario. The knowledge of these variables is not, at best, more than statistical information and we admit a representation in the form of random variables (Ditlevsen & Madsen 1996).

● System response – Structures with static linear and/or perfectly plastic responses were generally studied RBDO. Few RBDO studies considering dynamic analyses were reported in the literature. RBDO under stochastic loads was studied by (Kim & Wen 1990). However, hysteresis and cumulation of damage under dynamic stochastic loads have not been included in RBDO.

● Limit states – Another obvious need in RBDO is consideration of all relevant limit states of a structural system. Current design techniques for non-deterministic structural systems subjected to random loads almost invariably treat the serviceability and ultimate limit states separately without allowing for the interaction between them to modify the design solution. Usually, the serviceability performance constraints ensure acceptable elastic stresses and displacements while the ultimate performance constraints ensure adequate safety against collapse. The current approach to such reliability-based design problems is to proportion the structure to satisfy reliability requirements at one limit state and then to modify the design to satisfy reliability requirements at other limit states of concern. The major drawback to this approach is that design decisions at one limit state must be made in the absence of complete information as to their consequences at other limit states (Grierson & Schmit 1982). It may thus be stated that lack of knowledge of a probability-based design technique whereby performance constraints are satisfied simultaneously at both a specified service-load and a specified ultimate-load level is a major obstacle to reliable prediction of the best solution of structural systems considering all relevant limit states simultaneously. Studies addressing this problem were performed by (Cohn & Parimi 1973), Parimi & Cohn (1978), Frangopol (1985), and Akbora et al. (1993), among others.

● Basic approach – In general, when a probabilistic approach is used instead of a conventional deterministic approach, some of the uncertain quantities should be modeled as random variables. For the reliability-based optimization procedure we use two spaces: the physical space and the normalized space (Feng & Moses 1986). A lot of numerical calculations are required in the space of random variables to evaluate the system reliability. Furthermore, the optimization process itself is executed in the space of design variables which are deterministic. Consequently, in order to search for an optimum structure, the design variables are changed repeatedly, and each set of design variables corresponds to a new random variable space which then needs to be manipulated to evaluate the structural reliability at that point. Since a very large number of repeated searches are needed in the above two spaces, the computational time for such an optimization is a big problem.

● Solution technique – Many numerical optimization techniques for solving RBDO have been used, and many algorithms have been proposed in last decade. For a review of some of these techniques the reader is referred to (Arora 1990). On the other hand, system reliability software is available for various applications. The capability of a particular optimization algorithm coupled with particular system reliability software to solve a specific RBDO problem is of paramount importance. Several successful links between optimization software and system reliability software were reported in the literature by Frangopol & Hendawi (1994), and by Enevoldsen & Sorensen (1994) who performed reliability-based optimization in combination with finite element models. However, most of the solution techniques used in RBDO has been used for small or moderate size structural systems. Solution techniques for RBDO of large scale structures need to be developed.

3.2. Approaches

3.2.1 Classical approach

In the reliability-based optimization field, we distinguish between two kinds of variables: The design variables {x} which are deterministic variables to be controlled in order to optimize the design. The random variables {y} representing the structural uncertainties, identified by probabilistic distributions. The reliability-based optimization is usually calculated by the following nested problems (FrengY.1986):

- Optimization problem:

min : $f(\{x\})$

subject to: $g_k(\{x\}) \leq 0$ and $\beta(\{x\},\{u\}) \leq \beta_t$ (3)

where f({x}) is the objective function, gk({x}) $\leq$ 0 are the associated constraints, β({x},{y}) is the reliability index of the structure and βt is the target reliability index.

- Reliability analysis:

The reliability index β({x},{y}) is calculated by solving the minimization problem:

$$\beta = \min \; d(\{u\}) = \sqrt{\sum_{1}^{m} u_i^2} \quad \text{subject to: } H(\{x\},(u)) \leq 0 \qquad (4)$$

where d({x},{y}) is the distance in the normalized random space (figure 1), and H({x},{u}) is the performance function (or limit state function), defined as H({x},{u}) $\leq$ 0 means failure.

In general, when a probabilistic approach is used instead of a conventional deterministic approach, all the uncertain quantities should be modeled as random variables. A lot of analytical or numerical calculations are required in the space of random variables to derive the system reliability. Furthermore, the optimization process itself is executed in a space of design variables which are usually deterministic. Consequently, in order to search for an optimal structure, the design variables are repeatedly changed, and each set of design variables corresponds to a new random variable space which then needs to be manipulated to evaluate the structural reliability at that point. Because of too many repeated searches needed in the above two spaces, the computational time for such an optimization is a big problem. The structural designers performing deterministic optimization do not consider the RBDO model as a practical tool for the design of real structures. In order to overcome this difficulty, we use a hybrid method as an efficient approach which will be next presented.

3.2.1. Hybrid approach

The solution of the above nested problems leads to very large computational time, especially for large-scale structures. In order to improve the numerical performance, the hybrid approach consists in minimizing a new form of the objective function *F({x},{y})* subject to a limit state and to deterministic as well as to reliability constraints (Kharmanda ,and al):

min : $F(\{x\},\{y\}) = f(\{x\}) \,.\, d_\beta(\{x\},\{y\})$

subject to : $G(\{x\},(y)) \leq 0$, $g_k(\{x\}) \leq 0$ and $\beta(\{x\},\{y\}) \leq \beta_t$ (5)

Here, d_β({x},{y}) is the distance in the hybrid space between the optimum and the design point, d_β({x},{y})=d({u}). The minimization of the function *F({x},{y})* is carried out in the Hybrid Design Space (HDS) of deterministic variables {x} and random variables {y}. An example of this HDS is given in figure 1, containing design and random variables, where the reliability levels d_β can be represented by ellipses in case of normal distribution, the objective function levels are given by solid curves and the limit state function is represented by dashed level lines except for G({x},{y})=0. We can see two important points: the optimal solution P_x^* and the reliability solution P_y^* (i.e. the design point found on the curves G({x},{y})=0 and $d_\beta = \beta_t$).

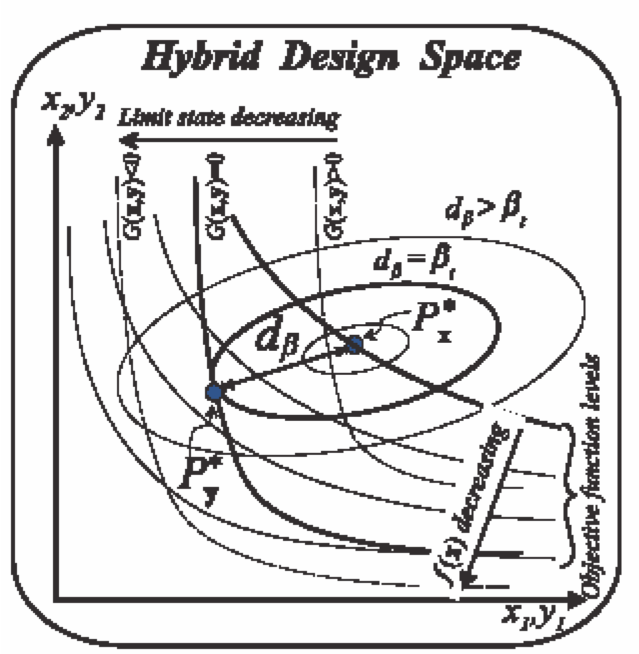

figure2: Hybrid design spaces

4. Synthesis modal

Nowa, the coupling between CAD and finite element software allows models of complex structure to be generated, and yields a nearly perfect representation of their topology. Applied to industrial cases, refined meshes can lead to accurate models that match the real dynamical characteristics of the structure. However, such precise discretizations of a whole structure results in very large finite element models which are not practically exploitable in an industrial context. To be usable, the model has to be simplified while preserving its dynamical behavior in a given frequency band.

Several methods can be applied in order to get reduced models. The most popular are condensation and component mode synthesis methods (Guyan 1965, Craig & Bampton 1968, Macneal 1971). They are based on the decomposition of the global complex structure into several substructures having a simpler geometry, which are individually condensed and then assembled.

The interested reader can refer to the recent works of (Corn and al. 2000). Their approach is based on the observation that many industrial structure components have in fact a beam-like behaviour for the first mode. So they simplified finite element models of structures having a beam-like global dynamical behaviour using finite element formulation. For example models for car bodies contain many beam-like substructures which are usually finely meshed with shell elements (ribs, pillars, stiffeners, etc). In this case if appropriate equivalent beam properties for each subpart can be identified, it can naturally lead to a significant reduction of the degrees of freedom. The resulting condensed models can then be assembled with remaining the global structure.

Furthermore, very few papers have been carried out with object of coupling the equivalent model with reliability-based design optimization. In Kharmanda and al. (2003), a simple cylinder has been simplified to a beam type and has been then reliability-based optimized using optimum safety factor approach. This simplification led to a drastic reduction in size while preserving the global dynamic properties (eigen-frequencies, mode shapes) at frequencies where the cross-sections remain undeformed.

In fact, when applying the RBDO procedures on real dynamic cases (for large-scale problems), the computing time becomes much higher. The use of the sequential RBDO process may be impossible to solve real cases because of its weak stability and high computing time. However, the hybrid RBDO can efficiently reduce the computing time and allows us coupling between different models. In order to increase its efficiency, we implement the synthesis modal technique and compare its results with those when applying on the real structure. This way we reduce the degrees of freedom (simplify the complex structures). The advantage of this technique is demonstrated on the numerical application in the following section.

5. Numerical Application

5.1 Aircraft wing under free vibrations

The studied wing has L=10in, as length and the cross-section dimensions are D1, D2, D3, D4, H1, H2 and H3, illustrated in the (figure 3). The used material is the construction steel with Young's modulus E=38000Psi, and, ρ=1.033E-3 lbf-sec2/in. The objective is to minimize the cross-section area subject to eigen-frequency constraint. The system must satisfy a predefined target reliability. The choice of the target index is usually carried out by statistical studies, but for several engineering applications, we consider the target reliability index as β_t=3.8. Figure 4 shows the relationship between the displacement (the eigen-vector is normal with respect to the mass) and the eigen-frequency f_c for the first eigen-mode. In order to guarantee a required safety level, the performance of the studied structure must be outside of the interval $[f_a,f_b]$. In this example, we compare the Deterministic Design Optimization (DDO) and the hybrid RBDO procedures.

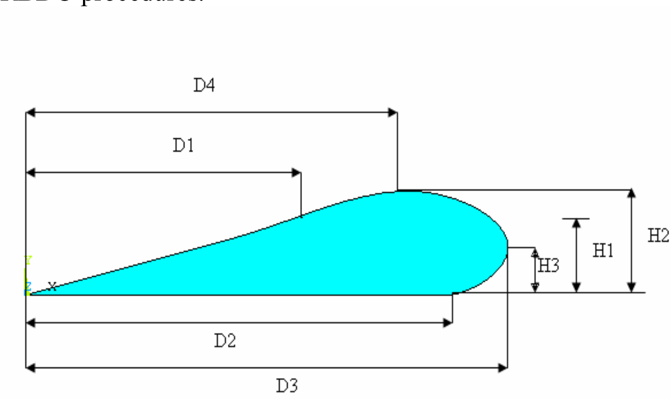

Figure 3: Cross-section of the studied wing.

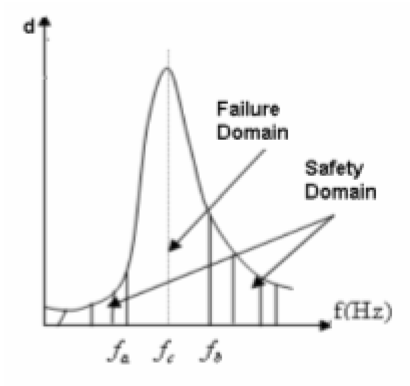

Figure 4: Displacement and eigen-frequency.

Figure 3: Cross-section of the studied wing.

5.1.1. DDO problem:

- Optimization problem:

We minimize the cross-section area subject to eigen-frequency constraint as the following:

$$\min \quad : Area(D1, D2, D3, H3)$$
$$\text{subject to} \; : f(D1, D2, D3, H3) - f_c = 0 \qquad \text{with } f_c\text{=3.5 Hz.}$$

- Reliability analysis of the optimal solution:

For a normal distribution, the normalized variable u has the following form:

$$u_i = \frac{x_i - m_{x_i}}{\sigma_{x_i}} \qquad \text{with } \{x_i\}=\{D1,D2,D3,H3\}.$$

In order to compute the reliability index introduced by Hasofer-Lind, we have to formulate two sub-problems

$$\beta_1 = \min \; d_1(\{u\}) = \sqrt{\sum_1^m u_j^2} \quad \text{subject to: } f_a(D1, D2, D3, H3) - f_c = 0 \quad \text{with } f_c\text{=3.5 Hz.}$$

$$\beta_2 = \min \; d_2(\{u\}) = \sqrt{\sum_1^m u_j^2} \quad \text{subject to: } f_b(D1, D2, D3, H3) - f_c = 0 \quad \text{with } f_c\text{=3.5 Hz.}$$

5.1.2. RBDO problem:

The classical RBDO approach leads to a weak stability of convergence but the hybrid method allows the coupling between the reliability analysis and the optimization problems (Kharmanda, Mohsine).

The hybrid RBDO problem can expressed as:

$$\min \quad : F(\{x\},\{y\}) = f(\{x\}) \,.\, d_{\beta_1}(\{x\},\{y\}) \,.\, d_{\beta_2}(\{x\},\{y\})$$
$$\text{subject to :} \; f(D1, D2, D3, H3) - f_c = 0 \,,\; \beta_1(\{x\},\{y\}) \le \beta_t \text{ and } \beta_2(\{x\},\{y\}) \le \beta_t$$

Where D1, D2, D3 and H3 are grouped in the random vector {Y} but to optimize the design, the means m_{D1}, m_{D2}, m_{D3} and m_{H3} are grouped in the deterministic vector {x}, and their standard-deviation equals to 0.1.

Table 1 shows the DDO results and RBDO ones using the hybrid RBDO. The solution when using DDO procedure is taken in the middle of the given interval of frequencies *[3,4]* Hz. This solution does not respect a required reliability levels and needs to be improved using the RBDO model. Here, we redefine a new interval *[3.34,3.68] Hz*, satisfying a required reliability index (see table 1). In practical cases, we cannot know if the given interval is large or small, this depends on the engineering experience, but the RBDO model is a good tool to control it.

Table I: DDO and RBDO results.

	DDO			RBDO		
variables	***Design Point (a)***	***Optimum Solution***	***Design Point (b)***	***Design Point (a)***	***Optimum Solution***	***Design Point (b)***
D1 [in]	*1.00*	*1.01*	*1.00*	*0.85*	*1.01*	*1.11*
D2 [in]	*1.65*	*1.99*	*1.99*	*1.81*	*1.98*	*2.07*
D3 [in]	*1.95*	*2.23*	*2.23*	*2.00*	*2.24*	*2.59*
H3 [in]	*0.53*	*0.35*	*0.35*	*0.49*	*0.32*	*0.39*
β	*4.7*	*-----*	*4.22*	*3.80*	*-----*	*3.80*
Frequencies [Hz]	*3.02*	*3.5*	*4.01*	*3.34*	*3.51*	*3.68*

5.2. Stator wing under free vibrations

In this work, we present a new study on the Reliability-Based Design Optimization of a piezoelectric engine with annular progressive wave SHINSEI USR 60. In this study interesting us that on the conception of a stator who will be subjected to the constraint of frequency (T.MORO, A 2002, A. El hahami, R. 2003).

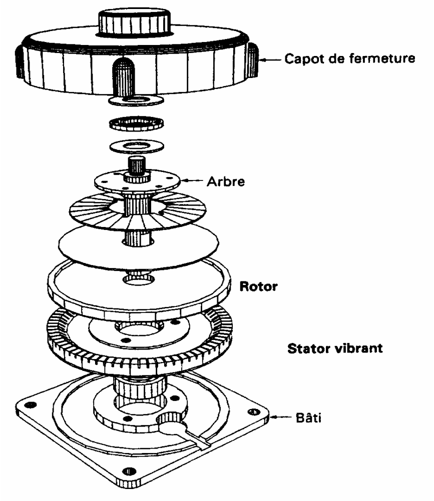

Figure3: Piezoelectric engine with annular progressive wave SHINSEI USR 60

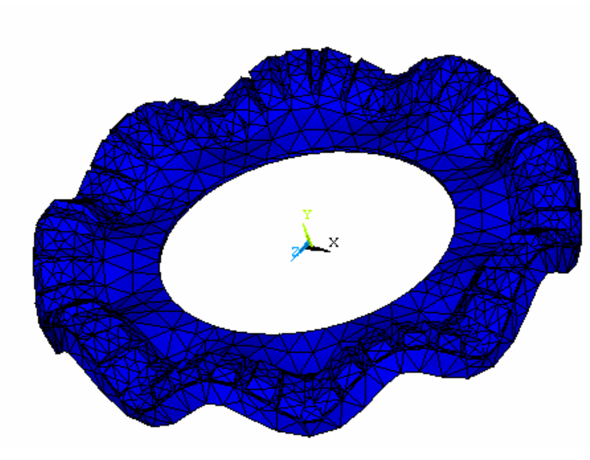

Figure 4: Mode of functioning of the stator

The objective is to minimize the volume of the stator subject to eigen-frequency constraint. The system must satisfy predefined target reliability. The choice of the target index is usually carried out by statistical studies, but for several engineering applications, we consider the target reliability index as β_t=3.6. Figure 4 shows the relationship between the displacements according to the frequency where we illustrate the first frequency of resonance. In order to guarantee a required safety level, the performance of the studied structure must be outside of the interval $[f_a, f_b]$. However the good functioning of the stator is on this band of frequency.

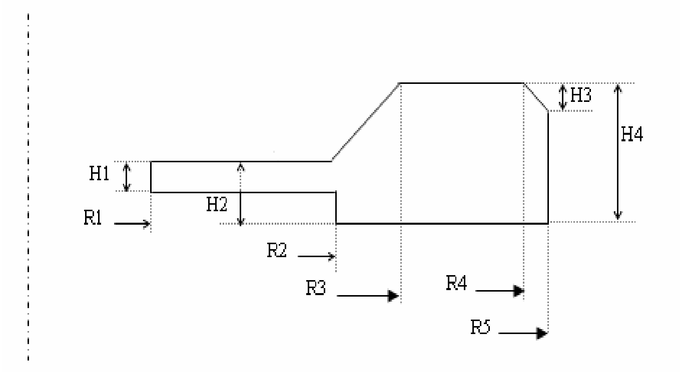

figure5 : Cross-section of the studied stator

The dimensions of the initial system are: R1 = 16.5 mm, R2=22.5mm, R3=25.5mm, R4=29mm,R5=30 mm,H1=0.6 mm,H2=1.5 mm,H3=1.4 mm et H4=3.7mm.

The mechanical properties of the material: E= 1.23e+11 N/m2, ρ=8250 kg/m3 et ν=0.3.

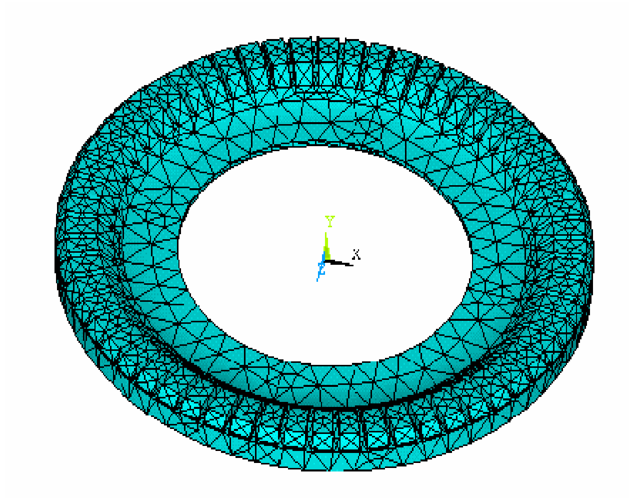

Figure 7: Modelling of the stator

5.2.1 RBDO problem:

The hybrid RBDO problem can expressed as:

$$\min \quad : F(\{x\},\{y\}) = f(\{x\}) \,.\, d_{\beta_1}(\{x\},\{y\}) \,.\, d_{\beta_2}(\{x\},\{y\})$$

$$\text{subject to :} \quad f(\mathrm{R2,R3,R4,H1,H2,H3,H4}) - f_c = 0 \,,\; \beta_1(\{x\},\{y\}) \le \beta_t \text{ and } \beta_2(\{x\},\{y\}) \le \beta_t$$

Where R2,R3,R4,H1,H2,H3and H4 are grouped in the random vector *{y}* but to optimize the design, the means m_{R2} , m_{R3}, m_{R4} , m_{H1} , m_{H2} , m_{H3} *and*, m_{H4} are grouped in the deterministic vector *{x}*, and their standard-deviation equals $0.1m_x$.

The resolution of problem RBDO will be done in two steps: the first step solution by solving our problem in a traditional way, the second step solution by integrating the method of Guyan to reduce the degrees of freedom of the system.

Table II: RBDO results.

variables	Complete structure *(13122)*			*Simplified model (2000)*		
	Design Point (a)	***Optimum Solution***	***Design Point (b)***	***Design Point (a)***	***Optimum Solution***	***Design Point (b)***
R2	22.211	21.214	20.76	22.3	21.213	20.76
R3	25.59	23.959	25.4	25.49	23.958	25.43
R4	29.66	27.925	28.33	29.67	27.92	28.32
H1	0.428	0.562	0.678	0.429	0.56	0.68
H2	1.345	1.483	1.549	1. 346	1.481	1.55
H3	1.5	1.318	1.66	1.45	1.32	1.67
H4	3.66	3.435	3.489	3.65	3.43	3.49
β	3.65	-------	3.6	3.62	-------	3.6
Frequencies [Hz]	37.2	39.5	41	37	39	40.7
Volume [mm^3]	-------	42.76 10^2	-------	-----	42.67 10^2	-------------

Table II shows the RBDO results for both cases: complete model and equivalent. For this example, we first studied the entire body for the complete model study and the simplified model case. The resulting volume and frequencies interval is almost the same with very small error with respect to a required target reliability level β_t=3.6. Using the equivalent model technique we could reduce the degrees of freedom by 2000 degrees for Guyan method.

6. Conclusion

The solution of DDO problem can be improved by the use of RBDO model in order to satisfy the safety requirements. When solving the reliability-based optimization problem in a hybrid space containing random and deterministic variables, we obtain all numerical information about the optimization process that leads to a good efficiency for practical cases. Although the hybrid RBDO method seems to be very efficient with respect to the sequential RBDO process, we need to improve its efficiency using the modal synthesis method. This way we reduce the degrees of freedom, especially for real dynamic cases (for large-scale problems). An application on an aircraft wing and Stator structure showed an improvement of the hybrid method when using modal synthesis technique.

7. Future Works

For real structures, it is difficult to define the safest (reliability-based optimum) point for a given interval. We cannot consider the middle of a given interval as the optimum point because the diagram (Displacement- Frequency) is not always symmetric. So in the future work, we have to develop an efficient algorithm being able to find an optimum structure (reliability-based optimum point) for a given interval.

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