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Adaptive Torque-Based Control of Tracked Mobile Robots with Unknown Longitudinal Slip Parameter

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Abstract. *This paper presents the design of an adaptive tracking control law that guarantees stability for a tracked mobile robot under unknown longitudinal slip conditions. The final control law is obtained using two independent control design method. First, a velocity controller is designed for the kinematic steering system to enforce the tracking error to asymptotically converge to zero. Second, a torque controller is designed such that the true mobile robot velocity follows the desired velocity generated by the first controller. Finally, an update rule is used to estimate the slip parameter in real time. The asymptotic stability of the global closed-loop system is ensured using an appropriate Lyapunov function. Numerical results shows the usefulness of the proposed modeling and control strategy.*

Keywords: *nonholonomic mobile robot, dynamic model, backstepping, adaptive control.*

1 INTRODUCTION

Recently, the interest in mobile robots has grown significantly because of its large applications in unstructured environments where a high degree of autonomy is required. Despite the fact that the kinematic model of tracked vehicles is in general similar to the models used for wheeled robots, the tracked mobile robots have much larger ground contact patches, that provides better stability and traction at various terrain conditions compared to the conventional wheeled robots (Nourbakhsh and Siegwart (2004)).

Tracked mobile robots are typical example of systems that has nonholonomic constraints. Much research effort has been carried out in order to solve the problem of the motion for nonholonomic mobile robot (d'Andréa Novel *et al.* (1995); Yang and Kim (1999); Oriolo *et al.* (2002)). Most existing methods usually assumes there exists a dynamic controller that is able to produce the velocity profile generated by the kinematic controller.

Control design method for kinematic and dynamic controllers for nonholonomic robots are presented in Mnif and Touati (2005); Wu *et al.* (2009); Ju *et al.* (2009). In these works, the control design is split in two parts. The first part provides a kinematic controller, based on the robot kinematics, and the second part provides a dynamic controller, based on the robot dynamics. The dynamic controller is capable of estimating some physical parameters of the robot (Martins *et al.* (2008)). However, most of the results on the control design of nonholonomic robots are based on the assumption that the kinematics of the system is exactly known and there are only uncertainties in the dynamics of the system. On the other hand, our work takes into account an unknown slip condition in the kinematic of the system.

Many researches have addressed the slip phenomenon in the navigation of wheeled mobile robots (Wang and Low (2008); Sidek and Sarkar (2008); Gonzales *et al.* (2009)) and of tracked mobile robots (Zhou *et al.* (2007); Zhou and Han (2008)). However, in most of those works, the slip parameters are assumed to be previously known or estimated through some filtering algorithm. Here, we propose an update rule to estimate the slip parameter, based on Fukao *et al.* (2000). The uncertainties in the dynamics of the system is not considered. For trajectory tracking, it is only required to show that the center of mass of the vehicle follows the desired trajectory (Eghtesada and Necsulescub (2006)).

In this paper, feedback velocity control inputs are designed, according to Fierro and Lewis (1997), for the kinematic steering system to assure the position error converges to zero. Next, a torque feedback control law is designed such that the velocities of the mobile robot follows the desired velocity profile. The velocity and torque controllers are designed independently. An update rule is also designed such that the estimated slip parameter converge to the true slip parameter of the tracked robot. The update rule is obtained using a Lyapunov function that guarantee close-loop stability.

The paper is organized as follows. In section 2, the kinematics of the tracked mobile robot model is derived. In section 3, an adaptive tracking controller is designed for the tracked mobile robot model and the stability of the proposed control system is shown using a Lyapunov function. Section 4 presents the numerical results. Conclusions are presented in section 5.

2 MODELING OF THE TRACKED MOBILE ROBOT

In this section, the model of the tracked mobile robot is derived. First, the kinematic equations of tracked vehicles under slip condition are presented. Next, the dynamic equations that govern the vehicle are obtained using the Lagrange formulation.

2.1 Kinematic Equations Under Slip Conditions

The slip is described by a time-varying parameter, under the assumption that the robot will operate at low velocities. In this work, we only consider the longitudinal slip. The lateral slip is zero for straight line motions, and it can be neglected when the vehicles turns "on the spot" or at low velocities. Fig. 1 depicts the mobile robot, its parameters and the variables of interest.

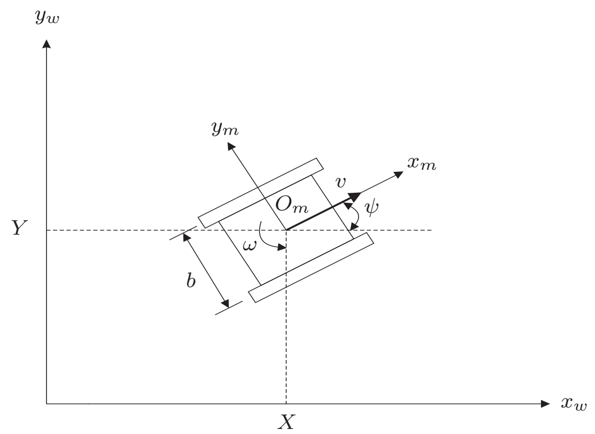

Figure 1. Tracked mobile robot representation.

In order to describe the motion of the tracked vehicle, its is defined a fixed reference frame $F_1(x_w, y_w)$ and a moving frame $F_2(x_m, y_m)$ attached to the vehicle body with origin at the geometric center O_m of the vehicle. The motion of the vehicle is composed of the translation velocity v and the rotational velocity $\omega = d\psi/dt$, where v is the velocity of the vehicle in the x_m-axis direction and ψ is the yaw angle. Furthermore, the motion of the vehicle is constrained in the y_m-axis direction, with $v_y = 0$ (nonholonomic constraint).

The longitudinal slip ratio of the two wheels is defined as follows

$$i = \frac{(r\omega_L - v_L)}{r\omega_L} = \frac{(r\omega_R - v_R)}{r\omega_R}, \quad 0 \le i < 1$$

where r is the radius of the wheels, ω_L and ω_R are the angular velocities of the left and the right wheels respectively and v_L and v_R are the linear velocities of the left and the right wheels in absence of the slip.

In the moving frame F_2, the model with longitudinal slip is given by

$$\begin{aligned} v &= \dot{x} = \frac{r\omega_L(1-i) + r\omega_R(1-i)}{2} \\ v_y &= \dot{y} = 0 \\ \omega &= \dot{\psi} = \frac{-r\omega_L(1-i) + r\omega_R(1-i)}{b} \end{aligned}$$

where b is the distance between the two tracks.

After applying some appropriate rotation matrix, from the reference frame F_2 to the reference frame F_1, the kinematic model can be written as

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \frac{r(1-i)}{2}\cos\psi & \frac{r(1-i)}{2}\cos\psi \\ \frac{r(1-i)}{2}\sin\psi & \frac{r(1-i)}{2}\sin\psi \\ \frac{-r(1-i)}{b} & \frac{r(1-i)}{b} \end{pmatrix} \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = S(q)\xi \tag{1}$$

where $q = (X, Y, \psi)^T$ denotes the coordinates of the tracked vehicle in the inertial Cartesian frame F_1. The angle ψ is assumed to be in $(-\pi, \pi]$.

The auxiliary velocity η is defined as $\eta = (v, \omega)^T$ and the effective velocity ξ for the model (1) is defined as $\xi = (\omega_L, \omega_R)^T$. Note that η is related to ξ according to the following equation

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \dfrac{r\omega_L(1-i) + r\omega_R(1-i)}{2} \\ \dfrac{-r\omega_L(1-i) + r\omega_R(1-i)}{b} \end{pmatrix} = T \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} \tag{2}$$

with

$$T = r \begin{pmatrix} (1-i)/2 & (1-i)/2 \\ -(1-i)/b & (1-i)/b \end{pmatrix}$$

We also have that $\xi = T^{-1}\eta$ is given by

$$\begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \frac{1}{r} \begin{pmatrix} \dfrac{1}{(1-i)} & -\dfrac{b}{2(1-i)} \\ \dfrac{1}{(1-i)} & \dfrac{b}{2(1-i)} \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{3}$$

Substituting (3) in (1), we arrive to the following model

$$\dot{q} = S_a(q)\eta$$

with

$$S_a(q) = \begin{pmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

Note that the nonholonomic constraint $\dot{y} = 0$ restrict the robot to move only in the direction normal to the axis of the driving tracks. This nonholonomic constraint can equivalently be written in the frame F_1 as

$$\begin{pmatrix} -\sin\psi & \cos\psi & 0 \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{pmatrix} = A(q)\dot{q} = 0 \tag{5}$$

2.2 Dynamic Equations of the Mobile Robot

The Lagrange formalism is used to derived the dynamic equations of the mobile robot. The trajectory of the mobile robot is constrained to the horizontal plane, thus its potential energy U remain constant. The kinematic energy T is given by

$$T(\dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q}$$

with

$$M = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{pmatrix}$$

where m is the total mass of the mobile robot and I is the moment of inertia about the vertical axis through O_m.

Using the fundamental nonholonomic form (Greenwood (2003)) of the Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i + C_i \quad (i = 1, ..., n)$$

where Q_is are the generalized applied forces and $C_i s$ are the generalized constraint forces for a system specified by the values of its n generalized coordinates, we obtain the following dynamics for the mobile robot:

$$M\ddot{q} = B(q)\tau - A^T(q)\lambda \tag{6}$$

where $q = (X, Y, \psi)^T$ has been defined in section (1), the input torque in left and right wheels is given by $\tau = (\tau_L, \tau_R)^T$, the vector of constrained forces is λ, the matrix $A(q)$ is given in (5) and the matrix $B(q)$ is given by

$$B(q) = \frac{1}{r} \begin{pmatrix} \cos\psi & \cos\psi \\ \sin\psi & \sin\psi \\ b/2 & b/2 \end{pmatrix}$$

The dynamics is now represented in a more appropriate form for control purposes. Note that we can eliminate the constraint matrix $A^T(q)\lambda$ by differentiating (1), substituting the result in (6) and finally multiplying by $S^T(q)$. The complete equations of motion of the nonholonomic mobile platform are thus given by

$$\dot{q} = S(q)\xi \tag{7}$$

$$\overline{M}\dot{\xi} = \overline{B}(q)\tau \tag{8}$$

where $\overline{M} = S^T(q)MS(q)$ and $\overline{B}(q) = S^T(q)B(q)$.

3 CONTROL DESIGN

In this section, we consider the tracking control problem of the tracked mobile robot subject to the dynamic part (8) and the kinematic part (7) with the slip considered as the time-varying parameter i. The design is divided in three steps as follows: first, a backstepping tracking control law is found neglecting the slip; next, an update rule is designed to estimate the slip parameters; and finally, closed-loop stability is shown using an appropriate Lyapunov function.

3.1 Backstepping Controller Design

In order to apply the proposed control design, it is necessary to represent the system (7)-(8) in the integrator backstepping form (Khalil (2001)). For this purpose, let u be an auxiliary control input for dynamic part, then by applying the law

$$\tau = \overline{B}(q)^{-1}\overline{M}u \tag{9}$$

we obtain the following form

$$\dot{q} = S(q)\xi \tag{10}$$

$$\dot{\xi} = u \tag{11}$$

To apply the procedure, we first determine a desired velocity control law ξ_d for (10) that drives to zero the error between the trajectory q and the reference trajectory q_r without slip. The reference $q_r = (X_r, Y_r, \psi_r)^T$, in the fixed frame F_1, is generated using the kinematic model

$$\dot{q}_r = S_a(q_r)\eta_r$$

that is

$$\begin{pmatrix} \dot{X}_r \\ \dot{Y}_r \\ \dot{\psi}_r \end{pmatrix} = \begin{pmatrix} \cos\psi_r & 0 \\ \sin\psi_r & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_r \\ \omega_r \end{pmatrix} \tag{12}$$

where $\eta_r = (v_r, \omega_r)^T$ is a desired linear and angular reference trajectory. It is assumed in (12), that the signal η_r is constructed to produce the desired motion and that the signals η_r, $\dot{\eta}_r$, q_r, $\dot{q}_r$ are bounded for all time t. It is also assumed that $\psi_r \in (-\pi, \pi]$ and that v_r does not go to zero as $t \to \infty$.

In order to analyze the tracking problem, we define the error $e_c = (e_1, e_2, e_3)^T$ in the frame F_1 as

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_r - X \\ Y_r - Y \\ \psi_r - \psi \end{pmatrix} \tag{13}$$

The dynamics of the error e_c, derived using (4) and (13), is given by

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} \omega e_2 + v_r \cos e_3 - v \\ -\omega e_1 + v_r \sin e_3 \\ \omega_r - \omega \end{pmatrix} \tag{14}$$

Neglecting the slip, Fierro and Lewis (1997) showed that the following control input

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r \cos e_3 + k_1 e_1 \\ \omega_r + v_r k_2 e_2 + k_3 \sin e_3 \end{pmatrix} \tag{15}$$

with $k_i > 0$, drive the error signal e_c to zero in the region $D = \{e_c \in \mathbb{R}^3 \mid -\pi < e_3 < \pi\}$, using the following Lyapunov function

$$V_0 = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \frac{(1 - \cos e_3)}{k_2} > 0$$

whose derivative satisfies the inequality

$$\begin{aligned}\dot{V}_0 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{k_2} \\ &= -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 \leq 0\end{aligned}$$

Under the conditions $V_1 > 0$, $\dot{V}_1 \leq 0$ and that $\dot{e}$ is bounded, Fierro and Lewis (1997) proved using the dynamics of the system and Barbalat's Lemma (Li and Slotine (1991); Khalil (2001)) that the error e converge to zero. Therefore, neglecting the slip, the feedback control law ξ_d that guarantee asymptotic stability for (10) is given by

$$\xi_d = \begin{pmatrix}\omega_{Ld} \\ \omega_{Rd}\end{pmatrix} = \frac{1}{r}\begin{pmatrix}1 & -\frac{b}{2} \\ 1 & \frac{b}{2}\end{pmatrix}\begin{pmatrix}v \\ \omega\end{pmatrix} \tag{16}$$

with v and ω given by (15).

It is now necessary to convert the velocity control law ξ_d into an input torque control τ that will be applied to the system. For this purpose, we define an auxiliary velocity error by

$$e_d = \begin{pmatrix}e_4 \\ e_5\end{pmatrix} = \xi - \xi_d \tag{17}$$

where $\xi - \xi_d$ represents the error between the vector of the true and desired angular velocities.

The input u, which assure that e_d converges to zero, is given by the following expression

$$u = \dot{\xi}_d + \begin{pmatrix}k_4 & 0 \\ 0 & k_4\end{pmatrix}(\xi_d - \xi) \tag{18}$$

where k_4 is a positive constant.

The derivative of the error e_d, using (11), (17) and (18), is given by

$$\begin{pmatrix}\dot{e}_4 \\ \dot{e}_5\end{pmatrix} = -\begin{pmatrix}k_4 & 0 \\ 0 & k_4\end{pmatrix}\begin{pmatrix}e_4 \\ e_5\end{pmatrix} \tag{19}$$

To show that the entire error $e = (e_c, e_d)^T$ goes to zero as $t \to 0$, we consider the following Lyapunov function candidate

$$V_1 = V_0 + \frac{1}{2k_4}\left(e_4^2 + e_5^2\right) = \frac{1}{2}\left(e_1^2 + e_2^2\right) + \frac{(1-\cos e_3)}{k_2} + \frac{1}{2k_4}\left(e_4^2 + e_5^2\right) > 0$$

whose derivative satisfies the inequality

$$\begin{aligned}\dot{V}_1 &= e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{k_2} + \frac{e_4}{k_4}\dot{e}_4 + \frac{e_5}{k_4}\dot{e}_5 \\ &= -k_1 e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - e_4^2 - e_5^2 \leq 0\end{aligned}$$

Under the conditions $V_1 > 0$, $\dot{V}_1 \leq 0$ and that $\dot{e}$ is bounded, it can be proved using similar ideas as before that the error e converges to zero.

3.2 Update Law and Lyapunov Analysis

If the parameter i in (1) is unknown, we cannot choose the desired velocity as given by (16). Hence, it is necessary to design an update rule to attain the control objective using the estimate for i. First, we redefine the slip parameter as

$$a = \frac{1}{(1-i)}, \quad 0 \leq i < 1$$

Then, the relation (16) can be written as a function of the new slip parameters a.

Since this parameter is not known, we use a formula for (16) considering now the estimate $\hat{a}$ for a, given by

$$\begin{pmatrix}\omega_{Ld} \\ \omega_{Rd}\end{pmatrix} = \frac{1}{r}\begin{pmatrix}\hat{a} & -\dfrac{b}{2}\hat{a} \\ \hat{a} & \dfrac{b}{2}\hat{a}\end{pmatrix}\begin{pmatrix}v \\ \omega\end{pmatrix}$$

where the estimates $\hat{a} = a + \tilde{a}$ is the true value a plus the estimate error $\tilde{a}$.

In order to derive the update rules, it is necessary to calculate (14) that depends on the auxiliary velocity (2), which is a function of the effective velocity (3). The derivative of the error $\dot{e}_c$ is given by:

$$\begin{aligned}
\dot{e}_1 &= -\frac{a+\tilde{a}}{a}\left(\frac{e_2}{b}+\frac{1}{2}\right)\left(v-\frac{b}{2}\omega\right)+\frac{a+\tilde{a}}{a}\left(\frac{e_2}{b}-\frac{1}{2}\right)\left(v+\frac{b}{2}\omega\right)+v_r\cos e_3\\
\dot{e}_2 &= \frac{a+\tilde{a}}{ba}\left(v-\frac{b}{2}\omega\right)e_1-\frac{a+\tilde{a}}{ba}\left(v+\frac{b}{2}\omega\right)e_1+v_r\sin e_3\\
\dot{e}_3 &= \omega_r+\frac{a+\tilde{a}}{ba}\left(v-\frac{b}{2}\omega\right)-\frac{a+\tilde{a}}{ba}\left(v-\frac{b}{2}\omega\right)
\end{aligned} \tag{20}$$

To obtain the update rule, we consider the following Lyapunov function candidate

$$V = V_1 + \frac{\tilde{a}^2}{2\gamma a} \tag{21}$$

with $a \geq 1$ and $\gamma > 0$.

The derivative of V is given by

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3\frac{\sin e_3}{k_2} + \frac{e_4}{k_4}\dot{e}_4 + \frac{e_5}{k_4}\dot{e}_5 + \frac{\tilde{a}}{\gamma a}\dot{\hat{a}} \tag{22}$$

Substituting equations (19) and (20) in (22), we obtain

$$\dot{V} = \dot{V}_1 + \frac{\tilde{a}}{a}\left[\frac{\dot{\hat{a}}}{\gamma} - \left(ve_1 + \frac{b\omega\sin e_3}{2k_2}\right)\right]$$

Now, choosing the update rule for $\hat{a}$ as

$$\dot{\hat{a}} = \gamma\left(ve_1 + \frac{b\omega\sin e_3}{2k_2}\right) \tag{23}$$

The equation for $\dot{V}$ take the form

$$\dot{V} = -k_1e_1^2 - \frac{k_3}{k_2}\sin^2 e_3 - e_4^2 - e_5^2 \leq 0 \tag{24}$$

It is now possible to guarantee closed-loop stability by showing that $e = 0$ is an asymptotically stable equilibrium. Let the domain D be given by $D = \{e \in R^3 \mid -\pi < e_3 < \pi\}$, then the Lyapunov function given in (21) is positive definite in $D - \{0\}$ with derivative $\dot{V} \leq 0$ in D. This implies that the error e and the estimate parameters are bounded. Since the reference velocity $\eta_r = (v_r, \omega_r)^T$ is assumed to be bounded, we known from (15) that the velocity η is also bounded. Thus, $\dot{e}$ is bounded by (14). After all, $\ddot{V}(e, \dot{e})$ given by

$$\ddot{V} = -2k_1e_1\dot{e}_1 - \frac{2k_3}{k_2}\sin e_3\cos e_3\dot{e}_3 - 2e_4\dot{e}_4 - 2e_4\dot{e}_4$$

is also bounded.

Since V is a nonincreasing function that converges to some constant value. Barbalat's Lemma shows that $\dot{V} \to 0$ as $t \to \infty$. Thus, from (24), we know e_1, e_3, e_4 and e_5 tend to zero as $t \to \infty$. This conclusion could have also been derived using LaSalle's invariance principle (Khalil (2001)).

It now remains to show that e_2 also converges to zero. From (14) and (15) we have

$$\dot{e}_3 = \omega_r - \omega = -v_rK_2e_2 - K_3\sin e_3 \tag{25}$$

and

$$\ddot{e}_3 = -v_rK_2\dot{e}_2 - K_3\cos e_3\dot{e}_3$$

Thus $\ddot{e}$ is bounded since e and $\dot{e}$ are both bounded. Barbalatt's Lemma shows that $\dot{e}_3 \to 0$ as $t \to \infty$. Since $e_3 \to 0$, we have that $v_re_2 \to 0$. If v_r does not go to zero as $t \to \infty$, then $e_2 \to 0$ as $t \to \infty$. Thus, the equilibrium $e = 0$ is asymptotically stable.

Theorem 1 *If we choose the control inputs as* (9) *and the parameter update rule as* (23) *for the dynamic model* (7)-(8) *of the mobile robot with unknown slip parameter i, the equilibrium* $e = (e_c, e_d)^T$ *is asymptotically stable. Thus, the robot configuration* q *asymptotically converge to the reference configuration* q_r.

4 NUMERICAL RESULTS

This section shows the effectiveness of the proposed adaptive tracking controllers presented in the previous sections. The numerical simulations were performed using MATLAB.

The physical parameters for the model, taken from Zhou *et al.* (2007), are given by $b = 0.65$ m, $r = 0.35$ m, $m = 0.80$ kg and $I = 0.0608$ kg.m^2. The total time of the simulation is chosen as $t = 80$ s. The control parameters of the controller are chosen as $k_1 = 6$, $k_2 = 8$, $k_3 = 6$ and $k_4 = 10$, and the parameter of the adaptive rule is chosen as $\gamma = 10$. The initial conditions are taken as $q_r(0) = (0, 0, 0)^T$, $\xi(0) = (0, 0)^T$ and $\hat{a}(0) = 1$. Two reference trajectories are used. First, a linear trajectory generated by the reference velocity $v_r = 0.5$ m/s and $\omega_r = 0$ rad/s. Second, a circular trajectory generated by the reference velocity $v_r = 0.5$ m/s and $\omega_r = 0.3$ rad/s. The initial conditions of the robot for the linear and circular trajectory are respectively given by $q(0) = (0, -1, \pi/8)^T$ and $q(0) = (0, 1, \pi/6)^T$. In order to demonstrate the tracking performance, the slip parameter changes from $i = 0$ to $i = 0.25$ during the time period 30 s $\leq t \leq$ 60 s.

Figure 2 shows the tracking error e in the fixed frame F_1 for the linear reference trajectory. Note that the posture error e_c and the velocity error e_d converge to zero. At the time instant $t = 30$ s and $t = 60$ s, these errors increase due to the step change in the slip parameter.

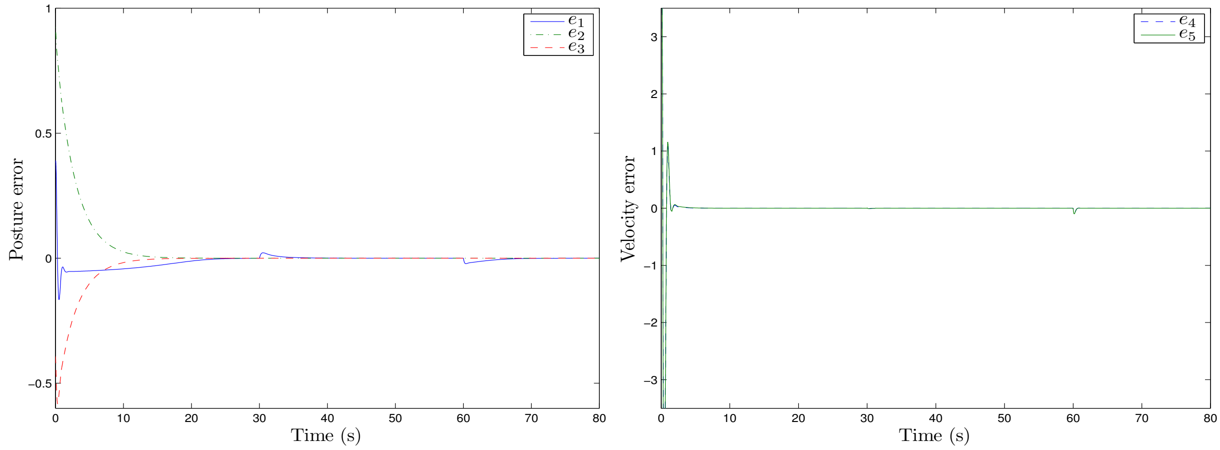

Figure 2. The posture error and velocity error for the linear reference trajectory.

Figure 3 shows the results for the linear reference trajectory in the inertial frame. The red solid line stands for the reference trajectory, while the blue circles stands for the robot trajectory.

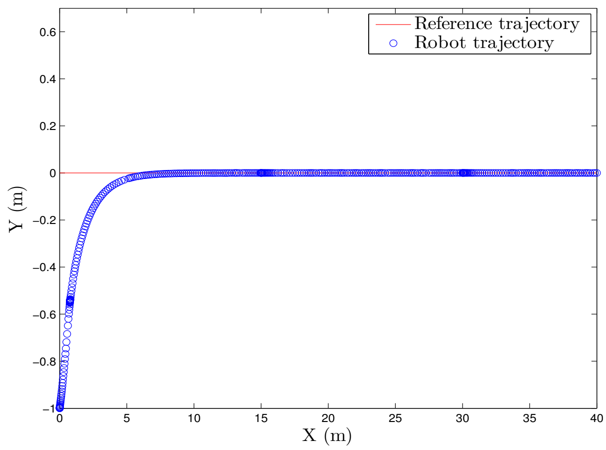

Figure 3. Results for the linear reference trajectory.

Figure 4 shows the estimate $\hat{a}$ of the slip parameter for the linear reference trajectory. The red dashed line represents the true value of the slip parameter and the blue solid line is the estimated value. For the time period 30 s $\leq t \leq$ 60 s the value of the slip parameter is $a = 4/3$ which correspond to the slip rate $i = 0.25$. The value $a = 1$ means that the slip rate is zero.

Figure 5 shows the tracking error e for the circular reference trajectory. Note that the posture error and velocity error converge to zero.

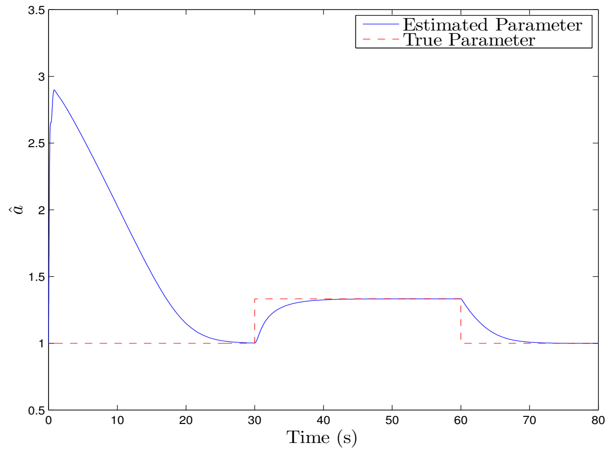

Figure 4. Estimated parameter $\hat{a}$ for the linear reference trajectory.

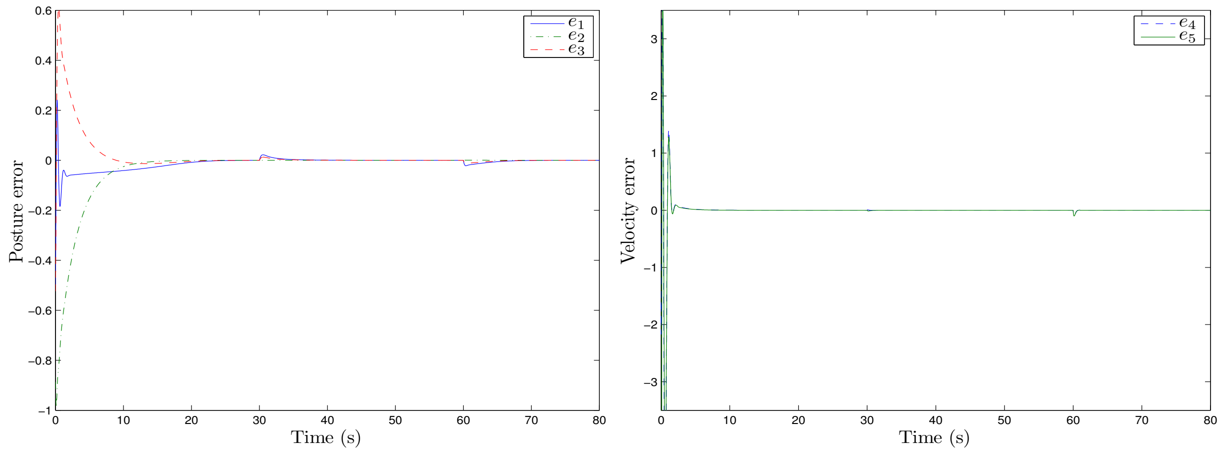

Figure 5. The posture errors and velocities errors for the circular reference trajectory.

Figure 6 shows the results for the circular reference trajectory in the inertial frame. The red solid line stands for the reference trajectory, while the blue circles stands for the robot trajectory.

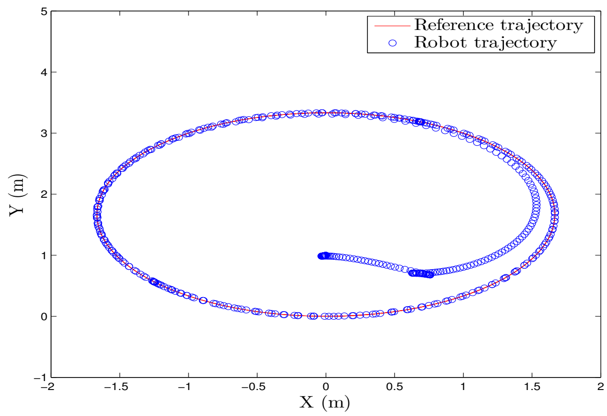

Figure 6. Results for the circular reference trajectory.

Figure 7 shows the estimate $\hat{a}$ of the slip parameter for the circular reference trajectory. The red dashed line represents the real value of the slip parameter and the blue solid line is the estimated value.

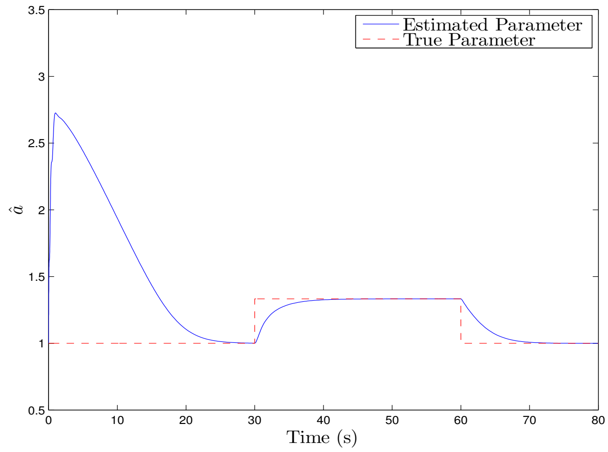

Figure 7. Estimated parameter $\hat{a}$ for a circular reference trajectory.

5 CONCLUSIONS

We have considered in this work the tracking control problem of a nonholonomic mobile robot in the presence of an unknown longitudinal slip parameter. A kinematic model containing the slip parameter was proposed. Neglecting the slip parameter, a preliminary velocity controller was designed for the kinematic model in order to enforce the tracking error to asymptotically converge to zero. Then, a backstepping control approach was used to design the torque needed to assure that the velocities of the mobile robot follows the desired velocity, generated by the previous velocity controller. Including the slip parameter in the kinematic model, an update rule was designed such that the estimated slip parameter converges to the true value for the mobile robot. Asymptotic stability of the global closed-loop system was guaranteed using an appropriate Lyapunov function. Numerical results showed that the performance of the proposed adaptive control was effective.

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7 REFERENCES

d'Andréa Novel, B., Bastin, G. and Campion, G., 1995. "Control of nonholonomic wheeled mobile robots by state feedback linearization". *International Journal of Robotics Research*, Vol. 14, No. 6, pp. 543–559.

Eghtesada, M. and Necsulescub, D.S., 2006. "Study of the internal dynamics of an autonomous mobile robot". *Robotics and Autonomous Systems*, Vol. 54, No. 4, pp. 342–349.

Fierro, R. and Lewis, F.L., 1997. "Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics". *Journal of Robotic Systems*, Vol. 14, No. 3, pp. 149–163.

Fukao, T., Nakagawa, H. and Adachi, N., 2000. "Adaptive tracking control of a nonholonomic mobile robot". *IEEE Transactions on Robotics and Automation*, Vol. 16, No. 5, pp. 609–615.

Gonzales, R., Fiacchini, M., Alamo, T., Gusman, J.L. and Rodriguez, F., 2009. "Adaptive control for a mobile robot under slip conditions using LMI-based approach". In *Proceedings of the European Control Conference*. Budapest, Hungary, pp. 1251–1256.

Greenwood, D.T., 2003. *Advanced Dynamics*. Cambridge University Press, Cambridge, UK.

Ju, G., Wu, Y. and Sun, W., 2009. "Adaptive output feedback asymptotic stabilization of nonholonomic systems with uncertainties". *Nonlinear Analysis*, Vol. 7, No. 1, pp. 5106–5117.

Khalil, H.K., 2001. *Nonlinear Systems*. Prentice-Hall, Upper Saddle River, NJ, USA.

Li, W. and Slotine, J.J.E., 1991. *Applied Nonlinear Control*. Prentice-Hall, Englewood Cliffs, NJ, USA.

Martins, F.N., Celeste, W.C., Carelli, R., Sarcinelli-Filho, M. and Bastos-Filho, T.F., 2008. "An adaptive dynamic controller for autonomous mobile robot trajectory tracking". *Control Engineering Practice*, Vol. 16, No. 11, pp. 1354–1363.

Mnif, F. and Touati, F., 2005. "An adaptive control scheme for nonholonomic mobile robot with parametric uncertainty". *International Journal of Advanced Robotic Systems*, Vol. 2, No. 1, pp. 59–63.

Nourbakhsh, I.R. and Siegwart, R., 2004. *Introduction of Autonomous Mobile Robots*. The MIT Press, London, UK.

Oriolo, G., Luca, A.D. and Vendittelli, M., 2002. "WMR control via dynamic feedback linearization: Design, implementation, and experimental validation". *IEEE Transactions on Control Systems Technology*, Vol. 10, No. 6, pp. 835–852.

Sidek, N. and Sarkar, N., 2008. "Dynamic modeling and control of nonholonomic mobile robot with lateral slip". In *Proceedings of the 7th WSEAS International Conference on Signal Processing, Robotics and Automation*. Cambridge, UK, pp. 66–74.

Wang, D. and Low, C.B., 2008. "Modeling and analysis of skidding and slipping in wheeled mobile robots: Control design perspective". *IEEE Transactions on Robotics*, Vol. 24, No. 3, pp. 676–687.

Wu, J., Xu, G. and Yin, Z., 2009. "Robust adaptive control for a nonholonomic mobile robot with unknown parameters". *Journal of Control Theory and Applications*, Vol. 7, No. 2, pp. 212–218.

Yang, J.M. and Kim, J.H., 1999. "Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots". *IEEE Transactions on Robotics and Automation*, Vol. 15, No. 3, pp. 578–587.

Zhou, B. and Han, J., 2008. "Dynamic feedback tracking control of tracked mobile robots with estimated slipping parameters". In *Proceedings of the IEEE World Congress on Computational Intelligence*. pp. 1991–1996.

Zhou, B., Peng, Y. and Han, J., 2007. "UKF based estimation and tracking control of nonholonomic mobile robots with slipping". In *Proceedings of the IEEE International Conference on Robotics and Biomimetics*. Sanya, China, pp. 2058–2063.

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