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CHAOS AND BIFURCATION ANALYSIS IN A COUPLED OSCILLATORS SYSTEM APPLIED IN THE LOCOMOTION OF A BIPEDAL ROBOT

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Abstract. *In this work, the central pattern generator (CPG), responsible for the production of rhythmic movements, is formed of a set of mutually coupled nonlinear oscillators of van der Pol. From a model of two-dimensional robot, oscillators with integer ratio of frequency were used for simulating the behavior of the hip angle and of the knees angles. Each oscillator has its own parameters and the link to the other oscillators is made through coupling terms. The objective of this work is to analyze the dynamics of this coupled oscillators system by using bifurcation diagrams and Poincaré maps. By means of the analysis and graphs generated in MATLAB®, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions, presence of strange attractors and other phenomena of the chaos, such as "crisis". Based on the results of the study, we conclude that although the use of coupled oscillators represents an excellent way for generating pattern signals of locomotion, its application in the control of a bipedal robot will only be possible with the correct choice of parameters, which must be done from the data provided by the analysis of bifurcation and chaos.*

Keywords*: bipedal locomotion, central pattern generator, chaos, nonlinear dynamics, oscillators.*

1. Introduction

The study of mechanisms that perform motor functions, in special, the study of mechanical members, intends not only construct autonomous robots, but also to help in the rehabilitation of people who have suffered some accident. The study of the locomotion is inserted in this context, and this has been intensively studied since the second half of century XX. An ample vision of the state of the technique up to 1990 can be found in works as Raibert (1986) and Vukobratovic *et al*. (1990).

In the course of many years the human being has been trying, in all forms, to recreate the complex mechanisms that form the human body. Such task is extremely complicated and the results are frequently unsatisfactory. However, with the greater technological advances each time, based on theoretical and experimental researches, the man gets, in a way, to copy or to imitate some systems of the human body. It is the case, for example, of the central pattern generator (CPG), responsible for the production of rhythmic movements, such as to swim, to walk, and to jump, that it can be modeled by means of mutually coupled nonlinear oscillators. There are some significant works about the locomotion of vertebrates controlled by central pattern generators. Amongst them, Grillner (1985), Collins and Stewart (1993), and Pearson (1993) are very important.

The human locomotion is partially controlled by a CPG, what can be evidenced in works such as Calancie *et al*. (1994) and Dimitrijevic *et al*. (1998). A correctly projected CPG can generate trajectories of reference for locomotion and can be used in the control of bipedal robots. In this work the CPG is formed of a set of mutually coupled nonlinear oscillators, in which each oscillator generates angular signals of reference for the movement of the legs. Each oscillator has its proper amplitude, frequency and parameters, and the linking to the other oscillators is made through the choice of coupling terms. We intend to evaluate the use of van der Pol oscillators. Some previous works about CPGs formed by van der Pol oscillators, applied in the locomotion of bipedal robots, can be seen in Bay and Hemami (1987), Dutra (1995), Zielinska (1996), Dutra *et al*. (2003) and Pina Filho (2004).

The objective of the this work is to analyze the dynamics of this coupled oscillators system by using bifurcation diagrams and Poincaré maps. By means of the analysis and graphs generated in MATLAB®, it was possible to evaluate some characteristics of the system, such as: sensitivity to the initial conditions, presence of strange attractors and other phenomena of the chaos, such as "crisis".

2. Oscillator of van der Pol

Balthazar van der Pol (1889-1959) was a Dutch engineer who made dynamic experimental studies in the beginning of century XX. He investigated electric circuits using vacuum tubes and verified that the circuits presented stable oscillations, later called limit cycles. Together with his colleague van der Mark, he was one of the first researchers to present an article with experimental studies about chaos. He also made some studies about the human heart, constructing models with the objective of studying the dynamical stability. More details about the work of van der Pol can be found in the site http://www.exploratorium.edu.

The equation of van der Pol that will be used in the analysis is:

$$\ddot{x} - \varepsilon\left(1 - p\left(x - x_0\right)^2\right)\dot{x} + \Omega^2\left(x - x_0\right) = 0 \qquad \varepsilon, p \geq 0 \tag{1}$$

where ε, p and Ω correspond to the parameters of the oscillator. Details and discussions about the equation of van der Pol, considering even force terms, can be seen in Jackson (1990) and Strogatz (1994).

3. Coupling of the oscillators

Systems of coupled oscillators have been used extensively in studies of physiological and biochemical modeling. Since the years of 1960, many researchers have studied the case of coupling between two oscillators, because this study is the basis to understand the phenomenon in a great number of coupled oscillators. In their works, French (1971) and Gaitskell (2002) present simpler analyses of the coupling between two linear systems mass-spring. Amongst other works about this subject, there can be cited Kozlowski *et al*. (1995) and more recently Wirkus and Rand (2002).

One of the types of oscillators that can be used in coupled systems is the auto-excited ones, which have a stable limit cycle without external forces. These will be the oscillators used in the analyses presented here. In relation to the type of coupling, considering a set of n oscillators, there are three basic schemes of coupling (Low and Reinhall, 2001). Figure 1 presents these schemes, knowing that the last configuration of coupling will be used in the analyses, since we want that each one of the oscillators has influence on the others.

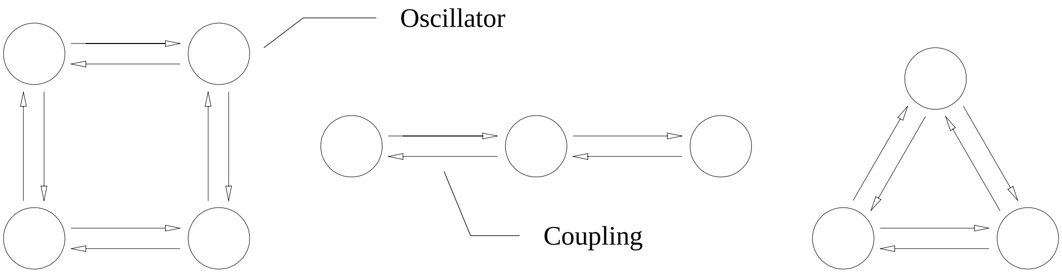

Figure 1. Basic schemes of coupling of oscillators: in ring *(a)*, in chain *(b)* and mutually coupled *(c)*.

Considering a net of n coupled van der Pol oscillators, from Eq. (1) and adding coupling terms that relate the velocities of the oscillators, we have:

$$\ddot{\theta}_i - \varepsilon_i\left[1 - p_i\left(\theta_i - \theta_{io}\right)^2\right]\dot{\theta}_i + \Omega_i^2\left(\theta_i - \theta_{io}\right) - \sum_{j=1}^{n} c_{i,j}\left(\dot{\theta}_i - \dot{\theta}_j\right) = 0 \qquad i = 1, 2, ..., n \tag{2}$$

which represents coupling between oscillators with the same frequency, where θ corresponds to the degrees of freedom of the system. In the case of coupling between oscillators with integer relation of frequency, the equation would be:

$$\ddot{\theta}_h - \varepsilon_h\left[1 - p_h\left(\theta_h - \theta_{ho}\right)^2\right]\dot{\theta}_h + \Omega_h^2\left(\theta_h - \theta_{ho}\right) - \sum_{i=1}^{m} c_{h,i}\left[\dot{\theta}_i\left(\theta_i - \theta_{io}\right)\right] - \sum_{k=1}^{n} c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right) = 0 \tag{3}$$

where the nonlinear term $c_{h,i}[\dot{\theta}_i\left(\theta_i - \theta_{io}\right)]$ is responsible for the coupling between oscillators with different frequencies, while the term $c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right)$, also seen in Eq. (2), makes coupling between oscillators with the same frequency. Both terms were defined by Dutra (1995).

3.1. Application of coupling in bipedal robot

Consider the model presented in Fig. 2. The angle of the hip θ_4 and the angles of the knees θ_3 and θ_5 will be determined by the system of coupled oscillators. The other angles are calculated by equations determined by the kinematical analysis of the mechanism. In this work we will not present details of this analysis, which can be seen in Pina Filho (2004). Besides the angles we have: x_t and y_t, which are the coordinates of the tip of the foot; ℓ_s is the length of the part of the foot responsible for the support (toes); ℓ_p is the length of the part of the foot that raises from the ground (sole); ℓ_t is the length of the tibia; and ℓ_f is the length of the femur.

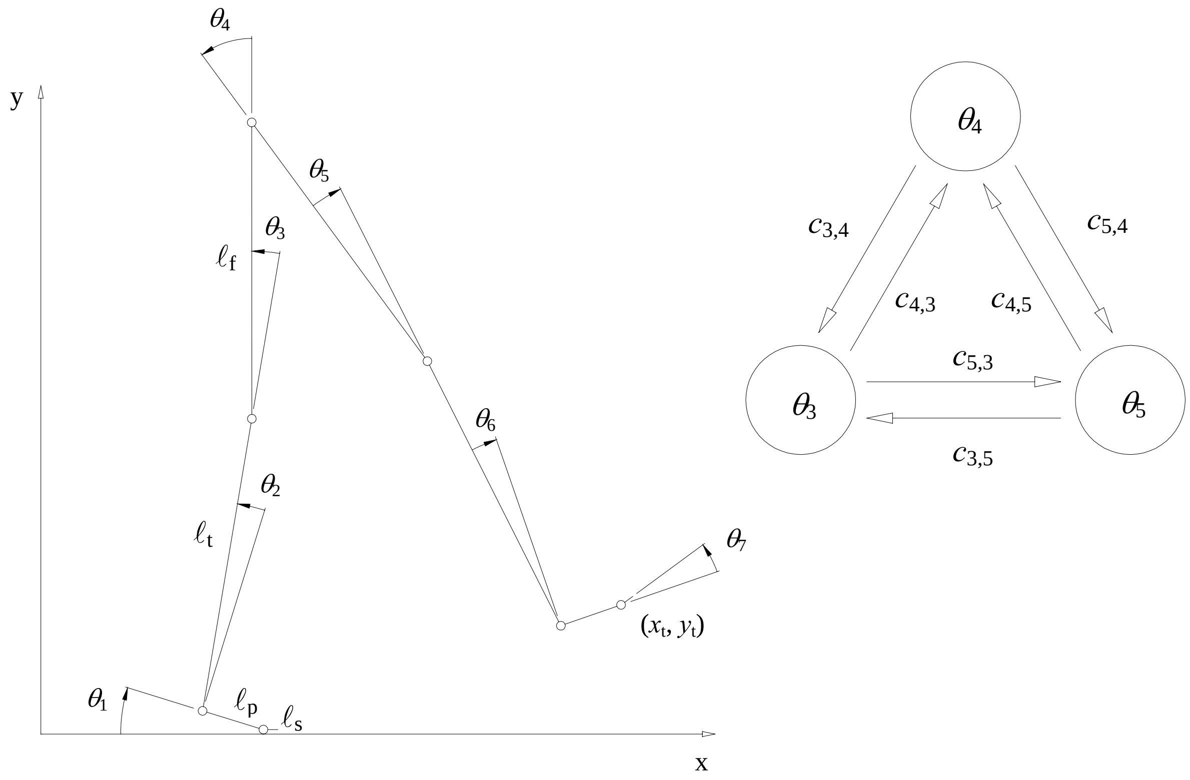

Figure 2. Model of the bipedal robot to be analyzed and structure of coupling between the oscillators.

Experimental studies of human locomotion (Braune and Fischer, 1987, and Raptopoulos, 2003) and of Fourier analysis of these data (Dutra, 1995) show that the movements of the angles θ_3, θ_4 and θ_5 can be described very precisely by their fundamental harmonic, whether the biped is in the double support phase, with the two feet on the ground, or in single support phase, with only one foot touching the ground.

The generation of the angles θ_3, θ_4 and θ_5 as a periodic attractor of a nonlinear net was intended, a set of three coupled oscillators have been used. These oscillators are mutually coupled by terms that determine the influence of an oscillator on the other oscillators, as seen in the Fig. (2). The lesser the value of these coupling terms is, the weaker is the relation between the oscillators.

Applying Eqs. (2) and (3) to the proposed problem, knowing that the frequency of θ_3 and θ_5 (angles of the knees) is the double of θ_4 (angle of the hip), one has the following equations:

$$\ddot{\theta}_3 - \varepsilon_3[1 - p_3(\theta_3 - \theta_{3o})^2]\,\dot{\theta}_3 + \Omega_3^2(\theta_3 - \theta_{3o}) - c_{3,4}\left[\dot{\theta}_4(\theta_4 - \theta_{4o})\right] - c_{3,5}\left(\dot{\theta}_3 - \dot{\theta}_5\right) = 0 \tag{4}$$

$$\ddot{\theta}_4 - \varepsilon_4[1 - p_4(\theta_4 - \theta_{4o})^2]\,\dot{\theta}_4 + \Omega_4^2(\theta_4 - \theta_{4o}) - c_{4,3}\left[\dot{\theta}_3(\theta_3 - \theta_{3o})\right] - c_{4,5}\left[\dot{\theta}_5(\theta_5 - \theta_{5o})\right] = 0 \tag{5}$$

$$\ddot{\theta}_5 - \varepsilon_5[1 - p_5(\theta_5 - \theta_{5o})^2]\,\dot{\theta}_5 + \Omega_5^2(\theta_5 - \theta_{5o}) - c_{5,4}\left[\dot{\theta}_4(\theta_4 - \theta_{4o})\right] - c_{5,3}\left(\dot{\theta}_5 - \dot{\theta}_3\right) = 0 \tag{6}$$

From Eqs. (4)-(6), using the parameters shown in Tab. 1 together with values supplied by Dutra *et al*. (2003), there have been generated the graphs in MATLAB® shown in Fig. 3, which present, respectively, the behavior of the angles in function of time and the stable limit cycles of the oscillators.

Table 1. Parameters of van der Pol oscillators.

$c_{3,4}$	$c_{4,3}$	$c_{3,5}$	$c_{5,3}$	$c_{4,5}$	$c_{5,4}$	ε_3	ε_4	ε_5
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01

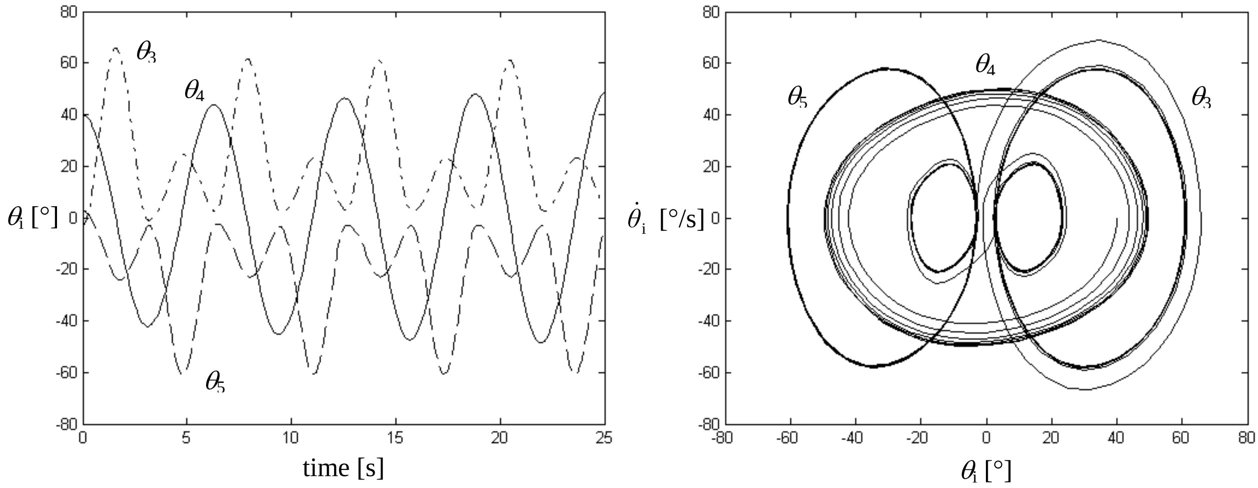

Figure 3. Graphs of θ_3, θ_4 and θ_5 in function of time and trajectories in the phase space.

In Fig. 3 the great merit of the system can be observed, i.e. if an impact occurs and the angle of a joint is not the correct or desired one, it returns to the desired trajectory in a small number of periods. Considering, for example, a frequency equal to 1 s^{-1}, with the locomotor quitting rest with arbitrary initial values: $\theta_3 = -3°$, $\theta_4 = 40°$ and $\theta_5 = 3°$, after some cycles we have: $\theta_3 = 3°$, $\theta_4 = 50°$ and $\theta_5 = -3°$.

4. Dynamical analysis of coupling

The dynamical analysis of the system presented here requires the definition of some usual concepts. The first one of them is the definition of the characteristics of a chaotic system. Usually, for some values of parameters, the behavior of the system is periodic, and for other values the behavior is chaotic. According to Moon (1998), a periodic system is that one that returns to its state after a finite time *t*. The trajectory in the phase space is represented by a closed curve. As for the chaotic system, it presents trajectories of not-closed orbits that are generated by the solution of a deterministic set of ordinary differential equations.

Two conditions must be satisfied to make possible that a system presents chaotic behavior: the equations of motion must include a nonlinear term; and the system must have at least three independent dynamic variables. The main consequence associated with the chaos is the so-called “sensitivity to the initial conditions”. In chaotic systems, a small change in the initial conditions results in a drastic change in the behavior of the system. This phenomenon was first recognized by Henri Poincaré (1854-1912), in the end of century XIX. More details about the Chaos theory and its characteristics can be found in many works, amongst them Strogatz (1994), Moon (1998) and Savi (2003).

4.1. Poincaré maps

The idea of the Poincaré map consists of the reduction of continuous systems in time (flows) in discrete systems in time (maps). Then, the construction of a Poincaré map allows the dynamics of the system to be represented in a space of a lesser dimension than that one of the original system, reducing the visualization of a *n*-dimensional space for $n-1$ dimensions.

Considering a section through the phase space of the original system, the intersection of the orbits with this section defines the Poincaré map. Figure 4 shows a section of Poincaré in a three-dimensional phase space. Notice that the Poincaré map is obtained from the phase space diagram by observing this “stroboscopically”, i.e. sample points in the phase space in regular intervals.

4.2. Bifurcation diagrams

Another important point of chaos relates to the study of “bifurcation”. The term bifurcation is associated with a qualitative change in the structure of a solution, as a consequence of a variation of the system parameters (Savi, 2003). The existence of bifurcation is related with the existence of chaos. In all chaotic system, it is possible to observe the phenomenon of bifurcation, however, not all system that presents bifurcation necessarily presents a chaotic response.

The influence of a given parameter in the response of a system can be identified by means of the bifurcation diagrams, which present the stroboscopic distribution of the system response from a slow variation of a given parameter (Thompsom and Stewart, 1986). This was the method applied here, which implies to simulate different parameter values that we want to analyze, evaluating the type of response in the section of Poincaré.

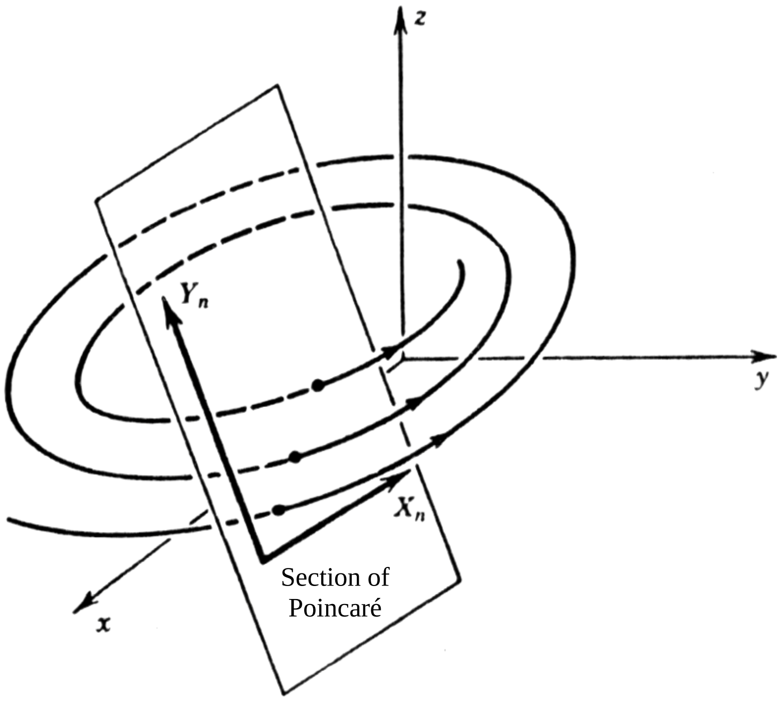

Figure 4. Section of Poincaré in a three-dimensional phase space (Moon, 1998).

4.3. Analysis and results

Considering different values for the parameters ε_3, ε_4 and ε_5, the tests have been performed using the MATLAB® to generate the bifurcation diagrams and Poincaré maps. In principle, keeping values of $\varepsilon_4 = 0.1$ and $\varepsilon_5 = 0.01$, the value of ε_3 was varied from 0 up to 10. All the other values of the system have been kept. Figure 5 presents the bifurcation diagram showing the behavior of the oscillator of the knee (θ_3) from the variation of the parameter ε_3, which represents the term of damping related with this oscillator.

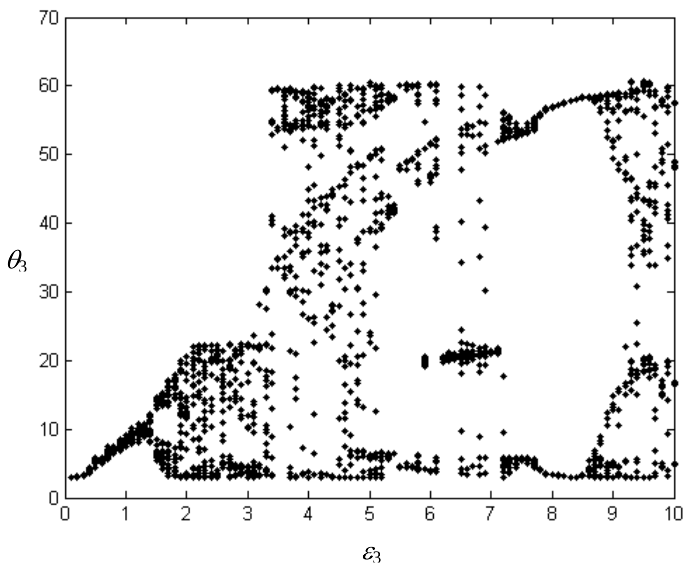

Figure 5. Bifurcation diagram for θ_3 with variation of ε_3.

Observe that a chaotic regime is already configured when $\varepsilon_3 = 2$. It is interesting to notice also that when $\varepsilon_3 = 8$ the regime is not chaotic anymore and starts to present a period-2 response, later returning to the chaotic regime. Sensitivity to the initial conditions can be verified considering two simulations with different conditions, for example, with $\varepsilon_3 = 3$ (chaotic regime), choosing initial values for the angles: $\theta_3 = 3^o$, $\theta_4 = 50^o$, $\theta_5 = -3^o$, and then changing $\theta_3 = 3.001^o$, it can be observed the influence of the initial conditions in the system response (Fig. 6).

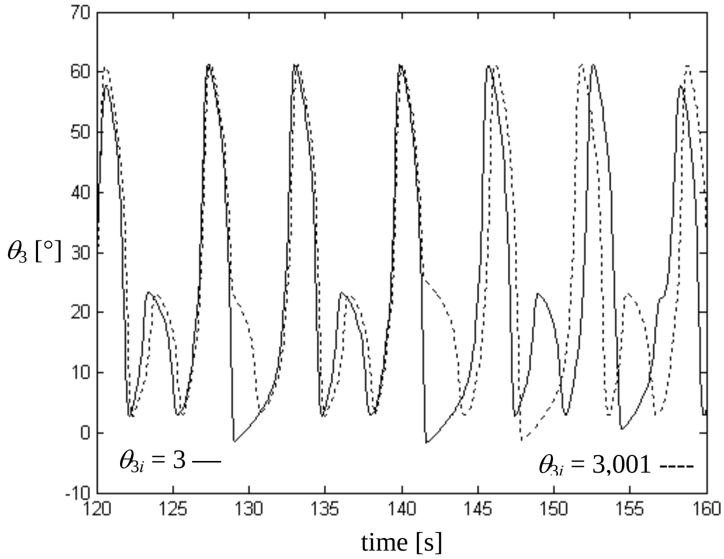

Figure 6. Sensitivity to the initial conditions in the chaotic response.

Another interesting point of the analysis of chaos is the presence of the so-called “strange attractor”, which can be observed through the Poincaré map. In dissipative systems the Poincaré map presents a set of points disposed in an organized form, with a preferential region in phase space that attracts the states of the dynamic system. Figure 7 presents the strange attractor generated in the analysis of the knee oscillator (θ_3).

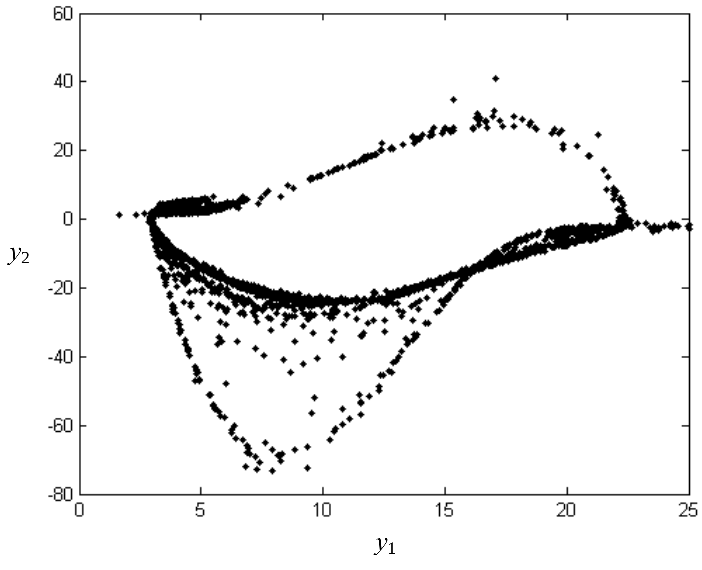

Figure 7. Strange attractor for θ_3.

Considering the coupling between the oscillators, the degree of influence of one on the others is defined by the coupling term. Then, a change of parameters of one oscillator must influence in the behavior of others.

Figure 8 presents the bifurcation diagram showing the behavior of the knee oscillator (θ_5) from the variation of the parameter ε_3. Observe that the behavior is similar to the other knee (θ_3), presenting a chaotic regime from $\varepsilon_3 = 2$; when $\varepsilon_3 = 8.5$ the regime is not chaotic anymore and starts to present a period-2 response, later returning to the chaotic regime.

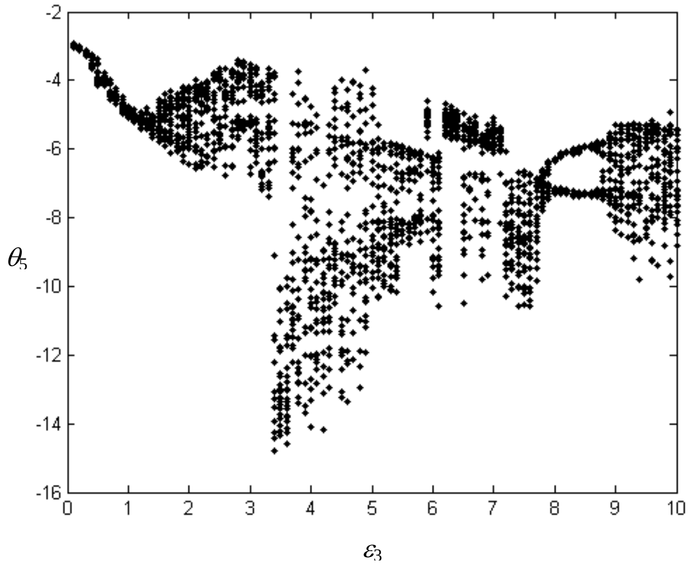

Figure 8. Bifurcation diagram for θ_5 with variation of ε_3.

Figure 9 presents the bifurcation diagram showing the behavior of the hip oscillator (θ_4) from the variation of the parameter ε_3. In this case, we observe that the influence of the knee oscillator (θ_3) on the hip (θ_4) is small, since the behavior of θ_4 does not suffer many alterations. This result was already expected due to the small value of the coupling term between the oscillators (c_{34} = c_{43} = 0.001). In relation to the knees, the coupling term is much greater (c_{35} = c_{53} = 0.1), configuring a more significant influence.

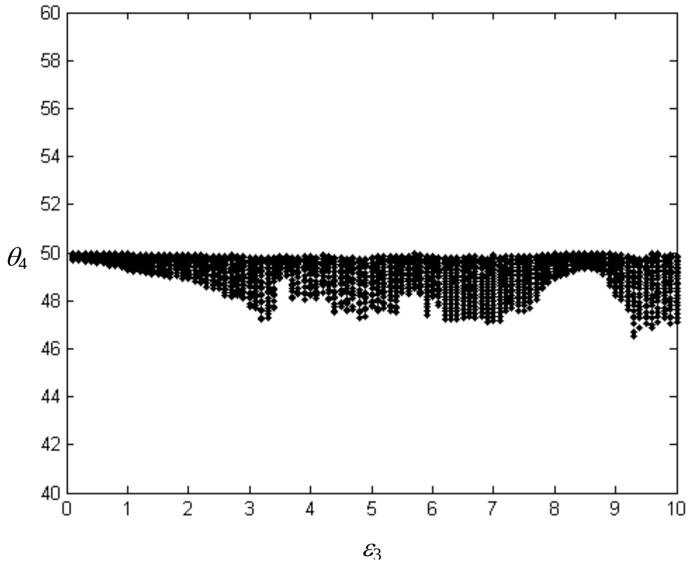

Figure 9. Bifurcation diagram for θ_4 with variation of ε_3.

In analogous form to what was made for ε_3, the response of the system can be analyzed by varying the values of ε_4 (from 0 up to 10) and keeping the other values fixed. Figure 10 presents the bifurcation diagram showing the behavior of the hip oscillator (θ_4) from the variation of the parameter ε_4, which represents the term of damping related with this oscillator. Figure 11 presents the strange attractor generated in the analysis of this oscillator.

As seen previously in the analysis of ε_3, the influence of the hip on the knees is small, then a variation of ε_4 does not bring about great changes in θ_3 and θ_5.

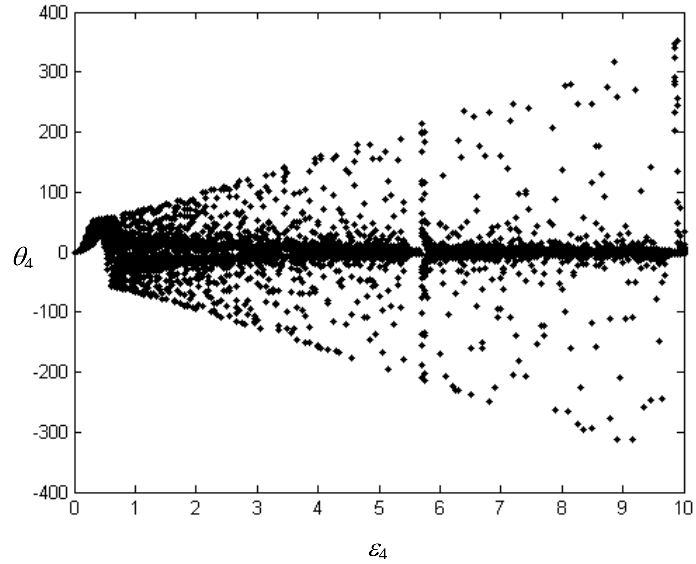

Figure 10. Bifurcation diagram for θ_4 with variation of ε_4.

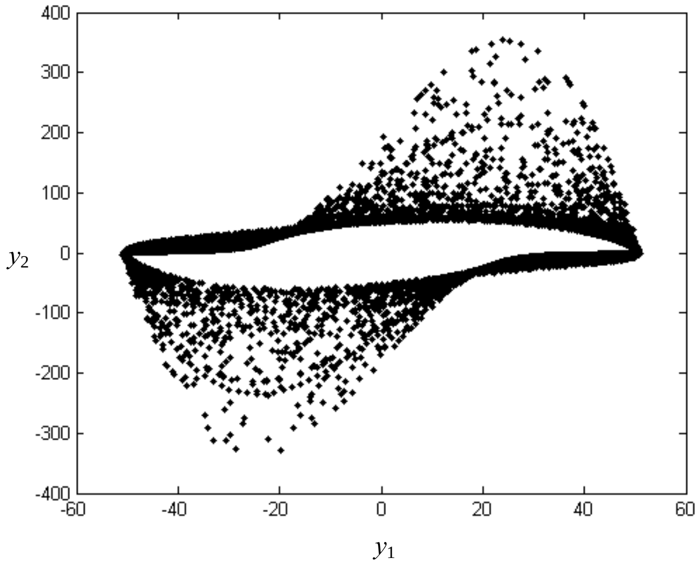

Figure 11. Strange attractor for θ_4.

Finally, the response of the system can be analyzed by varying the values of ε_5 (form 0 up to 10) and keeping the other system values fixed. Figure 12 presents the bifurcation diagram showing the behavior of the oscillator of the knee (θ_5) from the variation of the parameter ε_5, which represents the term of damping related with this oscillator. Figure 13 presents the strange attractor generated in the analysis of this oscillator.

In this analysis, another interesting particularity of the chaotic systems occurs, the "crisis". The crisis phenomenon is defined as the collision between a chaotic attractor and an unstable fixed point or an unstable orbit (Grebogi *et al.*, 1983). With this, the chaotic behavior can appear or disappear from a parameter change, what can be seen in Fig. 12 with $\varepsilon_5 \cong 5.2$ and later $\varepsilon_5 \cong 8.5$. The same behavior can be noticed in the oscillator of the other knee (θ_3), from the variation of the parameter ε_5 (it sees Fig. 14).

In relation to the hip, the oscillator of the knee (θ_5) presents a much lesser influence than that one of θ_3 on θ_4.

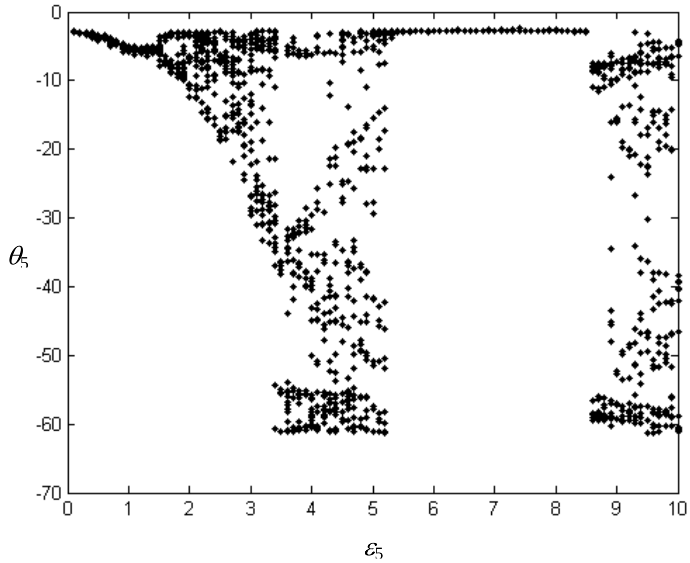

Figure 12. Bifurcation diagram for θ_5 with variation of ε_5.

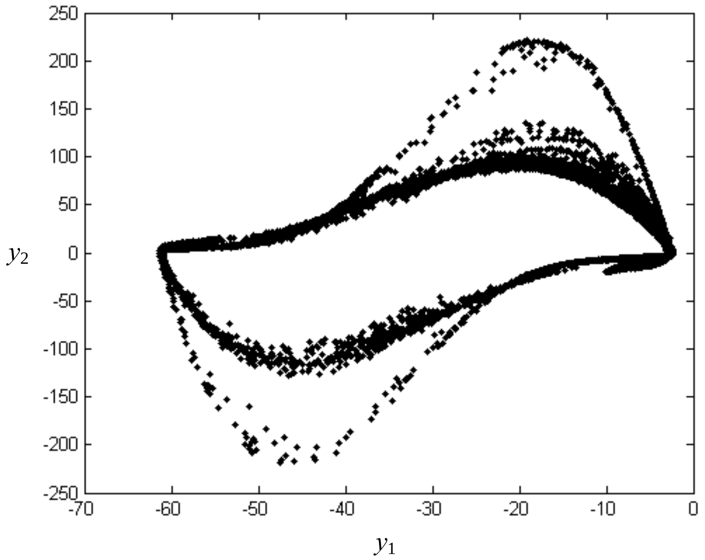

Figure 13. Strange attractor for θ_5.

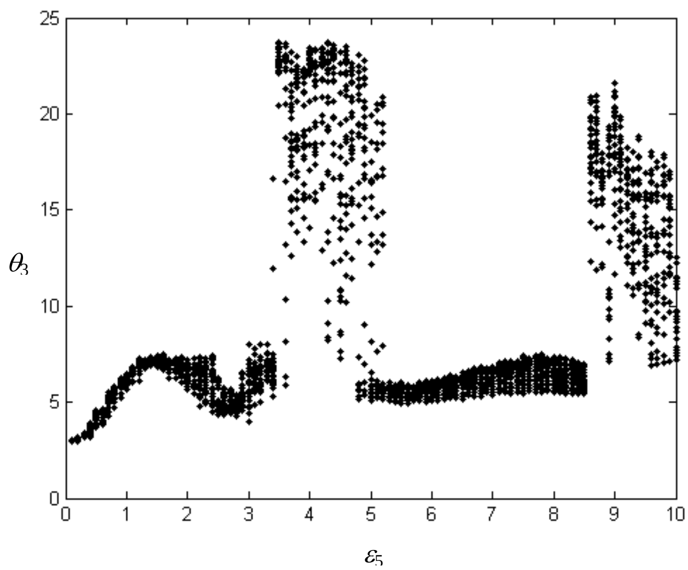

Figure 14. Bifurcation diagram for θ_3 with variation of ε_5.

5. Conclusion

From presented results and their analysis and discussion, we come to the following conclusions: the use of mutually coupled nonlinear oscillators of van der Pol can represent an excellent form generating locomotion pattern signals, allowing its application for the control of bipedal robot by the synchronization and coordination of the legs, once the choice of parameters is correct, which must be made from the data supplied for the analysis of bifurcation and chaos. Through the dynamic analysis it was possible to evidence a weak point of coupling systems. The influence of the oscillators of the knees on the hip, and vice versa, is very small, what can harm the functionality of the system, i.e. if one of the knees suffers some disturbance, it will be automatically felt by the other knee, but it is possible that no reaction occurs in the hip. The solution for this problem seems immediate: to increase the value of the coupling term between the hip and the knees. However, a fast test proves that this can make the system unstable. Then, it is necessary a more refined study of the problem.

6. Acknowledgements

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