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VIBRATION CONTROL OF LAMINATE COMPOSITE PLATES USING EMBEDDED ACTIVE-PASSIVE PIEZOELECTRIC NETWORKS

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Abstract. *This work presents results from a study of laminate composite plates with embedded piezoelectric patches connected to active-passive resonant shunt circuits, composed of resistance, inductance and voltage source. Applications to passive vibration control and active control authority enhancement are also presented and discussed. The finite element model is based on an equivalent single layer theory combined with a third-order shear deformation theory. The formulation results in a coupled finite element model with mechanical (displacements) and electrical (charges at electrodes) degrees of freedom. For a Graphite-Epoxy laminate composite plate, a parametric analysis is performed to evaluate optimal locations along the plate plane (xy) and thickness (z) that maximize the effective modal electromechanical coupling coefficient. Then, the passive vibration control performance is evaluated for a network of optimally located shunted piezoelectric patches embedded in the plate, through the design of resistance and inductance values of each circuit, to reduce vibration amplitude of first four vibration modes. A vibration amplitude reduction of at least 10 dB for all vibration modes was observed. Then, an analysis of the control authority enhancement due to the resonant shunt circuit, when the piezoelectric patches are used as actuators, is performed. It is shown that the control authority can indeed be improved near a selected resonance even with multiple pairs of piezoelectric patches and active-passive circuits acting simultaneously.*

Palavras-chave: *Piezoelectric materials, shunt circuits, active-passive piezoelectric networks, laminate plates*

1. INTRODUCTION

Piezoelectric materials have been widely used as distributed sensors and actuators for structural vibration control (Sunar and Rao, 1999). In particular, piezoelectric ceramics, such as the lead titanate zirconate (PZT), are interesting choices for integrated sensors and actuators since they can be found in the form of small and thin patches and, hence, may be bonded to or embedded in laminate composite structures leading to little structural modification combined to a relatively high electromechanical coupling. Depending on the electronics connected to the piezoelectric transducers, the vibration control can be passive, active or hybrid active-passive (Ahmadian and DeGuilio, 2001). Passive control is particularly interesting since there is no need to inject additional energy into the system so that the mechanical energy dissipation is due only to transformation into electrical energy and heat (Hagood and von Flotow, 1991). Shunted damping is based on the important electromechanical coupling it provides, converting part of the vibratory energy into electric energy which is then dissipated through the shunt circuit (Hagood and von Flotow, 1991; Viana and Steffen Jr., 2006).

More recently, research was redirected to combined active and passive vibration control techniques (Tang *et al.*, 2000). One of these techniques, so-called Active-Passive Piezoelectric Networks (APPN), integrates an active voltage source with a passive resistance-inductance shunt circuit to a piezoelectric sensor/actuator (Tsai and Wang, 1999). In this case, the piezoelectric material serves two purposes. First, the vibration strain energy of the structure can be transferred to the shunt circuit, through the difference of electric potential induced in the piezoelectric material electrodes, and then passively dissipated in the electric components of the shunt circuit (Hagood and von Flotow, 1991). On the other hand, the piezoelectric material may also serve as an actuator for which a control voltage can be applied to actively control the structural vibrations. This technique allows to simultaneously dissipate passively vibratory energy through the shunt circuit and actively control the structural vibrations. Combined active-passive control was shown to perform better with smaller cost than separate active and passive ones, provided the simultaneous action is optimized (Tsai and Wang, 1999).

In order to better understand the electromechanical interaction between piezoelectric sensors and actuators and vibrating structures, several research groups have focused on proposing and implementing modeling techniques for structures containing piezoelectric elements. In particular, usual modeling techniques for laminate structures were adapted and extended to include the piezoelectric effect. There are today several modeling techniques for laminate structures available that include piezoelectric elements. Most of them can be grouped in: i) three-dimensional elasticity theories, ii) two-dimensional equivalent single layer theories, and iii) discrete-layer or layerwise theories. Both analytical and numerical methods can be applied to approximate the solutions of the equations resulting from these models (Saravanos and Heyliger, 1999). Two-dimensional equivalent single layer theories yield equations of motion that are easier to solve but also contain a series of simplifying assumptions. In particular, some a priori assumptions for the kinematics along the thickness direction must be made. For that, there have been a number of propositions presented in the literature, imposing polynomial functions of different orders or piecewise continuous functions to interpolate the kinematics behavior across the thickness. The finite element formulation proposed by Reddy (1999) using equivalent single layer combined to first-order shear (FSDT) and third-order shear (TSDT) deformation theories was used in the present work.

However, in order to study laminate plates with piezoelectric materials connected to active and passive shunt circuits, it is also necessary to account for the coupling between piezoelectric elements and external circuits. There are mainly two techniques used in the literature to perform that task. The first consists in modifying the equivalent piezoelectric element impedance considering piezoelectric element, in unidirectional deformation, and electric circuit as impedances in parallel for which the admittance can be summed. This leads to a modification in the constitutive equations for the piezoelectric material and, in particular, its elastic coefficient becomes a function of the circuit impedance (Hagood and von Flotow, 1991). A second approach considers the equivalence between the charge generated/induced in the piezoelectric material electrodes and the charge flowing in the electric circuit to which it is connect. This allows to write independently the equations of motion for the structure with piezoelectric elements and the electric circuit before coupling the charge variables (Thornburgh and Chattopadhyay, 2002). Unlike the first one, this technique allows a time-domain analysis and does not require unidirectional deformation in the piezoelectric material.

This work presents results from a study of laminate composite plates with embedded piezoelectric patches connected to active-passive resonant shunt circuits, composed of resistance, inductance and voltage source. The formulation results in a coupled finite element model with mechanical (displacements) and electrical (charges at electrodes) degrees of freedom. For a Graphite-Epoxy laminate composite plate, a parametric analysis is performed to evaluate optimal locations along the plate plane (xy) and thickness (z) that maximize the effective modal electromechanical coupling coefficient. Then, the passive vibration control performance is evaluated for a network of optimally located shunted piezoelectric patches embedded in the plate, through the design of resistance and inductance values of each circuit, to reduce the vibration amplitude of the first four vibration modes. Then, an analysis of the control authority enhancement, when the piezoelectric patches are used as actuators, due to the resonant shunt circuit is performed.

2. FINITE ELEMENT MODEL

A laminated plate composed of elastic and piezoelectric layers is considered. Both elastic and piezoelectric layers are supposed to be thin, such that a plane stress state can be assumed, perfectly bonded and made of orthotropic piezoelectric materials. Elastic layers are obtained by setting their piezoelectric coefficients to zero. Upper and lower surfaces of piezoelectric layers are assumed to be fully covered by electrodes. The equivalent single layer theory is used so that the same displacement field is considered for all $2n$ layers of the laminate plate. Also, for simplifying purposes, only symmetric laminate plates are considered, however the electrical displacements in each layer are independent. The displacements fields for the laminate plate are defined according to a third-order shear deformation theory (TSDT) proposed by Reddy (1999). Linear orthotropic piezoelectric materials with material symmetry axes parallel to the plate ones are considered here. The potential energy of a given piezoelectric layer is described by its electric enthalpy density leading to h-form constitutive equations that are then reduced using the hypotheses of plane stress, $\sigma_3 = 0$, and unidirectional electric displacement, $D_1 = D_2 = 0$.

A rectangular finite element model with four nodes is considered using linear Lagrange's interpolation functions for the in-plane displacements, u_0 and v_0, and cross-section rotations at midplane, ψ_x and ψ_y, and cubic Hermite's interpolation functions, nonconforming, for the transversal displacement w_0. For the electrical displacements in each layer, linear Lagrange's interpolation functions were also considered. Therefore, a four-node finite element model with seven mechanical degrees of freedom, u_0, v_0, ψ_x, ψ_y, w_0, $w_{0,x}$ and $w_{0,y}$, and four (per piezoelectric layer) electrical degrees of freedom is obtained.

The equations of motion for the laminate plate composed of elastic and piezoelectric layers can then be obtained using Hamilton's principle and, then, assembling the equations for all finite elements such that

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{D}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_m & -\mathbf{K}_{me} \\ -\mathbf{K}_{me}^{\mathrm{t}} & \mathbf{K}_e \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{D} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ 0 \end{Bmatrix}, \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}_m$, $\mathbf{K}_{me}$ and $\mathbf{K}_e$ are mass and elastic, piezoelectric and dielectric stiffness global matrices, respectively. $\mathbf{u}$ and $\mathbf{D}$ are global vectors of nodal mechanical and electrical degrees of freedom. More details can be found in (Godoy, 2008).

These equations of motion account for the coupling between mechanical displacements of whole laminate plate, including piezoelectric layers, and electrical displacements induced in the piezoelectric layers. It is possible to consider different boundary conditions for both mechanical and electrical quantities, for instance open ($\mathbf{D} = 0$) and closed ($\mathbf{D} \neq 0$)

circuit conditions for the piezoelectric layers. However, in this work, it is supposed that each piezoelectric layer may be connected to an electrical circuit, composed of resistance, inductance and voltage source. For a series of independent resonant (resistive-inductive) electrical circuits coupled to voltage sources, the equations of motion are

$$\mathbf{L}_c\ddot{\mathbf{q}}_c + \mathbf{R}_c\dot{\mathbf{q}}_c = \mathbf{V}_c, \tag{2}$$

where $\mathbf{L}_c$ and $\mathbf{R}_c$ are diagonal matrices and $\mathbf{q}_c$ and $\mathbf{V}_c$ are vectors with elements L_c^j, R_c^j, q_c^j and V_c^j, respectively, which are the inductance, resistance, electrical charge and applied voltage for the j-th circuit.

Combining the equations of motion of the circuits with those of the structure with piezoelectric elements yields

$$\begin{bmatrix} \mathbf{M} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{L}_c \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{D}} \\ \ddot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{R}_c \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{D}} \\ \dot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_m & -\mathbf{K}_{me} & 0 \\ -\mathbf{K}_{me}{}^{\mathrm{t}} & \mathbf{K}_e & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{D} \\ \mathbf{q}_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ 0 \\ \mathbf{V}_c \end{Bmatrix}. \tag{3}$$

In order to couple the electrical circuits and piezoelectric structure, two additional steps should be undertaken. First, an equipotential surface representing the electrodes over the upper and lower surfaces of the piezoelectric layers must be reinforced. This is done by rewriting the global vector of nodal electrical displacements $\mathbf{D}$ in terms of a vector of electrical displacements in each independent piezoelectric patch or layer, such that

$$\mathbf{D} = \mathbf{L}_p\mathbf{D}_p,\ \mathbf{D}_p = \begin{bmatrix} D_{p1} & D_{p2} & D_{p3} & \cdots & D_{pm} \end{bmatrix}^{\mathrm{t}}, \tag{4}$$

where $\mathbf{L}_p$ is a binary matrix that imposes the equality between the electrical displacements at the nodes located in the region covered by an electrode. Then, the total electrical charge accumulated in an electrode is related to the electrical displacement by $\mathbf{q}_p = \mathbf{A}_p\mathbf{D}_p$, where $\mathbf{A}_p$ is a diagonal matrix containing the surface area of the electrodes covering each piezoelectric patch. Next, it is assumed that all electrical charge accumulated in an electrode will flow through the circuit to which it is connected. This allows to write an equality relation between the electrical charge $\mathbf{q}_p$, and thus the electrical displacement $\mathbf{D}_p$, in the patches and the electrical charge $\mathbf{q}_c$ in the circuits, such that $\mathbf{D}_p = \mathbf{A}_p^{-1}\mathbf{q}_p = \mathbf{A}_p^{-1}\mathbf{q}_c$. Replacing this relation into (4) leads to

$$\mathbf{D} = \mathbf{B}_p\mathbf{q}_c,\ \mathbf{B}_p = \mathbf{L}_p\mathbf{A}_p^{-1}. \tag{5}$$

Replacing (5) in a variational form of the equations of motion (3) and summing the terms related to $\mathbf{q}_c$, one gets the following reduced equations of motion in terms of the mechanical displacement $\mathbf{u}$ and circuit electrical charges $\mathbf{q}_c$

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{L}_c \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{R}_c \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{q}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_m & -\bar{\mathbf{K}}_{me} \\ -\bar{\mathbf{K}}_{me}{}^{\mathrm{t}} & \bar{\mathbf{K}}_e \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{q}_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_m \\ \mathbf{V}_c \end{Bmatrix}, \tag{6}$$

where $\bar{\mathbf{K}}_{me} = \mathbf{K}_{me}\mathbf{B}_p$ and $\bar{\mathbf{K}}_e = \mathbf{B}_p^{\mathrm{t}}\mathbf{K}_e\mathbf{B}_p$.

3. PASSIVE VIBRATION CONTROL USING RESONANT CIRCUITS

In this section, the TSDT finite element model is used to evaluate and optimize the performance of a passive vibration control system using piezoelectric patches connected to resonant (resistive-inductive) shunt circuits. This is done by first placing four pairs of piezoelectric patches in a laminate composite plate, so that energy conversion between plate and electrical circuits is maximized, and then designing shunt circuit components (resistance and inductance) so that reduction of vibration amplitude of the plate near first four resonances is maximized. The plate is made of 12 Graphite-Epoxy AS4/3501-6 layers (90º/ 90º/ 0º/ 0º/ 90º/ 90º// 90º/ 90º/ 0º/ 0º/ 90º/ 90º) with dimensions $(250 \times 350 \times 1.5)$ mm and is supposed to be clamped on all sides (CCCC). The piezoelectric patches are made of thickness-poled PZT-5A piezoceramic material with dimensions $(25 \times 25 \times 0.25)$ mm. The material properties for the GE, are: $c_{11} = 146$ GPa, $c_{22} = 9.71$ GPa, $c_{33} = 11.7$ GPa, $c_{12} = 2.91$ GPa, $c_{13} = 4.92$ GPa, $c_{23} = 4.76$ GPa, $c_{44} = 3.45$ GPa, $c_{55} = c_{66} = 4.14$ GPa, $\rho^{GE} = 1389$ kg m^{-3}. The material properties of through-thickness poled PZT-5A is: $c_{11}^E = c_{22}^E = 99.201$ GPa, $c_{33}^E = 86.856$ GPa, $c_{12}^E = 54.016$ GPa, $c_{13}^E = c_{23}^E = 50.778$ GPa, $c_{44}^E = c_{55}^E = 21.100$ GPa, $c_{66}^E = 22.600$ GPa, $e_{15} = e_{24} = 12.322$ Cm^{-2}, $e_{31} = e_{32} = -7.209$ Cm^{-2}, $e_{33} = 15.118$ Cm^{-2}, $\epsilon_{11}^\sigma = \epsilon_{22}^\sigma = 15.3$ nFm^{-1}, $\epsilon_{33}^\sigma = 15$ nFm^{-1}, $\rho^{pzt} = 7750$ kg m^{-3}.

3.1 Optimal Positioning of Piezoelectric Patches on the Plate

It is known that the squared modal electromechanical coupling coefficient (EMCC) is an adequate indicator of the amount of strain energy converted to electrical energy by a piezoelectric patch when the structure to which it is bonded/

embedded vibrates in a given mode shape (Trindade and Benjeddou, 2009). Therefore, the EMCC is used here as a metric to perform the positioning of four symmetric pairs of patches throughout the plate, via parametric analysis, such that each patch maximizes the EMCC of one of the first four vibration modes of a laminate plate.

In order to analyze the optimal positioning of piezoelectric patches that maximizes the electromechanical coupling, a parametric analysis were performed. This was done through variation of the x, y and z position of one symmetrical pair of patches and subsequent evaluation of the electromechanical coupling of the pair for the first four vibration modes. Three positions across the thickness z direction were chosen based on a previous analysis. In the first two positions, each piezoelectric patch replaces part of the first and last, starting from the midplane, two adjacent layers of graphite-epoxy, respectively. In the third case, the piezoelectric patches are bonded on the external (upper/lower) surfaces of the laminate plate. In the plane xy, 9 positions in x and in y are considered according to Fig. 1. 700 plate elements were used.

The EMCC of a given piezoelectric patch/layer used in the evaluations is defined as (Trindade and Benjeddou, 2009)

$$K_j^2 = ({f_{oc}^j}^2 - {f_{sc}^j}^2)/{f_{oc}^j}^2, \tag{7}$$

where f_{oc}^j and f_{sc}^j denote the j-th eigenfrequency of the structure evaluated with the piezoelectric patch/layer in open-circuit and in short-circuit, respectively.

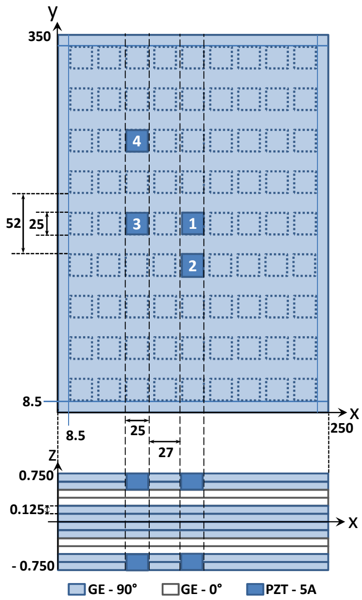

Figure 1. Representation of possible and optimal (highlighted) positions in x and y directions for piezoelectric patches.

Previous results (Godoy and Trindade, 2009) have shown that, as expected, the optimal position of the patch follows the mode shapes such that the coupling is maximum at the regions of maximum curvature, which are, for instance, at the center of the plate for the first mode. Then, four pairs of piezoelectric patches are considered, where each pair is considered to focus at one of the four vibration modes and, hence, is located where maximum coupling is obtained for the corresponding vibration mode. The resulting design is defined by considering all highlighted patches in Fig. 1. The coupling coefficients were then evaluated for each vibration mode by making all patches in open circuit or short circuit and show that, although each pair was located to optimize the coupling for only one vibration mode, the second and fourth pairs lead to non-null coupling with vibration modes other than the ones they were designed for. More details can be found in (Godoy, 2008; Godoy and Trindade, 2009).

3.2 Design of Resonant Shunt Circuits

This section presents a simplified formulation based on the theory of dynamic vibration absorbers for the design of a passive shunt circuit working similarly to mechanical vibration absorbers, but using piezoelectric patches as converters of mechanical energy into electrical energy and the shunt circuit as an electrical absorber. For that, the resistance and inductance of the circuit have to be properly tuned to optimize the energy absorption/dissipation. In order to use the standard theory of dynamic vibration absorbers, the equations of motion (6) are reduced to two degrees of freedom, one mechanical, corresponding to the modal displacement of one vibration mode of interest, and one electrical, corresponding to the electrical charge in the shunt circuit. This simplification implies that one or more piezoelectric patches must be connected to a single shunt circuit and that the contribution of other vibration modes to the structural response is neglected. Therefore, consider the following modal decomposition for the nodal displacements as

$$\mathbf{u}(t) = \boldsymbol{\phi}_n \alpha_n(t), \tag{8}$$

where $\boldsymbol{\phi}_n$ is the mass-normalized vibration mode of interest and α_n is the corresponding modal displacement. Moreover, as only one circuit is considered, the equations of motion (6) are reduced to

$$\ddot{\alpha}_n + \omega_n^2 \alpha_n - k_p q_c = b_n f \quad \text{and} \quad L_c \ddot{q}_c + R_c \dot{q}_c + \bar{K}_e q_c - k_p \alpha_n = 0, \tag{9}$$

with the following definitions: $k_p = \boldsymbol{\phi}_n^{\mathrm{t}} \bar{\mathbf{K}}_{me}$, $\mathbf{F}_m = \mathbf{b} f$ and $b_n = \boldsymbol{\phi}_n^{\mathrm{t}} \mathbf{b}$. Notice that ω_n is the n-th eigenfrequency of the structure considering open-circuit condition for all piezoelectric patch(es) that are connected to the electrical circuit.

In order to evaluate the frequency response function of the structure with piezoelectric patch(es) connected to the electrical shunt circuit, let us consider that the structure is excited through a harmonic mechanical force $f = \tilde{f} e^{\mathrm{j}\omega t}$, such that $\alpha_n = \tilde{\alpha}_n e^{\mathrm{j}\omega t}$ and $q_c = \tilde{q}_c e^{\mathrm{j}\omega t}$, and its response is measured by a displacement sensor that gives output $y = \mathbf{cu}$, where $\mathbf{c}$ is a row vector that defines the degree of freedom that is measured. Hence, $y = \tilde{y} e^{\mathrm{j}\omega t}$, where $\tilde{y} = c_n \tilde{\alpha}_n$ and $c_n = \mathbf{c}\boldsymbol{\phi}$. The equations of motion (9a) and (9b) then become

$$(\omega_n^2 - \omega^2)\tilde{\alpha}_n - k_p \tilde{q}_c = b_n \tilde{f} \quad \text{and} \quad (-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)\tilde{q}_c - k_p \tilde{\alpha}_n = 0. \tag{10}$$

Solving (10b) for $\tilde{q}_c$ and substituting in (10a), it is possible to write $\tilde{\alpha}_n$ and, thus $\tilde{y}$, in terms of the excitation amplitude $\tilde{f}$, such that $\tilde{y} = G_p(\omega)\tilde{f}$, where $G_p(\omega)$ is the frequency response function of the structure, subjected to a mechanical excitation, which results in

$$G_p(\omega) = c_n b_n \frac{-\omega^2 L_c + \bar{K}_e + \mathrm{j}\omega R_c}{\omega^4 L_c - \omega^2(\bar{K}_e + \omega_n^2 L_c) + \omega_n^2 \bar{K}_e - k_p^2 + \mathrm{j}\omega R_c(\omega_n^2 - \omega^2)}. \tag{11}$$

The amplitude of the frequency response function, for limited values of R_c, can be shown to have an anti-resonance at a frequency equal to the resonance frequency of the electrical circuit, defined as $\omega_c = (\bar{K}_e / L_c)^{1/2}$. One strategy to minimize the vibration amplitude of the structure at a resonance frequency of interest is to design the resonance frequency of the secondary system, the shunt circuit in the present case, such that it coincides with the resonance frequency of interest of primary system, the structure in the present case. Therefore, the following identity is defined $\omega_n = \omega_c$, which allows to evaluate directly the inductance of the shunt circuit as

$$L_c = \bar{K}_e / \omega_n^2. \tag{12}$$

Nevertheless, the designed anti-resonance at ω_n is accompanied by two resonances, before and after ω_n, which must have their amplitude controlled to minimize or prevent amplification of the original vibration amplitude. This can be done using the resistance parameter of the shunt circuit. One possible methodology based on the dynamic vibration absorbers theory, is to design the damping parameter, the resistance of the shunt circuit in the present case, such that the amplitude at the anti-resonance is approximately equal to that at one of the two invariant frequencies, which are independent of the damping parameter, found by the following relation

$$\lim_{R_c \to 0} |G_p(\omega)|^2 = \lim_{R_c \to \infty} |G_p(\omega)|^2, \tag{13}$$

and, in the present case, assume the form

$$\omega_{1,2}^2 = \frac{1}{2}\left[\omega_c^2 + \omega_n^2 \pm \sqrt{(\omega_c^2 - \omega_n^2)^2 + 2\omega_c^2 (k_p^2 / \bar{K}_e)}\right]. \tag{14}$$

Evaluation of vibration amplitudes at one of these frequencies ω_1 and at anti-resonance ω_n yields

$$|G_p(\omega_1)|^2 = \frac{R_c^2 \omega_n^2}{k_p^4} \text{ and } |G_p(\omega_n)|^2 = \frac{2\bar{K}_e}{k_p^2 \omega_n^2}, \tag{15}$$

and, then, equalized such that the following expression for the resistance R_c can be found in terms of the equivalent coupling stiffness k_p, electrical stiffness $\bar{K}_e$ and structural resonance frequency of interest ω_n,

$$R_c = \frac{k_p \sqrt{2\bar{K}_e}}{\omega_n^2}. \tag{16}$$

Table 1. Values for R_c and L_c calculated using (12) and (16) and updated after simulations.

	1 pair of PZT patches				4 pairs of PZT patches			
	R_c (Ω)		L_c (H)		R_c (Ω)		L_c (H)	
Circuit	calculated	updated	calculated	updated	calculated	updated	calculated	updated
1	987	988	6.30	6.20	678	650	5.13	5.11
2	104	730	1.99	1.99	136	310	1.93	1.94
3	375	350	1.48	1.44	306	468	1.54	1.47
4	133	280	0.97	0.96	63	580	1.02	0.98

3.3 Performance Results for Passive Vibration Control using Resonant Circuits

The methodology presented in the previous section was then applied to the clamped laminate composite plate with symmetrical pairs of piezoelectric patches represented in Fig. 1. Each pair is considered to be connected to a resonant shunt circuit, for which the resistance and inductance are evaluated according to the expressions (12) and (16) using the corresponding eigenfrequencies ω_n and eigenmodes $\boldsymbol{\phi}_n$.

The frequency response functions for the laminate plate were evaluated considering five cases and using 600 plate elements. For the first four cases, a single pair of piezoelectric patches, located to properly couple with one of the first four vibration modes according to Fig. 1, is considered. For the last case, the four pairs of piezoelectric patches are embedded simultaneously in the laminate plate and connected to the corresponding resonant shunt circuit to yield a multi-modal vibration reduction. In all cases, the frequency response function of the laminate plate with shunted piezoelectric patches was compared to the case of open-circuit condition to analyze the performance of vibration amplitude reduction. For comparison purposes, a modal damping of 0.5% was considered for the host structure.

A preliminary analysis showed that, for most cases, the values of R_c and L_c had to be adjusted to correct the hypothesis of single mode used in the previous section. While the values of L_c are only slightly changed, the values of R_c are normally very different from the ones predicted by (16), as shown in Tab. 1. Considering the updated values for R_c and L_c, an analysis is performed considering that the four pairs of piezoelectric patches are embedded simultaneously in the laminate plate and connected to the corresponding resonant shunt circuit to evaluate the case of a multi-modal vibration reduction. Fig. 2 shows the frequency response function of the laminate plate with four pairs of piezoelectric patches, distributed according to Fig. 1, in two electrical boundary conditions: i) all patches in open-circuit (OC) and ii) each pair of patches connected to a properly designed resonant shunt circuit, focusing one of the four first vibration modes. It is possible to observe in Fig. 2 that the vibration amplitude at the first four resonance frequencies is effectively reduced by the shunted piezoelectric patches, especially for the fourth resonance. In addition, the vibration amplitude at the fifth resonance, which was not included in the design, is also slightly reduced. In this case, a vibration amplitude reduction of more than 10 dB is also observed, but for the first four vibration modes simultaneously.

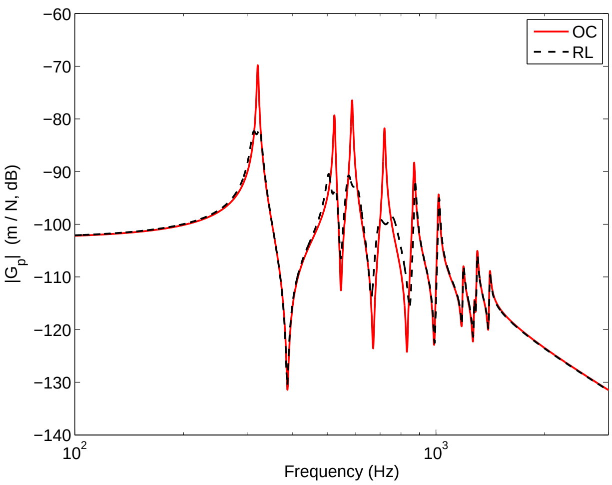

Figure 2. FRF of laminate plate with four shunted piezoelectric patches pairs designed for the first four vibration modes.

4. CONTROL AUTHORITY ENHANCEMENT USING RL SHUNTS

As an extension of the use of shunt circuits connected to piezoelectric elements for passive vibration control, it is also possible to include a voltage source to these circuits so that the piezoelectric elements may act both as vibration dampers/absorbers and actuators. Hence, passive, active and active-passive vibration control could be obtained. In the particular case of the circuit proposed previously, composed of a resistance, an inductance and a voltage source, it is worthwhile to analyze the active and active-passive action, since the passive action is obtained by eliminating the voltage source, which is the case treated in the previous section.

4.1 Evaluation of Control Authority of Piezoelectric Actuators

To evaluate the active action (control authority) of the piezoelectric elements, through an RL circuit, a harmonic excitation analysis is performed considering excitation through a single voltage source, that is a single piezoelectric element connected to a circuit composed of resistance, inductance and voltage source, such that

$$\mathbf{F}_m = 0,\ \mathbf{V}_c = \tilde{V}_c e^{\mathrm{j}\omega t},\ \mathbf{u} = \tilde{\mathbf{u}} e^{\mathrm{j}\omega t},\ \mathbf{q}_c = \tilde{q}_c e^{\mathrm{j}\omega t}, \tag{17}$$

and, thus, the equations of motion (6) can be rewritten as

$$\begin{aligned} (-\omega^2\mathbf{M} + \mathbf{K}_m)\tilde{\mathbf{u}} - \bar{\mathbf{K}}_{me}\tilde{q}_c &= \mathbf{0}, \\ -\bar{\mathbf{K}}_{me}^{\mathrm{t}}\tilde{\mathbf{u}} + (-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)\tilde{q}_c &= \tilde{V}_c. \end{aligned} \tag{18}$$

Solving the second equation of (18) for $\tilde{q}_c$ and replacing the solution in the first equation leads to

$$\left[-\omega^2\mathbf{M} + \mathbf{K}_m - \bar{\mathbf{K}}_{me}(-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)^{-1}\bar{\mathbf{K}}_{me}^{\mathrm{t}}\right]\tilde{\mathbf{u}} = \bar{\mathbf{K}}_{me}(-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)^{-1}\tilde{V}_c. \tag{19}$$

To evaluate the frequency response function of the structure subjected to an electrical excitation, it is supposed that a selected displacement in the structure is measured, such that

$$\tilde{y} = \mathbf{c}\tilde{\mathbf{u}}, \tag{20}$$

and the complex frequency response of the displacement output subjected to a voltage input is $\tilde{y} = G_c(\omega)\tilde{V}_c$, where

$$G_c(\omega) = \mathbf{c}\left[-\omega^2\mathbf{M} + \mathbf{K}_m - \bar{\mathbf{K}}_{me}(-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)^{-1}\bar{\mathbf{K}}_{me}^{\mathrm{t}}\right]^{-1}\bar{\mathbf{K}}_{me}(-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)^{-1}. \tag{21}$$

As in the passive case studied previously, the resistance and inductance of the electric circuits lead to a modification of the dynamic stiffness of the piezoelectric elements. The purely active case can be represented by considering $\mathbf{L}_c = \mathbf{R}_c = \mathbf{0}$. In this case, the frequency response function reduces to

$$G_c^V(\omega) = \mathbf{c}\left[-\omega^2\mathbf{M} + \mathbf{K}_m - \bar{\mathbf{K}}_{me}\bar{K}_e^{-1}\bar{\mathbf{K}}_{me}^{\mathrm{t}}\right]^{-1}\bar{\mathbf{K}}_{me}\bar{K}_e^{-1}, \tag{22}$$

where $\bar{\mathbf{K}}_{me}\bar{K}_e^{-1}$ indicates the equivalent force vector induced per voltage applied to the actuator and the reduction of piezoelectric stiffness accounts for the modification from a constant electric displacement to a constant electric field condition. The open-circuit case ($R_c \to \infty$) leads to the impossibility of actuation as expected.

For the most general case, the inductance and resistance not only modify the dynamic stiffness of the piezoelectric element, leading to vibration damping and/or absorption, but also affects the active control authority of the actuator due to the term $\bar{\mathbf{K}}_{me}(-\omega^2 L_c + \mathrm{j}\omega R_c + \bar{K}_e)^{-1}$ in (21). In particular, it may be possible to amplify active control authority near the resonance of the electric circuit (Tsai and Wang, 1999; Niezrecki and Cudney, 1994; Sirohi and Chopra, 2001).

4.2 Results for a Laminate Plate with Piezoelectric Actuators

In this section, the same laminate composite plate with four pairs of piezoelectric patches from the passive vibration control study is considered to evaluate the control authority of the piezoelectric patches, when they are used to excite the plate. This is done adding a voltage source in each one of the independent electrical circuits connected to the piezoelectric patches. To allow the analysis of the control authority enhancement provided by the RL elements of the circuit, the control authority $G_c(\omega)$ of a piezoelectric patch when connected to an electrical circuit with resistance, inductance and voltage source (RLV) is compared to the one obtained with a piezoelectric patch connected to an electrical circuit with only voltage source (V), $G_c^V(\omega)$, that is considering $L_c = R_c = 0$. This analysis is performed for each one of the four pair of piezoelectric patches, considering first only one pair of piezoelectric patches embedded into the plate and then with the four pairs of piezoelectric patches simultaneously.

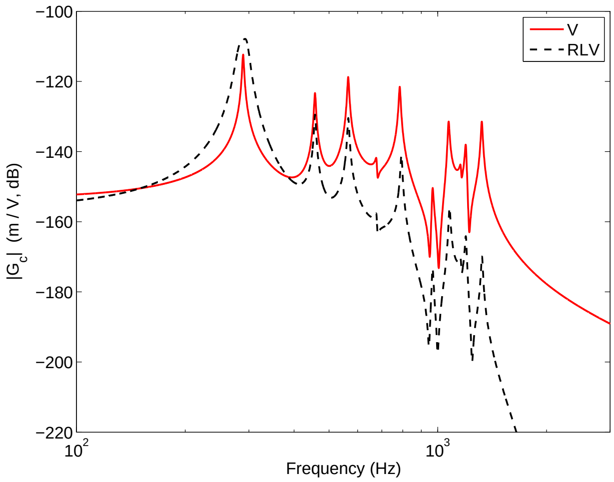

Figure 3. Control authority of first pair of piezoelectric patches with (RLV) and without (V) resonant circuit.

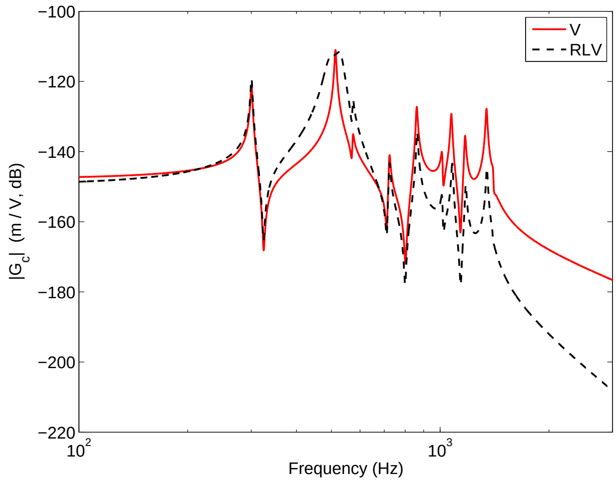

Figure 4. Control authority of second pair of piezoelectric patches with (RLV) and without (V) resonant circuit.

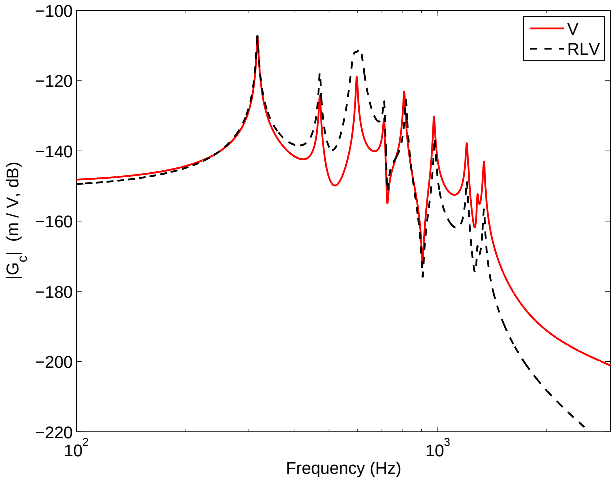

Figure 5. Control authority of third pair of piezoelectric patches with (RLV) and without (V) resonant circuit.

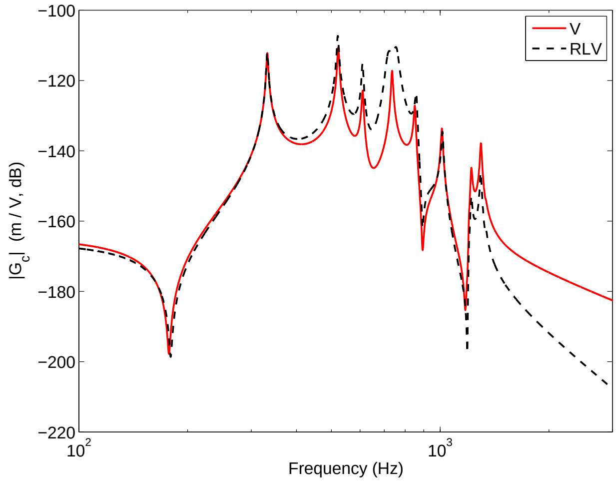

Figure 6. Control authority of fourth pair of piezoelectric patches with (RLV) and without (V) resonant circuit.

Figures 3, 4, 5 and 6 present the control authority of each one of the four pairs of piezoelectric patches, connected to RLV circuits tuned to the first four resonances, when only the active pair is embedded into the structure. In Figure 3, it is easy to notice that the control authority around the first resonance frequency is indeed enhanced at the cost of reducing the control authority at other frequencies. Unlike previous results published in the literature (Tsai and Wang, 1999; Santos and Trindade, 2009), for this case the control authority is enhanced even exactly at the first resonance frequency and not only around it. For the other three cases (Figures 4, 5 and 6), it becomes evident that the resonant circuit amplifies the control authority for a certain frequency range around the chosen resonance frequency. Indeed, for the third pair presented in Figure 5, the control authority is enhanced for the second, third and fourth resonance frequencies, although more effectively for the third one as expected. It should also be noticed that the location of the piezoelectric patch also affects the control authority. For instance, the second pair is not effective for controlling the third and fourth modes, even through the resonant circuit.

Figures 7, 8, 9 and 10 present the control authority of each one of the four pairs of piezoelectric patches, connected to RLV circuits tuned to the first four resonances, when the four pairs are embedded into the structure but only one of them is active. In this case, the inactive pairs are considered to be connected to a passive RL shunt circuit, that is without the voltage source. In this case, the control authority enhancement is still possible, but it can be noticed, in particular for the second and third pairs (Figures 8 and 9), that the passive circuits may affect the control authority of the active circuit. As an alternative analysis for simultaneous action of the four pairs, Figure 11 presents the control authority of the four pairs of piezoelectric patches, connected to RLV circuits tuned to the first four resonances, when the four pairs are embedded into the structure and connected to the same voltage source. In this case, it is possible to observe an enhancement on the control authority of the four pairs simultaneously.

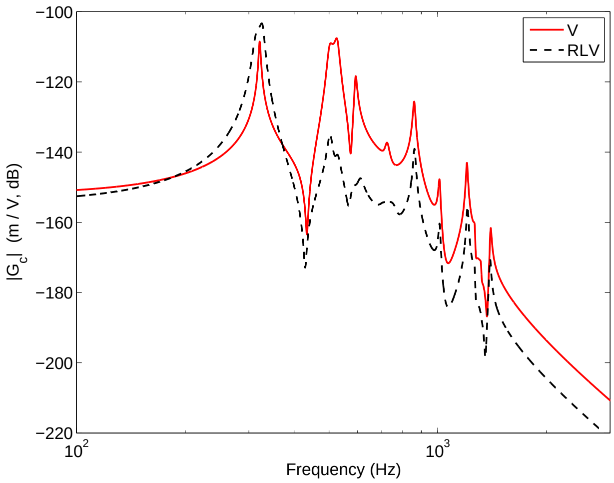

Figure 7. Control authority of first pair of piezoelectric patches with (RLV) and without (V) resonant circuit when the other three pairs are connected to a passive RL circuit.

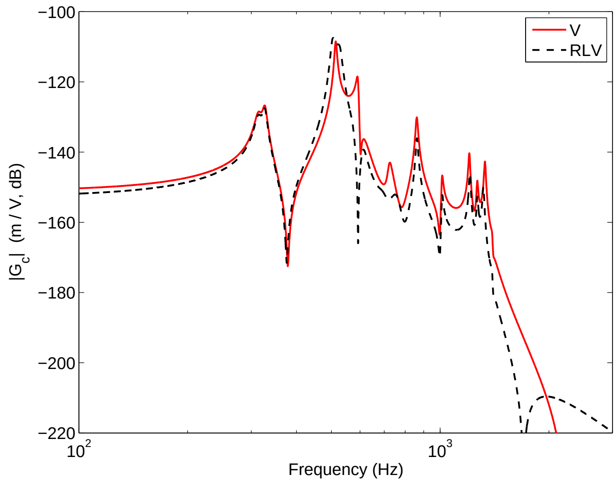

Figure 8. Control authority of second pair of piezoelectric patches with (RLV) and without (V) resonant circuit when the other three pairs are connected to a passive RL circuit.

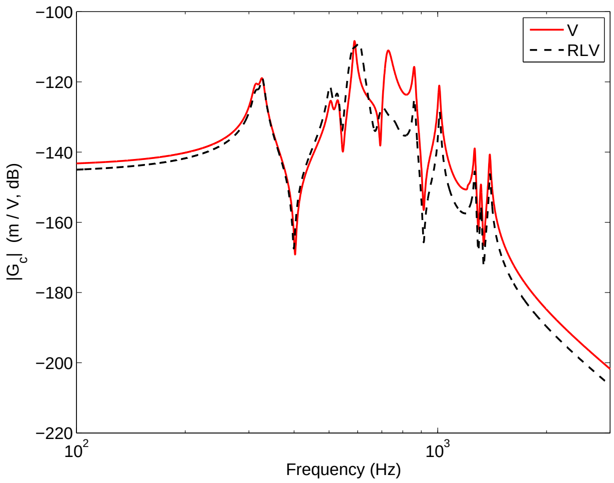

Figure 9. Control authority of third pair of piezoelectric patches with (RLV) and without (V) resonant circuit when the other three pairs are connected to a passive RL circuit.

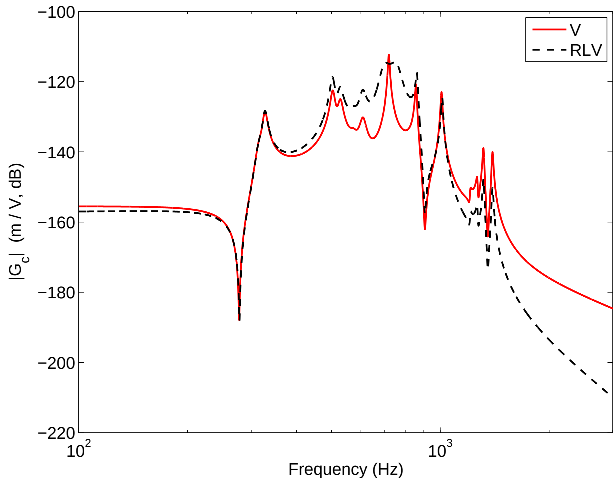

Figure 10. Control authority of fourth pair of piezoelectric patches with (RLV) and without (V) resonant circuit when the other three pairs are connected to a passive RL circuit.

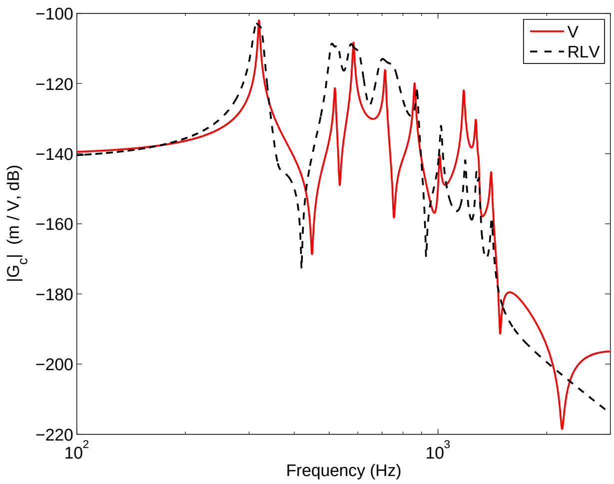

Figure 11. Control authority of four pairs of piezoelectric patches with (RLV) and without (V) resonant circuit when acting simultaneously.

5. CONCLUSIONS

The finite element modeling of laminate composite plates with embedded piezoelectric patches or layers connected to active-passive resonant shunt circuits, composed of resistance, inductance and voltage source, was presented. Applications to passive vibration control and active control authority enhancement were studied and discussed. An equivalent single layer theory combined with a third-order shear deformation theory was considered for the laminate plate and a stress-voltage electromechanical model was considered for the piezoelectric materials. Also, the dynamics of the electrical circuits connected to piezoelectric materials were accounted for resulting in a coupled finite element model with mechanical (displacements) and electrical (charges at electrodes) degrees of freedom. The model was then used to optimize the location of a set of piezoelectric patches, using the electromechanical coupling coefficient, and design the electrical circuit components for optimal passive vibration control and active control authority enhancement. A vibration amplitude reduction of at least 10 dB for the first four vibration modes was obtained. It was also shown that the control authority can indeed be improved near a selected resonance even with multiple pairs of piezoelectric patches and active-passive circuits acting simultaneously.

6. ACKNOWLEDGMENTS

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