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EXPERIMENTAL AND NUMERICAL DYNAMICAL ANALYSIS OF AN END MILLING PROCESS

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Abstract. *Machine tool vibration on end milling processes plays an important role concerning the cutting characteristics. The correct understanding of this process is important to improve the workpiece surface quality and avoid tool breakage. Chatter and squeal are some undesirable phenomena related to an improper functioning and it can be avoided changing the machining parameters. This article deals with experimental and numerical analysis of the end milling process. Basically, we try to establish a qualitative description of the machining process by considering a nonsmooth two-degree of freedom system. The cutting force is described by a composition of contact/non-contact of the tool and the workpiece. Stick-slip behavior is one of the important nonsmooth phenomenon guided by friction force and the prescribed velocity of the tool holder. Concerning experimental tests, tool displacement and cutting force are measured. Numerical simulations are carried out establishing a qualitative comparison with experimental data.*

Keywords: *End milling, force prediction, nonlinear dynamics, nonsmooth systems, chatter, machining.*

1. INTRODUCTION

Machining process is associated with a complex dynamics that involves the coupling of different phenomena. During cutting processes, for example, it is common to have temperature variations that induce dramatic changes in expected behaviors. Although all these complexity, some simple models may be used in order to obtain useful information concerning tool and workpiece behaviors as dynamic tool prediction.

The machine tool vibration during the cutting process plays an important role concerning the workpiece surface quality and also the tool durability. Chatter and squeal are some undesirable phenomena related to an improper functioning. During certain cutting conditions, motions of the workpiece-tool system are characterized by large amplitudes, which are not desirable for obtaining a good surface finish. Undesirable motions can result in wavy surfaces on the workpiece, inaccurate dimensions, and excessive tool wear.

The analysis of machining using a dynamical approach has been done in different research efforts. The first works in this way is due to Tobias (1965), Merritt (1965) and Tlusty (Altintas, 2000) that investigates the chatter on the process. Their research provided instability diagrams informing safe parameters to avoid chatter. After them, a new approach has been developed using nonlinear dynamic tools as bifurcation analysis, providing better comprehension of the process (Moon, 1978) .

Another references treated machining operations by dynamical approach. For turning operations, Chandiramani and Pothala (2006) analyzed the dynamics of cutting considering a two-degree of freedom system (2-dof) and orthogonal cutting modeling to predict chatter. Pratt and Nayfeh (1999) studied the boring bars for turning, which commonly presents chatter problems and experimentally determined modal properties of the tool. Also dedicated to turning, Kalmar-Nagy *et al* (2001) showed the existence of a subcritical Hopf bifurcation in the delay-differential equation used to describe the machining equations of motion. The stability of the milling process was investigated by Zhao and Balachandran (2001), Insperger *et al* (2003) and Gradisek *et al* (2005) by considering single and two degree of freedom systems for different experimental conditions. The mathematical model is represented by delay-differential equations.

Milling process has the peculiarity to have a contact/non-contact behavior due to its geometry and due to tool vibration. This kind of behavior is related to nonsmooth systems that are usually associated with either the friction phenomenon or the discontinuous characteristics as intermittent contacts (Savi *et al*, 2007) .

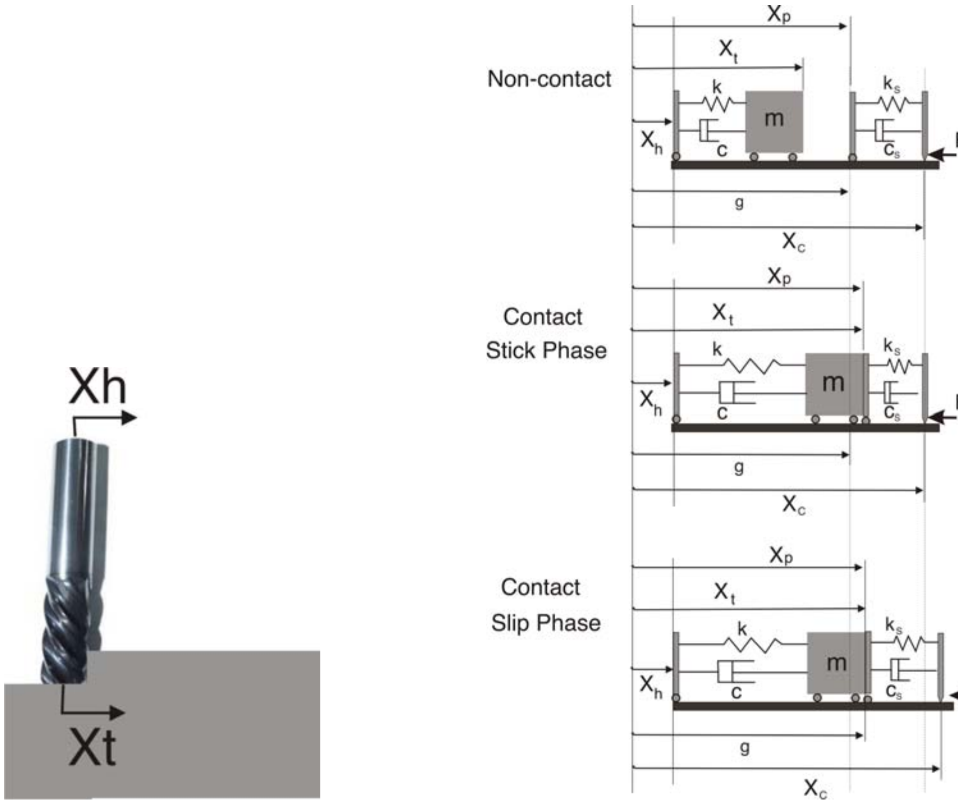
This article investigates the milling process either by experimental or by numerical approaches. The mathematical model assumes that the end milling tool is represented by a nonsmooth mass-spring-dashpot system (Araujo and Savi,

2008) . The process of cutting is related to a stick-slip behavior that defines whether the chip is being removed from the workpiece. The tool holder displacement is prescribed and the local force that allows the slip motion is related to the workpiece shear stress. Under these assumptions, the equation of motion is represented by a differential equation that is solved employing the Runge-Kutta method. Numerical simulations are carried out showing some situations related to proper and improper functioning during the cutting process.

2. DYNAMICAL MODEL

The milling process is of concern by assuming that it is a full immersed milling in the x direction (Araujo *et al*, 2009). Figure 1(a) shows the general view of the tool and the workpiece, where it is identified the prescribed displacement, X_h and the tool tip displacement, X_t. This process is modeled by assuming a nonsmooth system that is composed by a primary system that represents the tool and a secondary system, representing the workpiece. The primary system consists of a linear spring-dashpot-mass oscillator with parameters m, k, c and displacement X_t, and presents a gap that separates itself from the secondary system that represents the work-piece.

The workpiece system has weightless slider connected to a linear spring-dashpot system with parameters k_s, c_s that can present progressive motion when the acting force exceeds F_h, here represented by a dry friction, and displacement X_p, the tool-work piece contact position. Similarly to the stick-slip phenomena reported by Marian (2001), the progressive motion X_c occurs when the force acting on the slider exceeds the threshold of the dry friction force. Figure 1(b) presents a schematic picture of the cutting process system related to the milling.



(a) Tool (b) System model

Figure 1 - Tool and system modeling in contact and non-contact situations

As the system may operate in contact/non-contact modes, its dynamical behavior is nonlinear and its equations should consider each case. The system dynamics may be understood as a three-stage motion as represented in Figure 2: non-contact, contact in stick and contact in slip. The non-contact stage may be understood as two independent systems representing the situation where the tool is not in contact with the workpiece. Figure 2(a) shows that the displacements X_t and X_h are not connected with X_c and X_p by the mass.

The gap g is used to locate the work-piece position related to the tool position. This variable remains equal to X_t (as in Figure 2a) in non-contact case and in stick phase, as shown in Figure 2. The displacement g changes only when the slider slips simulating the chip removal. As long as g is greater than X_t, the system remains separate. When there is no more gap between X_t and X_p, both systems are in contact and the dynamic changes, as shown in Figure 2(b), for stick stage, and Figure 2(c) for slip stage. In stick stage the slider displacement does not change and the system is restricted to a fixed boundary. In slip stage this boundary is free and X_c displacement changes. The equations of motion are formulated by considering each kind of response separately.

2.1 Non-contact Stage

In non-contact stage, when $X_t < g$, the secondary system has no movement. Its displacement does not change $\dot{X}_p = \dot{X}_c$ and the primary system is described by the dynamic equation of a single-degree of freedom system, as follows:

$$m(\ddot{X}_t - \ddot{X}_h) + c(\dot{X}_t - \dot{X}_h) + k(X_t - X_h) = 0 \quad (1)$$

The holder force is calculated by:

$$F_h = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) \quad (2)$$

In order to rewrite the equation 2 for either contact and non-contact situations, the Heaviside function $H(.)$ is included. Note that the damping coefficient when the tool retracts from the workpiece ($\dot{X}_t < 0$) is not considered.

$$F_h = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) - H[X_t - g]\left(k_s(X_t - X_c) + H[\dot{X}_t]c_s\dot{X}_t\right) \quad (3)$$

The difference between chip displacement X_c and work-piece displacement X_p when the secondary system is free is constant and it is defined as δ, used to calculate the chip movement X_p on slip phase.

2.2 Contact - Stick Stage

The chip, represented by the slider, is not removed in stick stage. This case occurs when the friction force F_f is less than a maximum friction force F_f supported by the workpiece material, which can be considered as a function of the shear area and the maximum shear stress.

$$F_f = A_s\tau_s \quad (4)$$

The friction force is calculated, in stick phase, by:

$$F_{cont} = H[X_t - g]\left(k_s(X_t - X_c) + H[\dot{X}_t]c_s\dot{X}_t)\right) \quad (5)$$

2.3 Contact - Slip Stage

During slip stage, when the chip is being removed, the maximum friction force is achieved. The chip position X_c follows the workpiece displacement, $X_c = X_p + \delta$, and g assumes the work-piece position $g = X_p$. The friction force is equal to the maximum friction force $F_h = F_f$.

The force applied in the tool tip cannot be calculated by equation (3) due to the difference between $k_s(X_t - X_c) + H[\dot{X}_t]c_s\dot{X}_t)$ and F_f. Therefore, during slip phase the force F_h is:

$$F_h = -k(X_t - X_h) - c(\dot{X}_t - \dot{X}_h) - F_f \quad (6)$$

2.4 Equations of motion

Under these assumptions, the system dynamics may be represented by a nonsmooth system as follows, using F_{cont} described on Eq (5) and F_f on Eq (4):

$$m(\ddot{X}_t - \ddot{X}_h) + k(X_t - X_h) + c(\dot{X}_t - \dot{X}_h) = \left| \begin{array}{ll} F_{cont} & \text{if } F_{cont} < F_f \\ F_f & \text{elseif} \end{array} \right. \quad (7)$$

2.5 System parameters

The machine tool is represented by the tool holder and it has a prescribed displacement X_h defined by the feed velocity v_f, the spindle speed w and the run-out distance ρ between the the tool axis and the spindle speed axis. All these aspects are considered by the following equation:

$$X_h(t) = v_f t + \rho\sin(wt) + X_{h0} \quad (8)$$

where X_{h0} is constant and the feed velocity is writen as:

$$v_f = f_t w z \quad (9)$$

The run-out, ρ, is calculated based on the tool tip displacement. Therefore, the tool is only twisting without any tool deflection nor cutting force. The milling tool is considered as a flexible body with linear elastic behavior, therefore, the tool parameters m, k and c are calculated by considering solid mechanics principles from the tool geometry and material properties.

The stiffness of the primary system k is measured in laboratory and an the workpiece stiffness is evaluated by assuming that the workpiece is a compression beam, therefore:

$$k_s = \frac{E_c dt}{50L} \tag{10}$$

where workpiece material Young modulus E_c is considered as 200 GPaand t is the depth of cut.

The uncut chip thickness is not considered as in Martellotti equation (Araujo *et al*, 2006). The dry fiction is evaluated from the shear strength calculated by the shear area as a function of the depth of cut t and approximated by:

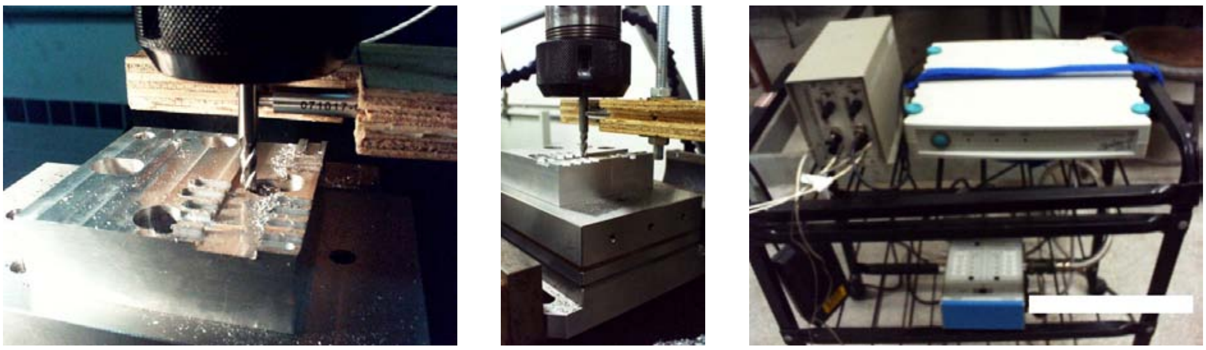
$$A_s = \int_0^\pi t f_t sin\phi d\phi \tag{11}$$

The chip thickness is given by the simulation as the displacement steps that the work-piece material let the tool cut. In order to improve the surface quality, the chip steps must be as smaller as possible. Also, the tool cannot touch and leave the work-piece with large displacements, because it causes impacts on the tool.

3. EXPERIMENTAL ANALYSIS

3.1 Experimental Set-up

An experimental set-up shown in Fig. (2) is employed to measure cutting force and tool tip displacement. A capacitive sensor (Lion Precision) is fixed to the machine in an inertial base, measuring the vibration of the tool tip on the feed direction, as illustrated in Fig. (2a). The feed component of the cutting force F_{cont} is measured by a piezoelectric dynamometer (Kistler Type 9257BA), as indicated in Fig. (2b). Both signals are amplified and conditioned in different equipments, being converted to digital data on a Spider A/D system (Fig. 2c).



(a) **(b)** **(c)**

Figure 2 - Experimental Set-up

The spindle speed is defined as 2500 rpm and the feed per tooth 0.007 mm. The tool has diameter of 5 mm and the tool is fixed in 39.9mm from the tool tip. The elastic modulus of the tool and those parameters are presented in Table 1.

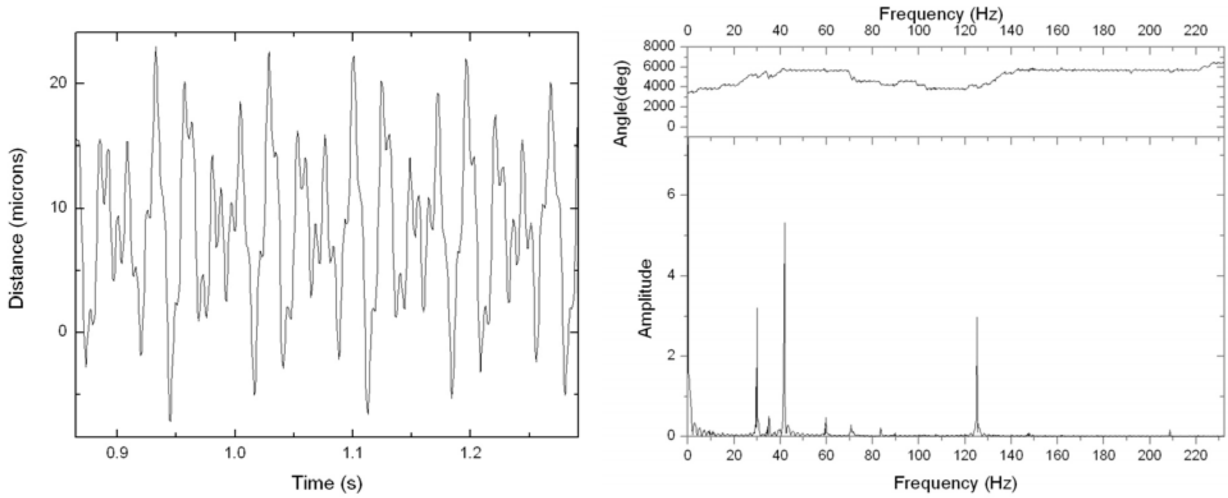
Table 1 - Tool, workpiece and machine tool properties.

Diameter d	Height L	Elastic Modulus E_t	Spindle Speed (w)	Feed per tooth f_t
5 mm	39.9 mm	200 GPa	2500 rpm	0.007 mm

3.2 Experimental Results

Initially, it is considered the non-cutting dynamics of the system. By imposing the spindle speed of 2500 rpm, the system response is shown in Fig. 3. The runout calculated is assumed to be as half signal amplitude of free tool tip. Displacement has amplitudes of approximately 20 microns.

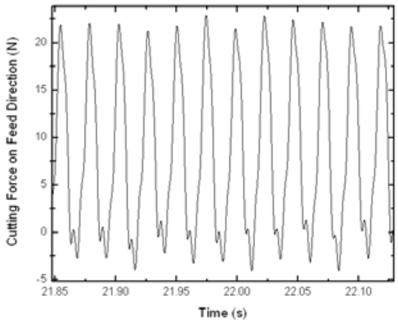
After this first approach, the stiffness of the primary system k is measured in laboratory by imposition of a measured force and measuring the tip tool displacement. The measured value is 640 N/m.



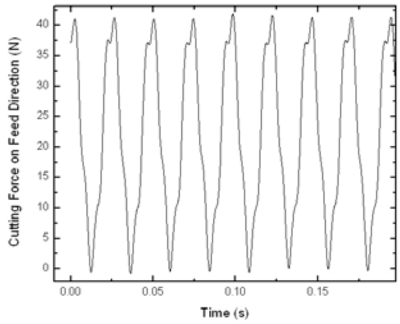
(a) Displacement **(b) FFT amplitude spectrum**

Figure 3 - Free Tool Tip Displacement

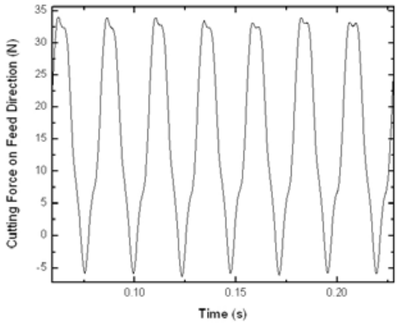
At this point, the influence of depth of cut on the stability and dynamic behavior of the cutting process is of concern. Basically, different situations are of concern varying the depth of cut from 0.25mm to 2.25mm. Fig. 4 shows the cutting force for all experiments while Fig. 5 presents the tool tip displacement. The phase space for all experiments is presented on Fig. 6. Results have a periodic aspect being related to sub-harmonic oscillations.



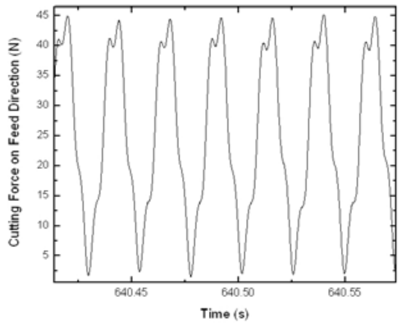
(a) $p_f = 0.25$mm



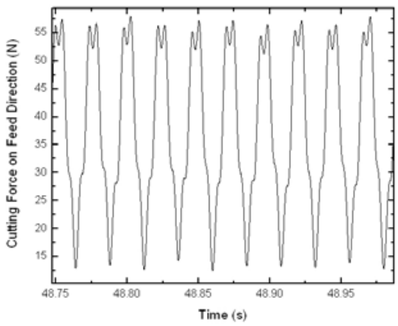
(b) $p_f = 0.5$mm



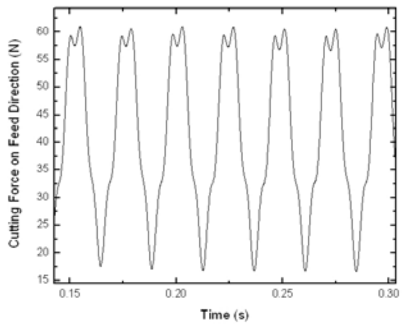
(c) $p_f = 0.75$mm



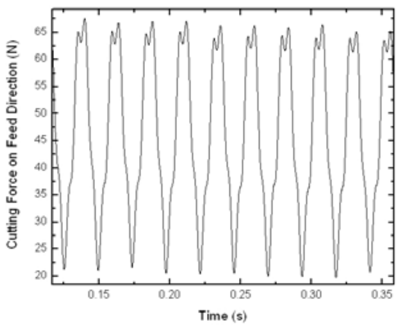
(d) $p_f = 1.0$mm



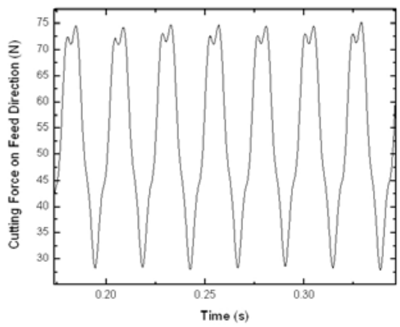
(e) $p_f = 1.5$mm



(f) $p_f = 1.75$mm

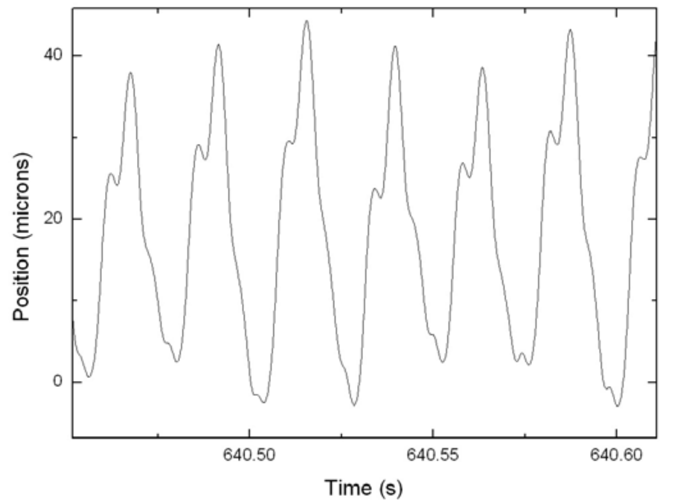


(g) $p_f = 2.0$mm

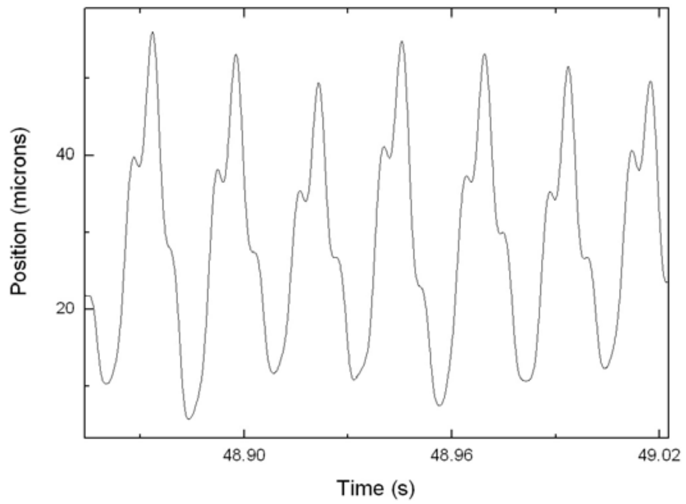


(h) $p_f = 2.25$mm

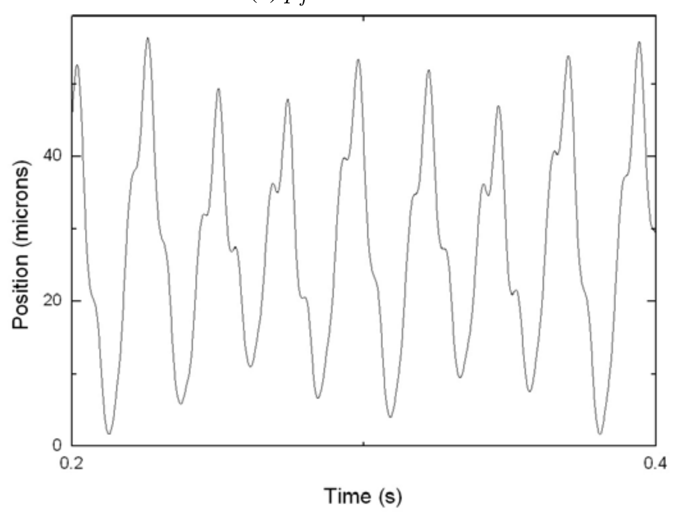
Figure 4 - Experimental Cutting Force - Depth of cut from 0.25 to 2.25 mm



(a) $p_f = 1.0$mm

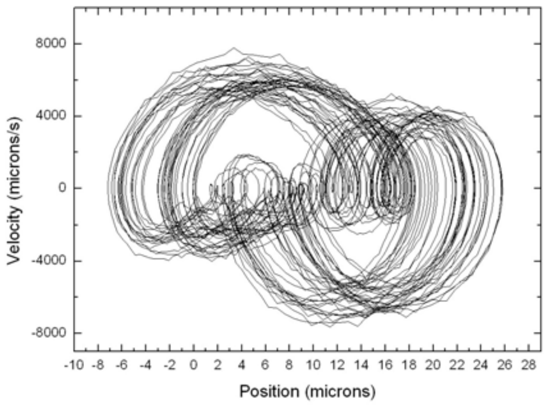


(a) $p_f = 1.5$mm

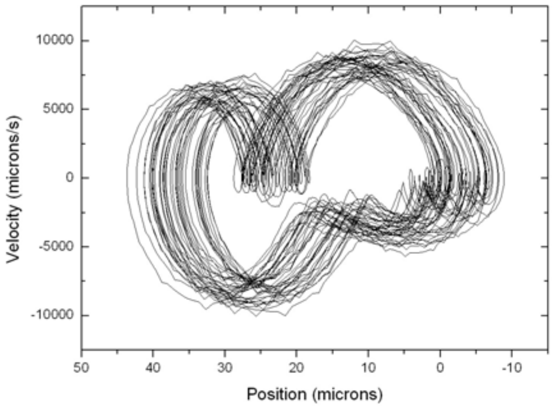


(c) $p_f = 1.75$mm

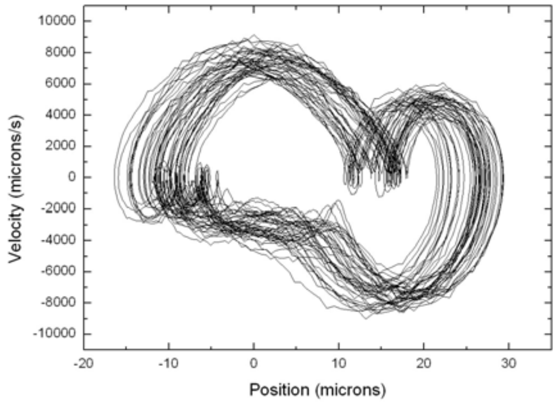
Figure 5 - Experimental Tool Tip Displacement



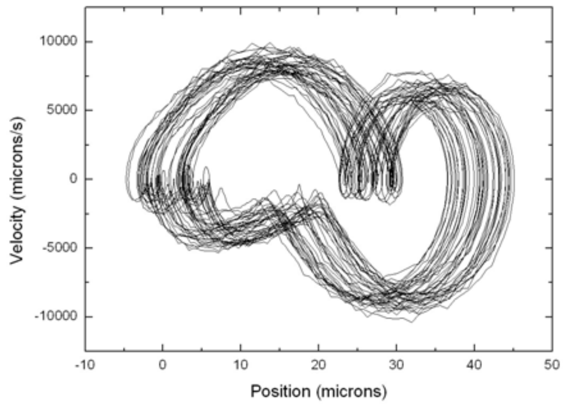
(a) $p_f = 0.25$mm



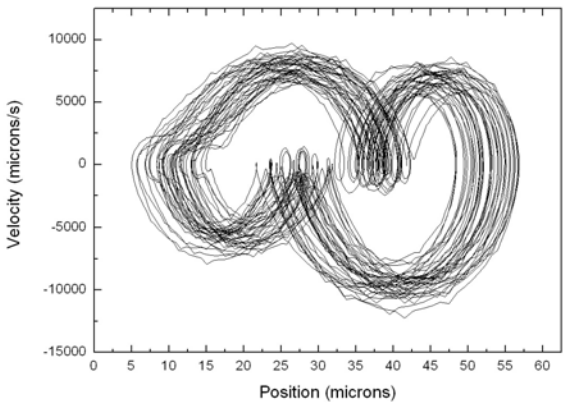
(b) $p_f = 0.5$mm



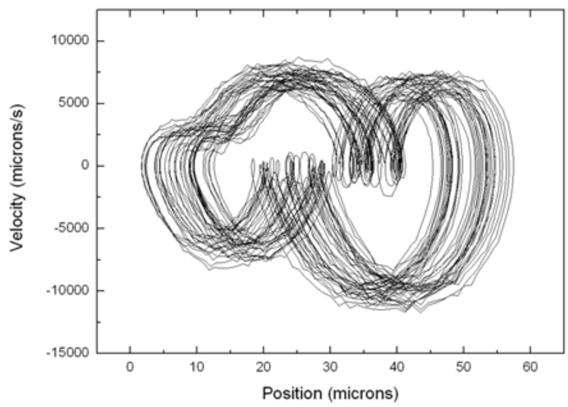
(c) $p_f = 0.75$mm



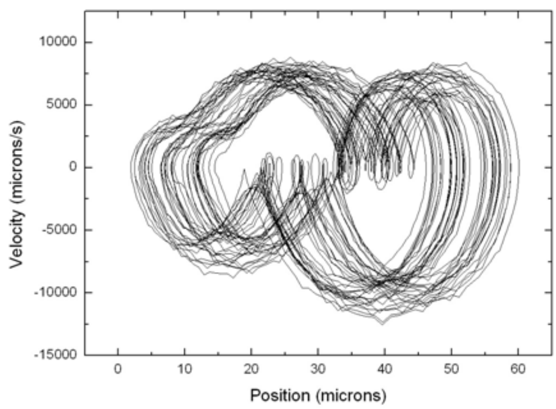
(d) $p_f = 1.0$mm



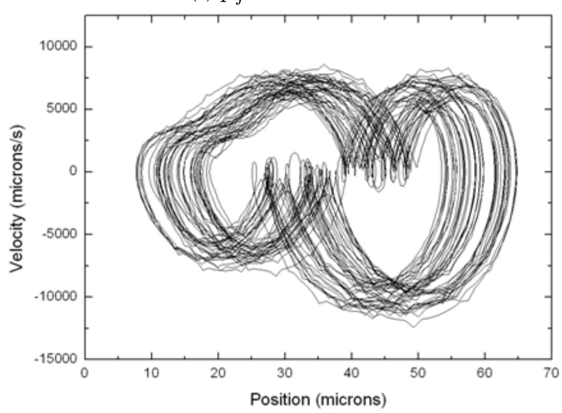
(e) $p_f = 1.5$mm



(f) $p_f = 1.75$mm



(g) $p_f = 2.0$mm



(h) $p_f = 2.25$mm

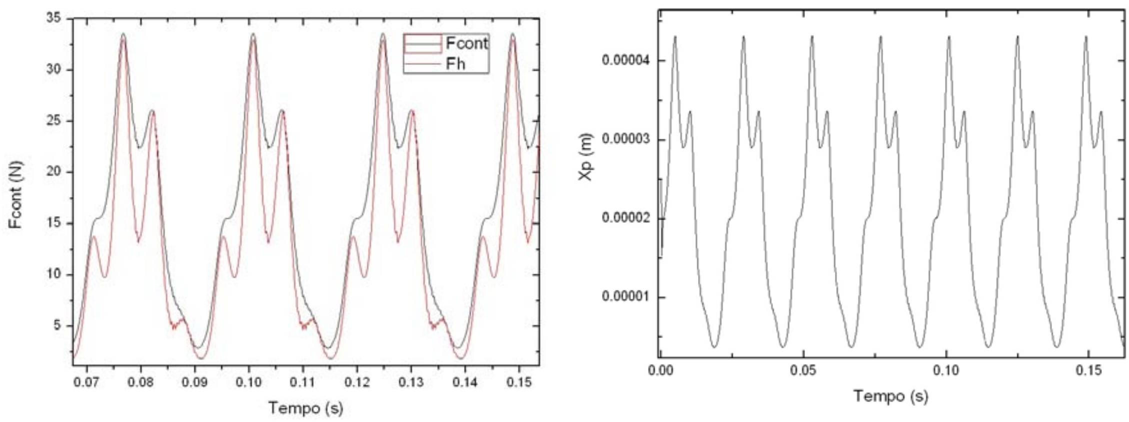
Figure 6 - Experimental Phase Space - Depth of cut from 0.25 to 2.25 mm

4. NUMERICAL SIMULATION

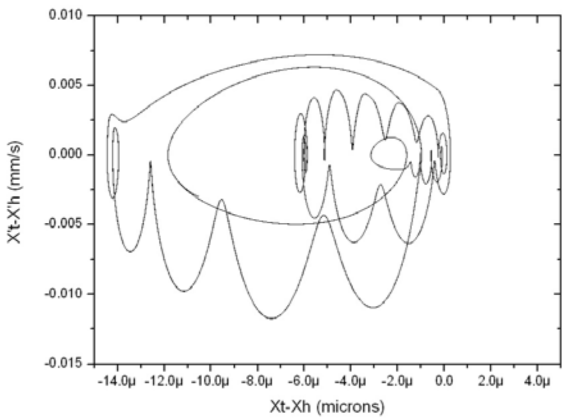
For numerical simulation it was assumed that X_{h0} is 25 microns. The distance between the tool holder and the workpiece in $t = 0$ is 0.001 mm. Moreover, it is assumed that the workpiece material presents plastic behavior. Under this assumption, the damping coefficient for the workpiece is defined as a function of the damping tool coefficient ($c_s = 105c$). Tool damping is considered to be 0.002 N s/m (Mann, 2005). Note that for $\dot{X}_t < 0$, when the tool move back from the workpiece, c_s vanishes. The material considered presenting yield shear stress as $\tau_s = 1000$MPa.

Numerical simulations are carried out in order to evaluate the model capability to describe milling process. The governing equations are solved by considering the fourth order Runge-Kutta method with time steps less than 5.10^{-4}s. Basically, we want to establish a qualitative comparison between the proposed model with experimental data presented in the preceeding section.

Fig. 7 presents numerical results for 1.5 mm of depth of cut. Basically, it presents the time-history of force and displacement, together with the phase space. Concerning the cutting force analysis, it presents values that are two times lower than experimental one but the force profile is close to the chip tool area profile, showing that the model needs improvements but is not far from modeling machining cutting process. The analysis of displacements (X_h, X_t, X_p and X_c) and the tool tip displacement ($X_t - X_h$) shows that the model is capturing the process behavior. They presents the same periodic aspects, being related to sub-hamonic response. The amplitude of the motion has the similar values, showing a qualitative agreement with experimental data.



(a) Cutting Force F_{cont} and Holder Force F_h (b) Secundary System Displacement - X_p



(c) Phase Space

Figure 7 - Numerical Results

5. CONCLUSIONS

This article presents a dynamical investigation of the milling process. Experimental and numerical results of cutting force and tool tip displacement for end milling processes are of concern. The cutting mechanics is modeled as a 2-dof nonsmooth system. The experimental data is collected in different depth of cuts maintaining the spindle speed and cutting velocities. Numerical results are developed in order to establish a qualitative comparison with experimental data. In general, the proposed model is capturing the general behavior of experimental data. Nevertheless, it is important to highlight that the model needs to be improved, especially concerning the cutting force behavior, in order to be used to predict the dynamic behavior of machining processes. The authors agree that this kind of model can be useful to choose important parameters to improve tool behavior.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Altintas, Y., 2000, "Manufacturing automation", 1st edn., Cambridge University Press, New York, 2000.

Araujo, A.C., Pacheco, P.M.L.C. and Savi, M.A., 2009, "Dynamical Analysis of an End Milling Process", International Congress of Mechanical Engineering - COBEM.

Araujo, A.C. and Savi, M.A., 2008, "Analysis of an End Milling Process- Encontro de Modelagem Computacional - Nova Friburgo.

Chandiramani, N. K. and Pothala, T., 2006, "Dynamics of 2 dof regenerative chatter during turning", Journal of Sound and Vibration, V.290, pp. 448-464.

Ehmann, K. F., Kapoor, S. G., DeVor, R.E. and Lazoglu, I., 1997, "Machining process modeling: a review", Journal of Manufacturing Science and Engineering, V.119, pp.655-663.

Gradisek, J., Kalveram, M., Insperger, T., Weinert, K., Stepan, G., Govekar, E. and Grabec, I., 2005, "On stability prediction for milling", International Journal Of Machine Tools and Manufacture, V. 45, pp.769-781.

Insperger, T., Mann, B. P., Stepan, G. and Bayly, P. V., 2003, "Stability of up-milling and down-milling, part 2: experimental verification", International Journal of Machine Tools and Manufacture", V. 43, pp. 35-40.

Mann, B.P., Garg, N.K., Young, K.A. and Helvey, A.M, 2005, "Milling bifurcation from structural asymmetry and nonlinear regeneration", Nonlinear Dynamics, V. 42, pp. 319-337.

Pavlovskaia, E., Wiercigroch, M. and Grebogi, C., 2001, "Modeling of an impact system with a drift", Physical Review E, v.64, pp. 1-9.

Merritt, H., 1965, "Theory of self-excited machine tool chatter", Journal of Engineering for Industry V.87(4), pp. 447-454.

Moon, F., 1998, "Dynamics and chaos in manufacturing processes", J. Wiley, New York.

Pratt, J. R. and Nayfeh, A. H., 1999, "Design and modelling for chatter control", Nonlinear Dynamics,V.19 pp.49-69.

Savi, M.A., Divenyi, S., Franca, L.F.P. and Weber, H.I., 2007, "Numerical and experimental investigations of the nonlinear dynamics and chaos in non-smooth systems", Journal of Sound and Vibration, V.301 pp. 59-73.

Smith, S. and Tlusty, J., 1991, "An overview of modeling and simulation of the milling process", Journal of Engineering for Industry, V.113 pp. 169-175.

Kalmar-Nagy, T., Sthepan , G. and Moon, F., 2001, "Subcritical Hoft bifurcation in the delay equation model for machine tool vibrations", Nonlinear Dynamics, V.26 pp.121-142.

Tobias, S. A., 1965, "Machine tool vibration", Blackie, London.

Zhao, M. X. and Balachandran, B., 2001, "Dynamics and stability of milling process", International Journal of Solids and Structures, V.38, pp. 2233-2248.

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