Proceedings of the XI DINAME, 28th February - 4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

USE OF SIMPLIFIED MODELS IN THE DYNAMIC STUDY OF HYDROGENERATORS

Felipe J. Vinaud
Universidade de Brasília - UnB
Campus Universitário Darcy Ribeiro
Brasília - DF - Brasil CEP 70910-900
fvinaud@unb.br

Alberto C.G.C. Diniz
Universidade de Brasília - UnB
Campus Universitário Darcy Ribeiro
Brasília - DF - Brasil CEP 70910-900
adiniz@unb.br

Abstract. *The purpose of this work is the use of simplified models in the rotordynamic analysis of a hydroelectric rotor-generator assembly. The analysis of different methodologies will allow for acquaintance with a phonomenological sense of the runing conditions of the assembly. Thus, the methodology will enable the detection of unstable vibrations and suply a control parameter to be used in more complex simulations. The Rayleigh-Ritz and Finite Element Methodologies are employed and produced results such as transversal vibration amplitudes, whirling orbits and Campbell Diagrams in which it is possible to locate critical speeds. The preliminar model to be simulated is a simplified reproduction of the working conditions at the Coaracy Nunes power plant under Eletronorte administration.*

Keywords: *rotor-generator assembly, rotordynamic analysis, vibration, Rayleigh-Ritz, Finite Elements.*

1. Introduction

During recent years it has been observed an increase in the demand for electrical power due to reduced raining seasons and the necessity of neighbouring countries of importing Brazilian energy. Thus, a more efficient genertation of power is to be assessed in order to optimize the operation of old hydro power plants and increase the extraction of energy from the water. With this aim, this work was proposed as a preliminar study of the running conditions of rotor-generator assemblies such as the Coaracy Nunes assembly in a rotordynamic standpoint.

Furthermore, traditional dynamic analyses are performed through the introduction of complex CAD drawings into a comercial and closed Finite Element package. This procedure may sometimes make it difficult to understand the general physical phenomena involved due to the complexity of the whole structure. To validate a modelling methodology it is usefull to build a simplified model to be used as a reference where the different aspects of the rotordynamic theory and the numerical procedures can be assessed and understood.

This article presents a simple study of the Rayleigh-Ritz and Finite Element methodologies and their use in an adapted model of the assembly under study. Both methodologies allow the calculation of the general behaviour of the assembly.

The Rayleigh-Ritz methodolgy was used with the intent of achieving a description of such behaviour using few degrees of freedom allowing the understanding of the physical phenomena involved. One of these phenomena is the evolution of the natural frequency of the system with the increase of rotational speed. The Finite Element formulation makes possible a more practical description of such phenomena due to the possibilities of its use in large scale problems.

2. The Rayleigh-Ritz methodology

The rotor-generator assembly being modeled in this work is comprised of two discs, one shaft and two bearings. For the use of the Rayleigh-Ritz methodology the energy equations of each component is needed (Lalanne, 1996). The unbalance, which cannot be completely avoided, needs to be taken into account as well. Figure 1 depicts the modeled assembly.

The kinetic energy expressions are used for the discs, shaft and unbalance models. The strain energy is used in the shaft model. Bearings are represented by the virtual work exerted on the shaft. Hence, the movement equations of a rotor are attained by the insertion of those energy expressions in Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Fq_i \quad (1 \leq i \leq N) \tag{1}$$

where N is the number of degrees of freedom, q_i are the independent generalized coordinates and Fq_i are the generalized forces.

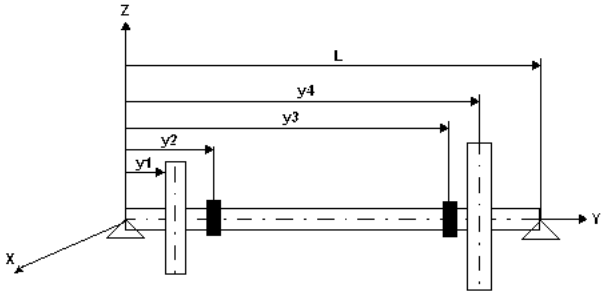

Figure 1: Modeled assembly

2.1. The discs

The discs are regarded as rigid strctures and can be represented by their kinetic energy. The kinetic energy of one disc is attained by its rotation around its center of mass and is calculated through displacement variables relating a coordinate system fixed on the disc and rotating with it and a fixed reference coordinate system. In the reference coordinate system, u and w are the coordinates of the centre of mass along the X and Z axes respectively. The angles θ and ψ are the angles of rotation relating the inclination of the disc with respect to the reference coordinate system. The kinetic energy is expressed finally, for each disc, by:

$$T_d = \frac{1}{2} M_d \left((\dot{u})^2 + (\dot{w})^2 \right) + \frac{1}{2} I_{dx} \left((\dot{\theta})^2 + (\dot{\psi})^2 \right) + \frac{1}{2} I_{dy} \left(\Omega^2 + 2\Omega\dot{\psi}\theta \right) \tag{2}$$

where the term $\frac{1}{2} I_{dy} \Omega^2$ represents the kinetic energy of the disc rotating at an angular speed Ω. This term is constant and will not modify the equations of motion in Eq. (1). The term $I_{dy}\Omega\dot{\psi}\theta$ represents the gyroscopic effect.

2.2. The shaft

The kinetic energy expression for the shaft is an extension of that obtained for the discs. For an element of length L and constant cross section the kinetic energy expression is:

$$T_s = \frac{\rho S}{2} \int_0^L \left((\dot{u})^2 + (\dot{w})^2 \right) dy + \frac{\rho I}{2} \int_0^L \left((\dot{\psi})^2 + (\dot{\theta})^2 \right) dy + \rho I L \Omega^2 + 2\rho I \Omega \int_0^L \dot{\psi}\theta dy \tag{3}$$

where ρ is the density, S the cross-sectional area and I the moment of inertia of the shaft. The first integral represents the classical problem of a beam under bending. The second integral represents the effects of rotational inertia. The term $\rho I L \Omega^2$ is constant and will not influence the equations of motion.

For obtaining the strain energy, it is necessary to consider a point $B(x, y)$ in the cross-section of the beam. A coordinate system fixed to the shaft, anf thus rotating with it, is considered. u^* and w^* are the coordinates of the geometric centre of the cross section with respect to the rotating axes x and z respectively. The strain energy of point B in the cross section of the disc is calculated in the rotating coordinate system and second order terms are neglected. The longitudinal deformation at point B and the strain energy can be written as:

$$\epsilon = -x \frac{\partial^2 u^*}{\partial y^2} - z \frac{\partial^2 w^*}{\partial y^2} \quad \text{and} \quad U_a = \frac{1}{2} \int_\tau \epsilon^t \sigma d\tau \tag{4}$$

In this expression, the stress-strain relationship and coordinate system transformations such as $u^* = u \cos\Omega t - w \sin\Omega t$ and $w^* = u \sin\Omega t - w \cos\Omega t$ must be used. The diametral moments of inertia in the x and z directions have to be inserted as well. After some calculations, one can obtain the following expression for the strain energy in the reference coordinate system:

$$U_s = \frac{E}{2} \int_0^L \left[I_z \left(\cos\Omega t \frac{\partial^2 u}{\partial y^2} - \sin\Omega t \frac{\partial^2 w}{\partial y^2} \right)^2 + I_x \left(\sin\Omega t \frac{\partial^2 u}{\partial y^2} - \cos\Omega t \frac{\partial^2 w}{\partial y^2} \right)^2 \right] \tag{5}$$

Finally, for a symetric shaft ($I = I_x = I_z$) the strain energy is writen as:

$$U_s = \frac{EI}{2} \int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 \right] \tag{6}$$

2.3. Bearings

The bearing properties are suposed known. Thus, the virtual work δW of the external forces acting on the shaft are found for each bearing in the following manner:

$$\delta W = -k_{xx}u\delta u - k_{xz}w\delta u - k_{zz}w\delta w - k_{zx}u\delta w - c_{xx}\dot{u}\delta u - c_{xz}\dot{w}\delta u - c_{zz}\dot{w}\delta w - c_{zx}\dot{u}\delta w \tag{7}$$

or in the compact form: $\delta W = F_u\delta u + F_w\delta w$. F_u and F_w are the generalized force components and are expressed in the matricial form as:

$$\begin{bmatrix} F_u \\ F_w \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{bmatrix}\begin{bmatrix} u \\ w \end{bmatrix} - \begin{bmatrix} c_{xx} & c_{xz} \\ c_{zx} & c_{zz} \end{bmatrix}\begin{bmatrix} \dot{u} \\ \dot{w} \end{bmatrix} \tag{8}$$

2.4. The unbalance

The unbalance is due to a mass m_u located at a distance d from the geometric centre of the shaft and its kinetic energy is to be calculated. The mass is located on a perpendicular plane with respect to the y shaft and its coordinate with respect to y is constant. The angular diplacement is Ωt. In the reference coordinate system, the mass velocities and positions are:

$$\overrightarrow{OD} = \begin{vmatrix} u + d\sin\Omega t \\ \text{Cte.} \\ w + d\cos\Omega t \end{vmatrix}, \quad V = \frac{d\overrightarrow{OD}}{dt} = \begin{vmatrix} \dot{u} + d\Omega\cos\Omega t \\ 0 \\ \dot{w} - d\Omega\sin\Omega t \end{vmatrix} \tag{9}$$

and the kinetic energy is:

$$T_u = \frac{m_u}{2}\left[(\dot{u})^2 + (\dot{w})^2 + \Omega^2 d^2 + 2\Omega\dot{u}d\cos\Omega t - 2\Omega\dot{w}d\sin\Omega t\right] \tag{10}$$

The term $m_u\Omega^2 d^2/2$ is constant and will not modify the equations of motion. The mass m_u is a different measure for each disc and, hence, the kinetic can be simplified as in:

$$T_b \approx m_b\Omega d(\dot{u}\cos\Omega t - \dot{w}\sin\Omega t) \tag{11}$$

2.5. Equations of motion

The Rayleigh-Ritz method is characterized by the replacement of displacements u and w by two functions: one time dependent generalized coordinate $q_i(t)$ and one displacement function $f(y)$ depending on coordinate y along the shaft representing the shape of the first vibration mode of a bi-supported beam. Hence, u, w and their time derivatives can be expressed as:

$$\begin{aligned} u &= f(y)q_1\,, \quad w = f(y)q_2 \\ \dot{u} &= f(y)\dot{q}_1\,, \quad \dot{w} = f(y)\dot{q}_2 \\ f(y) &= \sin\frac{\pi y}{L}\,, \quad \frac{df(y)}{dy} = g(y) = \frac{\pi}{L}\cos\frac{\pi y}{L} \end{aligned}$$

Angles θ and ψ are supposed small and can be approximated by:

$$\theta = \frac{\partial w}{\partial y} = \frac{df(y)}{dy}q_2 = g(y)q_2 \tag{12}$$

$$\psi = -\frac{\partial u}{\partial y} = -\frac{df(y)}{dy}q_1 = -g(y)q_1 \tag{13}$$

The displacements and angles can be replaced in the energy expressions for the calculation of the global energy expressions. The global kinetic energy can be writen as:

$$T = T_{d1} + T_{d2} + T_s + T_{u1} + T_{u2} \tag{14}$$

where T_{di} are the expressions for the kinetic energy of the discs. T_s is the kinetic energy of the shaft and T_{ui} are the kinetic energies for the unbalance on each disc. Thus:

$$T = A\left(\dot{q}_1^2 + \dot{q}_2^2\right) + B\left(\dot{q}_1 q_2\right) + C\left(\dot{q}_1\cos\Omega t - \dot{q}_2\sin\Omega t\right) \tag{15}$$

where A, B and C are respectively:

$$A = \frac{1}{2}\left[M_{d1}f(y_1)^2 + M_{d2}f(y_4)^2 + Ix_{d1}g(y_1)^2 + Ix_{d2}g(y_4)^2 + \rho SL + \frac{\pi\rho I_a}{4L}\right] \tag{16}$$

$$B = \Omega\left[Iy_{d1}g(y_1)^2 + Iy_{d2}g(y_4)^2 + \rho I_a L\pi\right] \tag{17}$$

$$C = \Omega\left[m_{b1}d_1 f(y_1) + m_{b2}d_2 f(y_4)\right] \tag{18}$$

The global strain energy is expressed by the strain energy of the shaft as in Eq. (6):

$$U = \frac{\pi E I_a}{4}\left(\frac{\pi}{L}\right)^3 (q_1^2 + q_2^2) \tag{19}$$

The virtual work is calculated as the sum of the virtual works exerted by the two bearings on the shaft. It is also considered the simetry of cross coupling terms ($k_{xz} = k_{zx}$ and $c_{xz} = c_{zx}$). The generalized forces obtained are:

$$\begin{aligned} \left[\begin{array}{c} F_u \\ F_w \end{array}\right] = & -\left[\begin{array}{cc} k_{xx1}f(y_1)^2 + k_{xx2}f(y_2)^2 & k_{xz1}f(y_1)^2 + k_{xz2}f(y_2)^2 \\ k_{zx1}f(y_1)^2 + k_{zx2}f(y_2)^2 & k_{zz1}f(y_1)^2 + k_{zz2}f(y_2)^2 \end{array}\right]\left[\begin{array}{c} q_1 \\ q_2 \end{array}\right] \\ & -\left[\begin{array}{cc} c_{xx1}f(y_1)^2 + c_{xx2}f(y_2)^2 & c_{xz1}f(y_1)^2 + c_{xz2}f(y_2)^2 \\ c_{zx1}f(y_1)^2 + c_{zx2}f(y_2)^2 & c_{zz1}f(y_1)^2 + c_{zz2}f(y_2)^2 \end{array}\right]\left[\begin{array}{c} \dot{q}_1 \\ \dot{q}_2 \end{array}\right] \end{aligned} \tag{20}$$

indexes 1 and 2 represent bearings 1 and 2. More bearings can be added to the system by the same manner.

The system equations of motion are obtained by plugging the energy expressions into Lagrange's equation and solving for the time derivatives of the generalized displacements:

$$2A\ddot{q}_1 + \tfrac{B}{\Omega}\Omega\dot{q}_2 + \tfrac{\pi^4 E I_a}{2L^3} q_1 = C\Omega \sin \Omega t \tag{21}$$

$$2A\ddot{q}_2 - \tfrac{B}{\Omega}\Omega\dot{q}_1 + \tfrac{\pi^4 E I_a}{2L^3} q_2 = C\Omega \cos \Omega t \tag{22}$$

If one is to analyze the non-forced behaviour of the system, then the homogeneous equations are:

$$M_{eq}\ddot{q}_1 + C_{eq}\Omega\dot{q}_2 + K_{eq}q_1 = 0 \tag{23}$$

$$M_{eq}\ddot{q}_2 - C_{eq}\Omega\dot{q}_1 + K_{eq}q_2 = 0 \tag{24}$$

with $M_{eq} = 2A$, $C_{eq} = B/\Omega$ and $K_{eq} = \pi^4 E I_a/(2L^3)$. Initial conditions are used in the integration of the equations of motion allowing one to attain the precession orbits of the shaft. Depending on the inital conditions, it is possible to obtain forward or backward whirling orbits.

Natural frequencies are obtained assuming a solution such as $q_1 = Q_1 e^{rt}$ and $q_2 = Q_2 e^{rt}$ and using it in Eqs. (23) and (24) leading to the following homogeneous system of equations:

$$\left[\begin{array}{cc} M_{eq}r^2 + K_{eq} & C_{eq}\Omega r \\ -C_{eq}\Omega r & M_{eq}r^2 + K_{eq} \end{array}\right]\left\{\begin{array}{c} Q_1 \\ Q_2 \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 0 \end{array}\right\} \tag{25}$$

The non-trivial solution for the system will lead to the following characteristic equation:

$$M_{eq}r^4 + (2M_{eq}K_{eq} + C_{eq}^2\Omega^2)r^2 + K_{eq}^2 = 0 \tag{26}$$

When $\Omega = 0$ (static rotor) the solution to Eq. (26) is:

$$r_{1stat}^2 = r_{2stat}^2 = j^2\omega_{1stat}^2 = j^2\omega_{2stat}^2 = -\frac{K_{eq}}{M_{eq}} \tag{27}$$

with $j^2 = -1$. The frequencies are: $\omega_{1stat} = \omega_{2stat} = \sqrt{K_{eq}/M_{eq}}$. When the system is under rotation, roots r_1 and r_2 and the corresponding frequencies are:

$$r_1^2 = j^2\omega_1^2\,, \quad \omega_1 = \sqrt{\omega_{1stat}^2 + \frac{C_{eq}^2\Omega^2}{2M_{eq}^2}\left(1 - \sqrt{1 + \frac{4M_{eq}^2\omega_{1stat}^2}{C_{eq}^2}\Omega^2}\right)} \tag{28}$$

$$r_2^2 = j^2\omega_2^2\,, \quad \omega_2 = \sqrt{\omega_{1stat}^2 + \frac{C_{eq}^2\Omega^2}{2M_{eq}^2}\left(1 + \sqrt{1 + \frac{4M_{eq}^2\omega_{1stat}^2}{C_{eq}^2}\Omega^2}\right)} \tag{29}$$

Equations (28) and (29) show the evolution of the natural frequencies of modes 1 and 2 as the rotation speed increases. With those equations it is possible to build the Campbell diagram that will indicate critical rotational speeds of the system (speeds that coincide with natural frequencies causing resonance).

3. The Finite Element method

The Finite Element methodology used here employs a Timoshenko beam element with four degrees of freedom at each node. The basis for the system matrices are the matrices obtained for the saft. Elements such as discs and bearings are added to the shaft matrices in order to build the system matrices. In this problem, some important assumptions have to be made: there are no displacements in the y direction; angles and displacements are considered to be small; gyroscopic effects will happen only around the y direction; angles around the x and z directions are much smaller than Ω; plane cross-sections remain plane after defformation; element displacements are represented by Hermitian shape functions.

Discs and bearings are concentrated at nodal positions. This means that their behavior will be added to the system matrices in specific nodal positions and this will be done after the shaft matrices are built.

3.1. Discs

Each node possesses four degrees of freedom: two displcements u and w and two angles θ and ψ. The δ vector of nodal displacements of the centre of the disc is $\delta = [u, w, \theta, \psi]^t$. The use of Eq. (2) into Lagrange's equation leads to:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \delta}\right) - \frac{\partial T}{\partial \delta} = \begin{bmatrix} M_d & 0 & 0 & 0 \\ 0 & M_d & 0 & 0 \\ 0 & 0 & I_{dx} & 0 \\ 0 & 0 & 0 & I_{dx} \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} + \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_{dy} \\ 0 & 0 & I_{dy} & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} \tag{30}$$

where the first matrix is the mass matrix and the second matrix a gyroscopic matrix.

3.2. Shaft

The shaft is modelled as a beam with circular cross-section. Each element has two nodes and eigth degrees of freedom. Thus, the elementary matrices have eight degrees of freedom on its span. The relations between angles and displacements are:

$$\theta = \frac{\partial w}{\partial y}\ , \qquad \psi = -\frac{\partial u}{\partial y} \tag{31}$$

The nodal displacement vector is $\delta = [u_1, w_1, \theta_1, \psi_1, u_2, w_2, \theta_2, \psi_2]^t$, which can be split into two vectors:

$$\delta u = [u_1, w_1, \theta_1, \psi_1]^t \quad \text{and} \quad \delta w = [u_2, w_2, \theta_2, \psi_2]^t \tag{32}$$

Displacements u and w can be expressed as functions of the hermitian shape functions ($N_i(y)$) and nodal displacements δu and δw as in $u = N_1(y)\delta u$ and $w = N_2(y)\delta w$. $N_i(y)$ are the calssical shape functions for a beam in bending:

$$N_1(y) = \left[1 - \frac{3y^2}{L^2} + \frac{2y^3}{L^3}; -y + \frac{2y^2}{L} - \frac{y^3}{L^2}; \frac{3y^2}{L^2} - \frac{2y^3}{L^3}; \frac{y^2}{L} - \frac{y^3}{L^2}\right] \tag{33}$$

$$N_2(y) = \left[1 - \frac{3y^2}{L^2} + \frac{2y^3}{L^3}; y - \frac{2y^2}{L} + \frac{y^3}{L^2}; \frac{3y^2}{L^2} - \frac{2y^3}{L^3}; -\frac{y^2}{L} + \frac{y^3}{L^2}\right] \tag{34}$$

The kinetic energy is attained from Eq. (3) which gives:

$$\begin{aligned} T_s =& \frac{\rho S}{2}\int_0^L \left[\delta \dot{u}^t N_1^t N_1 \delta \dot{u} + \delta \dot{w}^t N_2^t N_2 \delta \dot{w}\right] dy + \frac{\rho I}{2}\int_0^L \left[\delta \dot{u}^t \frac{dN_1^t}{dy}\frac{dN_1}{dy}\delta \dot{u} + \delta \dot{w}^t \frac{dN_2^t}{dy}\frac{dN_2}{dY}\delta \dot{w}\right] dy \\ &- 2\rho I \Omega \int_0^L \left[\delta \dot{u}^t \frac{dN_1^t}{dy}\frac{dN_2}{dy}\delta w\right] dy + \rho I L \Omega^2 \end{aligned} \tag{35}$$

Replacing the shape functions (33) and (34) and their derivatives into Eq. (35) it is possible to reach the compact form:

$$T_s = \frac{1}{2}\delta \dot{u}^t M_1 \delta \dot{u} + \frac{1}{2}\delta \dot{w}^t M_2 \delta \dot{w} + \frac{1}{2}\delta \dot{u}^t M_3 \delta \dot{u} + +\frac{1}{2}\delta \dot{w}^t M_4 \delta \dot{w} + \Omega \delta \dot{u}^t M_5 \delta w + \rho I L \Omega^2 \tag{36}$$

where matrices M_1 and M_2 are the classical mass matrices, M_3 and M_4 represent the rotational inertia effects and M_5 represents the gyroscopic effets. The last term, which is constant, will not present any contribution when Lagrange's equation is applied to (36) as follows:

$$\frac{d}{dt}\left(\frac{\partial T_s}{\partial \dot{\delta}}\right) - \frac{\partial T_s}{\partial \delta} = (M + M_s)\ddot{\delta} + C\dot{\delta} \tag{37}$$

where M and M_s are deduced directly from M_1, M_2, M_3 and M_4 and C from M_s. The matrices are:

$$C = \frac{\rho I \Omega}{15L}\begin{bmatrix} 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 3L & 0 & 0 & -4L^2 & -3L & 0 & 0 & L^2 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 3L & 0 & 0 & L^2 & -3L & 0 & 0 & -4L^2 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \end{bmatrix} \text{gyroscopic matrix} \tag{38}$$

$$M_s = \frac{\rho I}{30L}\begin{bmatrix} 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ -3L & 0 & 0 & 4L^2 & 3L & 0 & 0 & -L^2 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \\ -3L & 0 & 0 & -L^2 & 3L & 0 & 0 & 4L^2 \end{bmatrix} \text{ rotational inertia matrix} \quad (39)$$

$$M = \frac{\rho SL}{420}\begin{bmatrix} 156 & 0 & 0 & -22L & 54 & 0 & 0 & 13L \\ 0 & 156 & 22L & 0 & 0 & 54 & -13L & 0 \\ 0 & 22L & 4L^2 & 0 & 0 & 13L & -3L^2 & 0 \\ -22L & 0 & 0 & 4L^2 & -13L & 0 & 0 & -3L^2 \\ 54 & 0 & 0 & -13L & 156 & 0 & 0 & 22L \\ 0 & 54 & 13L & 0 & 0 & 156 & -22L & 0 \\ 0 & -13L & -3L^2 & 0 & 0 & -22L & 4L^2 & 0 \\ 13L & 0 & 0 & -3L^2 & 22L & 0 & 0 & 4L^2 \end{bmatrix} \text{ translational inertia matrix} \quad (40)$$

The strain energy of the shaft is obtained by means of Eq. (6) leading to:

$$U_s = \frac{EI}{2}\int_0^L \left[\delta u^t \frac{d^2 N_1^t}{dy^2}\frac{d^2 N_1}{dy^2}\delta u + \delta w^t \frac{d^2 N_2^t}{dy^2}\frac{d^2 N_2}{dy^2}\delta w\right] dy \quad (41)$$

After integration, the compact form is:

$$U_s = \frac{1}{2}\delta u^t K_1 \delta u + \frac{1}{2}\delta w^t K_2 \delta w \quad (42)$$

where K_1 and K_2 are the classical stiffness matrices.

It is frequently necessary to take into account the effect of shear, which will modify the classical stiffness matrix. The stiffness matrix with shear effects being taken into account is writen as:

$$K = \frac{EI}{(1+a)L^3}\begin{bmatrix} 12 & 0 & 0 & -6L & -12 & 0 & 0 & -6L \\ 0 & 12 & 6L & 0 & 0 & -12 & 6L & 0 \\ 0 & 6L & (4+a)L^2 & 0 & 0 & -6L & (2-a)L^2 & 0 \\ -6L & 0 & 0 & (4+a)L^2 & 6L & 0 & 0 & (2-a)L^2 \\ -12 & 0 & 0 & 6L & 12 & 0 & 0 & 6L \\ 0 & -12 & -6L & 0 & 0 & 12 & -6L & 0 \\ 0 & 6L & (2-a)L^2 & 0 & 0 & -6L & (4+a)L^2 & 0 \\ -6L & 0 & 0 & (2-a)L^2 & 6L & 0 & 0 & (4+a)L^2 \end{bmatrix} \quad (43)$$

In Eq. (43) a represents the effect of shear and is given by $a = (12EI)/(GS_rL^2)$. G is the shear modulus, ν Poisson´s coefficient and $S \approx S_r$ the cross sectional area. If the shear effect is not to be taken into account, a must be set to zero.

3.3. Bearings

The bearings' stiffness and damping features are due to the displacements and velocities. The influence of angles and moments is usually not taken into account (Lalanne, 1996). Using Eq. (8) it is possible to achieve:

$$\begin{Bmatrix} F_u \\ F_w \\ F_\theta \\ F_\psi \end{Bmatrix} = -\begin{bmatrix} k_{xx} & k_{xz} & 0 & 0 \\ k_{zx} & k_{zz} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} u \\ w \\ \theta \\ \psi \end{Bmatrix} - \begin{bmatrix} c_{xx} & c_{xz} & 0 & 0 \\ c_{zx} & c_{zz} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} \quad (44)$$

The first matrix is a stiffness matrix and second matrix a viscous damping matrix. These matrices are not usually symetric and the terms can vary as a function of the rotating speed.

3.4. Unbalance

The general kinetic energy expression due to unbalance is given by expression (11). The application of Lagrange's equation with $\delta = [u, w]^t$ gives:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\delta}}\right) - \frac{\partial T}{\partial \delta} = -m_u d\Omega^2 \begin{bmatrix} \sin \Omega t \\ \cos \Omega t \end{bmatrix} \quad (45)$$

3.5. Assembly of equations of motion

In building the global matrices, the elemental mass, damping, stiffness and force entries are then added to the corresponding rows and columns of the global mass, damping, stiffness and force matrices as in the following equation:

$$[M]\ddot{\delta} + [C(\Omega)]\dot{\delta} + [K]\delta = m_u d \sin \Omega t + m_u d \cos \Omega t \tag{46}$$

The elemental matrices can be added together to form the global matrices shown in Fig. 2.

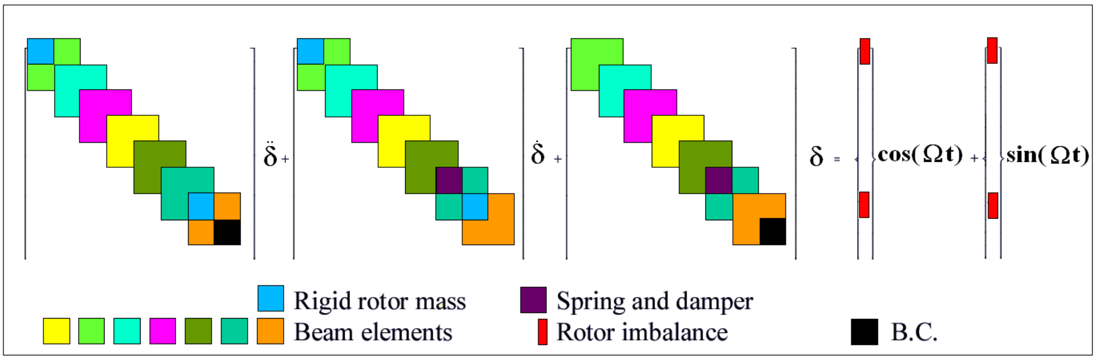

Figure 2: Assembly of system matrices

3.6. Natural Frequencies and time Integration

For the calculation of natural frequencies the pseudo-modal method was used. In this method, the system matrices are reduced allowing the calculation of the first lower frequency natural frequencies without loss of accuracy. A modal space is defined from the solutions of:

$$[M]\ddot{\delta} + [K^*]\delta = 0 \tag{47}$$

where K^* is the actual stiffness matrix with supressed k_{xz} and k_{zx} terms in order to maintain simetry. The first $n << N$ modes $\phi_1 \dots \phi_n$ are used to build a reduction matrix:

$$\phi = (\phi_1 \dots \phi_n) \tag{48}$$

used for the change of base ($\delta = \phi p$). p is the vector of modal variables. The change of base is performed by pre-multiplying Eq. (46) by ϕ^t, leading to:

$$\phi^t M \phi \ddot{p} + \phi^t C(\Omega) \phi \dot{p} + \phi^t K \phi p = \phi^t F(t) \tag{49}$$

The transformed matrices ($m = \phi^t M \phi,\ c = \phi^t C(\Omega)\phi,\ k = \phi^t K \phi,\ f = \phi^t F(t)$) are then used to build a reduced equation of motion:

$$m\ddot{p} + c\dot{p} + kp = f \tag{50}$$

The natural frequencies are obtained by assuming a solution such as $p = Pe^{rt}$. Replacing this solution into Eq. (50) produces:

$$[r^2 m + rc + k]p = 0 \tag{51}$$

This equation can be expressed as:

$$\begin{bmatrix} 0 & I \\ -k^{-1}m & -k^{-1}c \end{bmatrix} \begin{Bmatrix} rP \\ P \end{Bmatrix} = \frac{1}{r} \begin{Bmatrix} rP \\ P \end{Bmatrix} \tag{52}$$

The solution of the eigenproblem will produce complex conjugate frequencies that are modified as the value of Ω changes. The frequencies are in the form:

$$r_i = -\frac{\alpha_i \omega_i}{\sqrt{1-\alpha_i^2}} \pm j\omega_i \tag{53}$$

where α_i is also known as the damping factor. The orbits are obtained by the Newmark integration method.

4. Results

In this section the results for both methodologies are shown. Simulations with the twin-rotor simplified model of Coaracy Nunes power plant, shown in Fig. 1, were carried out with and without mass unbalance conditions on both rotors. In the mass unbalance case, the larger rotor was considered with a mass unbalance of 100 kg located at 2.5 m from its centre. The smaller rotor was considered with a mass unbalance of 50 kg located at 1.8 m from its centre.

4.1. Rayleigh-Ritz methodology

With the Rayleigh-Ritz methodology, it was possible to build the Campbell diagram and the whirling orbits for forward whirl and backward whirl depicted in the following figures by the red lines. The blue line represents points on which the rotating frequency is equal to the natural frequency. The green line depicts the responce to unbalance conditions.

The unbalance response was obtained using the equations of motion and plotted in the same figure of the Campbell Diagram. One can note that the peak amplitude is located at the same rotational speed of the first critical speed which is 1280 rpm. The rotational speed of the assembly is 150 rpm which assures safety against resonance. If anisotropic bearings were to be considered, the two curves (in red) would separate at the point where the shaft is stopped ($\Omega = 0$ rpm) and the green curve would show two peaks with the second peak coinciding with the second critical speed.

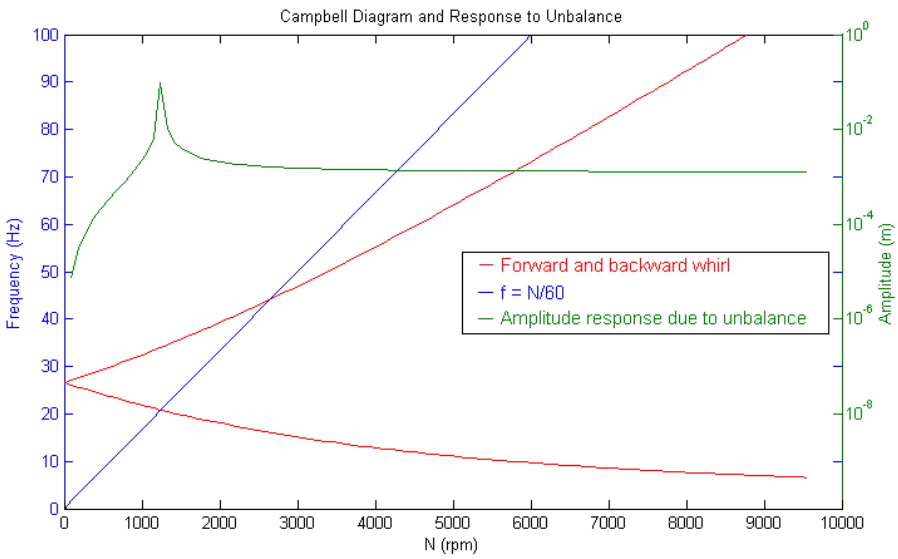

Figure 3: Campbell Diagram for Coaracy Nunes assembly showing unbalance response

The whirling orbit for forward whirl in the balanced and unbalanced cases are shown in Fig. 4. It is possible to note that, in the unbalanced case, the orbits are somewhat more coarse than in the balanced rotor case.

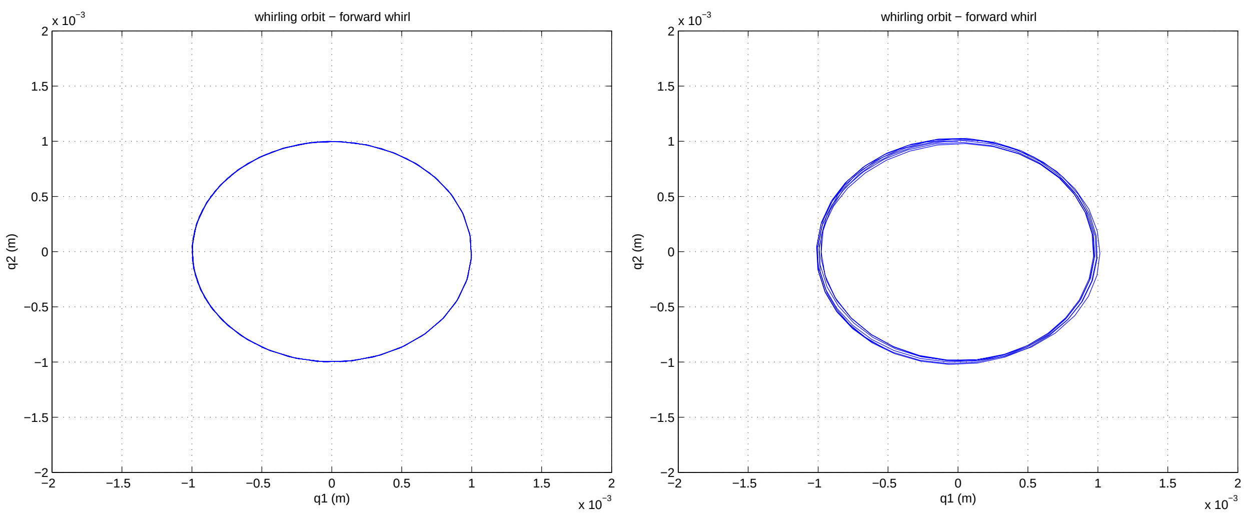

Figure 4: Forward whirl orbit for balanced and unbalanced rotor

The orbits for the Rayleigh-Ritz simulation were obtained by means of a fourth order Runge-Kutta integration procedure. The forward and backward configurations can be achieved by changing the initial conditions for each generalized displacement and speed.

4.2. The Finite Element methodology

For the Finite Element method, the evolution of natural frequencies with the increase of rotational speed was obtained by means of an eigenvalue procedure of a system including a damping matrix. As a result, all the frequencies obtained are of complex form. The mode frequencies used in the campbell diagram are complex conjugate frequencies as shown in Eq. (53). Due to this behaviour, the forward and backward natural frequency curves present a more symetric shape as shown in Fig. 5.

The whirling orbits were obtained using the Newmark integration procedure. This procedure is a single step integration formula (Géradin, 1997) where the system state vector at a subsequent time is deduced from the state vector at the actual time through a taylor series expansion of displacement and velocities. In the Newmark scheme, the errors of the taylor series expansion are represented by means of integrals being aproximated by the gaussian quadracture. The whirling orbit for the balanced case is shown in Fig. 6.

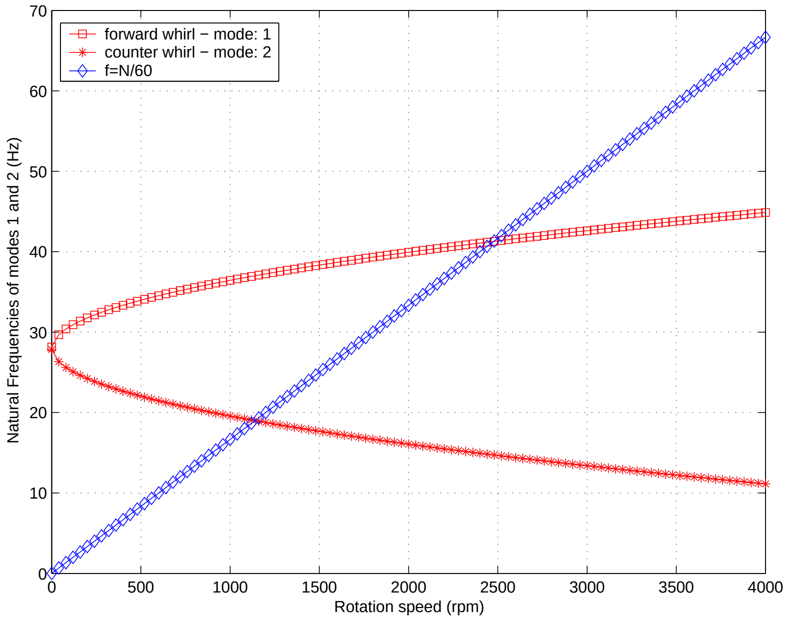

Figure 5: Campbell Diagram for Coaracy Nunes assembly - Finite Element

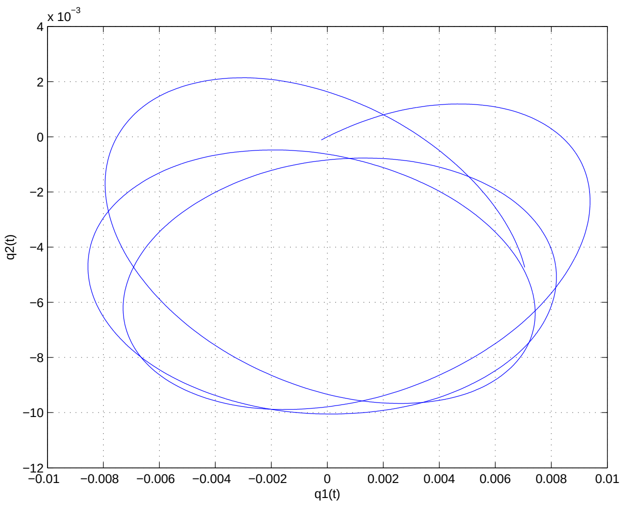

Figure 6: Forward whirl orbit for balanced rotors

The whirling orbit obtained shows a pathline that is not as circular as the one obtained for the Rayleigh-Ritz methodology. This feature is a result of the initial conditions necessary for each degree of freedom. In this case, the displacement initial condition was the first natural mode shape with frequency of 1089 rad/s (5.25 Hz). The velocity initial condition was the first mode shape multiplied by its frequency. The trend towards a circular pathline for the shaft observed in Fig. 6 shows a right direction for the choice of such initial conditions. This direction would not be regarded if the Rayleigh-Ritz results were not at hand.

5. Conclusions

This work presented the use of classical methodologies in the rotordynamic analysis of a hydro turbine rotor-generator assembly. To this end, the Rayleigh-Ritz methodology was applied in order to obtain a general direction for the Finite Element procedure. Due to the use of few degrees of freedom the Rayleigh-Ritz results allowed a better understanding of the general behaviour of the rotor-generator assembly through the assessment of the evolution of its natural frequencies with the increase of rotational speed. The response due to unbalanced conditions was applied to confirm the critical speed obtained with the Campbell diagram. The whirling orbit obtained showed a circular orbit which indicates that the shaft is rotating in its own natural frequency speed.

The Finite Element assessment carried out in this work showed an approximation to the results of the Rayleigh-Ritz procedure because the former results were already known and allowed a certain calibration of the Finite-Element routines.

The routines built for the Finite Element analysis present a general scope of applications and can be used in the analysis of many rotordynamics problems taking into account, naturally, their level of simplicity. The use of Timoshenko beam elements is particularly interesting due to the inclusion of shear effects into the stiffness matrix.

The use of classical and traditional methodologies in the analysis of a simplified model of complex structures is extremely important because it will suply valuable information necessary in the analysis of the same problem using comercial packages and more complex geometries. This comprehensive approach will allow the assessment of mode shapes and natural frequencies that are already known at some extent. Hence, errors due to lack of information about the natural physical behaviour of such structure will be greatly diminished.

6. References

Almeida, M. T., 1987, “Vibrações Mecânicas para Engenheiros”, Ed. Edgard Blücher, S.Paulo, Brazil, 400 p.

Betting, B., 2000, “Rotordynamics”, Notes on website www.me.mtu.edu/ mdrl, Michigan Technological University, Michigan, USA.

Géradin, M., Rixen, D., 1997, “Mechanical Vibrations, Theory and Application to Structural Dynamics”, John Wiley & Sons, Chichester, England, 425 p.

Hoffman, J. D., 2001, “Numerical Methods for Scientists and Engineers”, Marcel Dekker, New York, 823 p.

Krodkiewski, J. M., 2002, “Dynamics of Rotors”, Course Material, University of Melbourne, Department of Mechanical and Manufacturing Engineering, Melbourne, Australia.

Lalanne, M., Ferraris, G., 1990, “Rotordynamics Prediction in Engineering”, John Wiley & Sons, London, England, 198 p.

Lalanne, M., Ferraris, G., 1996, “Dynamique des Rotors en Flexion”, Results of research problems, Institut National des Sciences Appliquées (INSA) de Lyon, Lyon, France.

Mariz, L.G. R., 2003, “Análise Modal do Conjunto Rotor-Gerador de Turbina Hidráulica Usando Elementos Finitos”, Projeto de Graduação, Universidade de Brasília, Departamento de Engenharia Mecânica, Brasília, Brasil.

Meirovitch, L., 1986, “Elements of Vibration Analysis”, McGraw Hill, New York, USA, 560 p.

Pereira, J. C., 2003, “Introdução à Dinâmica de Rotores”, Lecture Notes, Universidade Federal de Santa Catarina, Departamento de Engenharia Mecânica, Florianópolis, Brasil.

Sobral, Y. D., 2002, “Modelos Para o Estudo do Comportamento Dinâmico de Rotores de Turbinas Hidráulicas”, Projeto de Graduação, Universidade de Brasília, Departamento de Engenharia Mecânica, Brasília, Brasil.

7. Responsibility notice

The authors are the only responsible for the printed material included in this paper.