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AIRCRAFT GROUND ROLL DYNAMIC MODELING

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Abstract: *Common threads to any aircraft mission profile are the takeoff and landing flight phases. Although great attention is given in these terminal flight phases to aircraft performance, handling qualities, and limitations associated with crosswinds, the dynamic characteristics of the landing ground roll are often ignored. Several parameters interfere in the aircraft ground roll dynamics, such as weight, CG position, tire pressure, aerodynamic configuration, shock absorbers' damping ratio, landing gears' stiffness, ground effect, runway's slope and friction coefficient. Besides all these variables, the coupling between shock absorbers and tire movements and the changes in the aerodynamic forces caused by the aircraft attitude alteration may result in an unexpected behavior of the aircraft rolling dynamics. Using the aerodynamic longitudinal model of the one regional aircraft with nose wheel tricycle landing gear coupled with nose and main landing gears considered as 1 DOF 2nd order dynamic models, the ground roll dynamic is obtained. Simulation of take-off running is performed in order to analyze the behavior of the aircraft and landing gear given by this model. Furthermore, a comparison between the results of take-off runway length obtained in the numerical simulation and analytical calculation is performed.*

Keywords: *Aircraft; ground; roll; modeling; landing gear; ground dynamics*

1. INTRODUCTION

This paper describes the development of ground roll dynamic model of a regional sized aircraft used to asses the influence of the parameters variation, such as shock absorber's damping ratio, gas spring curve and friction coefficient between tires and ground.

For modeling the ground roll dynamic effect, the usual approach was adopted which is to have detailed independent components and geometry models that produce open loop dynamics. This level of detail is required to achieve the objective of evaluate the landing gear performance (Ragsdale, 2000).

In the development herein presented, the characteristics of the aircraft, with the detailed components and geometry above mentioned, are used to model the landing gears reaction forces and moments contribution during the ground rolling.

The landing gear model considers the typical tricycle landing gear configuration with two main wheels including brakes and a nose steerable wheel. However, since this paper's purpose is only to analyze the longitudinal behavior the bicycle hypothesis simplification is adopted.

Landing gear model's input parameters are essentially shock absorbers stroke, compression rate and compression acceleration which are calculated from aircraft attitude and rotational rates and the body frame velocity vector of the center of gravity.

Special attention had been given to take into account the effects in aircraft dynamics due to attitude and attitude's rate and acceleration, which directly affects aerodynamics forces and moments, engine thrust vector direction and landing gear reactions. So that, the model developed consider not only the landing gear reaction force, but also aircraft's pitch angle cause by its compression. Whereas the tires deflection is much smaller than those found in the shock absorbers, this effect in aircraft attitude had been neglected.

Aerodynamic coefficients as function of wind incidence angle were used, so that the pitch angle changes influence lift and drag coefficient. The lift force increase due to ground effect was not modeled in the aerodynamic module.

2. LANDING GEAR MODEL

Landing gear model is one of the most difficult information to obtain, due to the fact it is considered industrial property. (Turbuk, Marcio Caetano, 2009). The classical model depends on several values, such as tire dynamics, weight and stiffness of the landing gear components. Due to this fact, a simplified model was considered for this study.

The landing gear's modeling started by defining the physical model which meets the needs for this simulation. The proposal was to represent the landing gear by a 1 DOF model, considering its dynamics as a 2nd order system. In this approach, the tire deflection was ignored due to the fact that its compression magnitude is much lower than those found in the shock absorbers. However, the ground reactions caused by this deformation were considered. Another assumption is that all metallic components were considered rigid, in such a way that the landing gear dynamics is driven by the shock absorber dynamics.

2.1. Shock Strut

The spring-damping characteristic varies according to the type of the landing gear / shock absorber. For this study and telescopic oleo pneumatic was selected because it is commonly used in the aircraft class that is being considered.

The following physical model was adopted to model the landing gear, where:

M_1 Landing gear sprung mass
M_2 Landing gear unsprung mass
FHID Hidraulic force due to the oil laminage
FPOL Politropic force due to the gas spring compression
FTIRE Force due tire compression

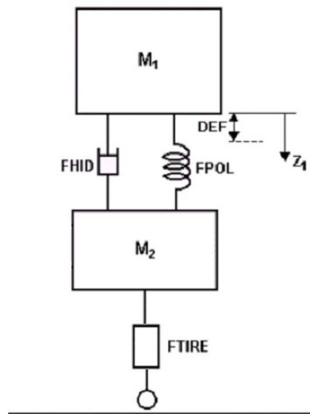

Figure 1: Landing gear physical model

This model has one degree of freedom, which is the shock absorber compression. Note that the tire was modeled as rigid body, so that, none degree of freedom has been added. In this physical model the internal friction forces were neglected for simplification.

The air spring force (FPOL) parameter is the force due to the air compression, also called polytrophic force. Its derivation is based on the gases law with polytrophic coefficient (Silva, 2008). Figure (2) shows a typical curve profile of a polytrophic compression.

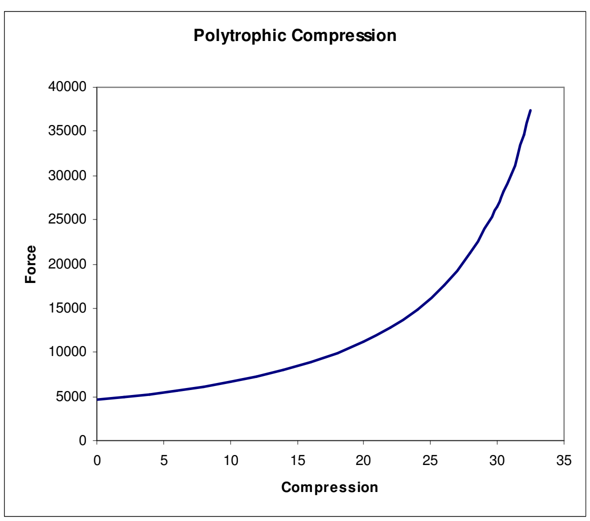

Figure 2: Polytrophic Compression Curve

$$FPOL = K(DEF) \cdot DEF \tag{1}$$

The hydraulic force generated in the shock absorber is function of the compression velocity, fluid density (ρ), hydraulic area (A_h), discharge coefficient (C_d) and opening area of the orifice (A_n). For the FHID parameter, a quadratic law presented in (Silva, 2008) relating the force with damper deflection speed was adopted. The hydraulic reaction force of the shock absorber is different during compression and expansion, due to the presence of an unidirectional valve, which opens or close additional orifices depending on the flow direction.

$$FHID = B \cdot D\dot{E}F^2 \quad (2)$$

Besides the forces just mentioned, there is the force caused by inertia of the landing sprung mass:

$$FI = M_1 \cdot D\ddot{E}F \quad (3)$$

Hence, the landing gear normal force is given by joining each reaction component:

$$FN = M_1 \cdot D\ddot{E}F + B \cdot D\dot{E}F^2 + K(DEF) \cdot DEF \quad (4)$$

2.2. Tire Model

One of the main sources of the nonlinearities in vehicle dynamics is the tire, whose behavior can be extraordinarily complex. The vehicle forces, in this case the aircraft, depends on the tire forces and at the same time tire forces depends on the vehicle motion (S. Sadeghi and M.T. Ahmadian, 2001)

The tire model's adopted for this study results in a horizontal force, tangent to the ground at the contact point between tire and ground and parallel to the tire orientation. Due to the fact that only the longitudinal movement is being considered, the lateral force and self-aligning torque were neglected.

The longitudinal forces are related to the normal force acting in the tire and to the longitudinal Wheel Slip (S), which is defined as (Véras, 2008):

$$S \triangleq \frac{R_{tire} \cdot \omega_R - V_x}{V_x} \quad (5)$$

Where:

R_{tire} is the tire radius, ω_R is the tire angular velocity and V_X is the longitudinal direction velocity. The force and wheel slip can be related by empirical model, such as Pacejka Magic Formula (Carlson, 2003), as shown in Fig. (3)

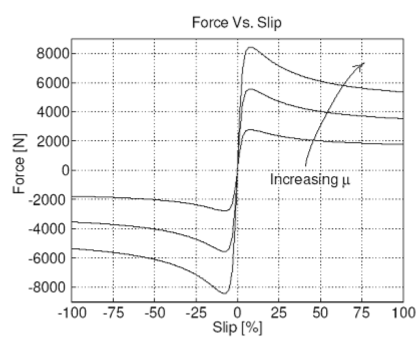

Figure 3: Force vs Slip

For operation conditions where the slip does not exceed 2%, this behavior can be approximated by Eq. (6), on which C_{slip} is called Longitudinal Stiffness. Since this equation presents a linear behavior, the longitudinal force can also be calculated by the friction coefficient (μ), as shown in Eq. (7).

$$F_{brake}(S) = C_{Slip}S \quad (6)$$
$$F_X = \mu_a F_z \quad (7)$$

3. MOTION EQUATIONS

Aircraft's motion equations consist in the result for applying Newton's 2^{nd} law, assuming the aircraft as a rigid body. Prior to applying the forces acting in the aircraft, the inertial (x_o,y_o), aerodynamic (x_a,y_a) and body (x,y) frames were defined, as shown in Fig. (4).

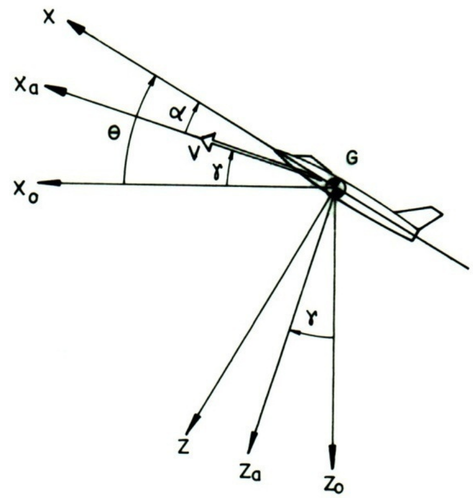

Figure 4. Aircraft coordinates system

Usually, the aerodynamics coefficients are expressed in terms of α, in other words, the aerodynamic forces are obtained in the aerodynamic frame system. The force and moment from the propulsion system are better expressed in body frame system, since it depends on the geometry of the aircraft, as well as landing gear reaction forces by the same reason. Thus, in order to calculate the motion equation, all the efforts shall be represented at the same coordinates system, however the equations here in presented are shown in the coordinate system that better explain its deduction.

3.1. Aircraft on Ground

The aircraft on ground is subject to the reaction forces from the landing gears and tires friction. Besides these forces, propulsion's and aerodynamics' forces are also present, since there will be an increase of the lift force as long as the aircraft speed increases. Figure (5) shows the forces and moments acting on the aircraft:

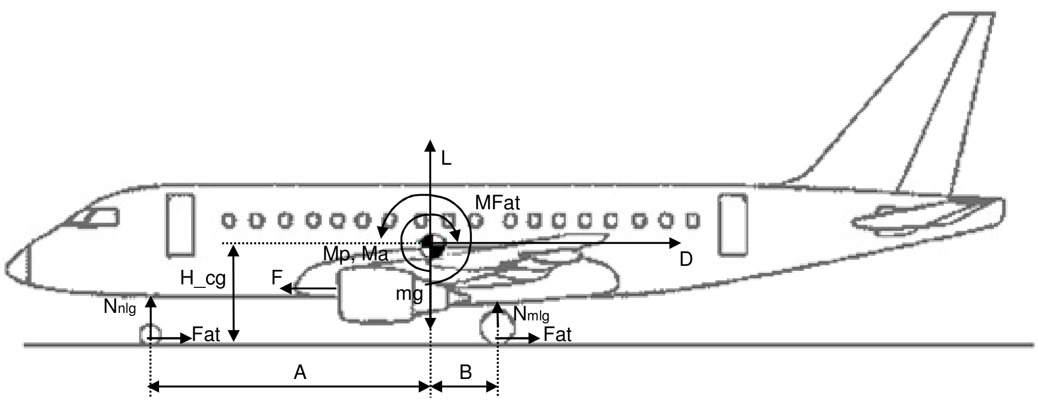

Figure 5: Aircraft's acting forces and moments

Motion equations were obtained considering Fig. (5) forces and moments and based on some simplification hypothesis. The bicycle hypothesis consists in a simplification of the main landing gear which is reduced to a single tire. This approximation is quite useful since it greatly simplifies the expressions of local sideslip angles from which friction forces are evaluated. As long as specific maneuvers, such as differential braking for example, are not to be studied, it will be shown that this approximation is not restrictive. Another simplification was to assume that both main and nose landing gears installation angle is equal to zero. This approximation makes the reaction forces due to the shock absorber normal to the body frame system.

Below are the resulting motion equations in the aerodynamics frame:

$$\dot{V} = \frac{F\cdot\cos(\alpha+\alpha_f) - D - m\cdot g\cdot sin(\alpha) - Fat\cdot\cos(\gamma) + (N_{NLG}+N_{MLG})\cdot\sin(\gamma)}{m} \quad (8)$$

$$\dot{\gamma} = \frac{F\cdot sin(\alpha+\alpha_f) + L - m\cdot g\cdot\cos(\gamma) + Fat\cdot sin(\gamma) + (N_{NLG}+N_{MLG})\cdot\cos(\gamma)}{m\cdot V} \quad (9)$$

$$\dot{q} = \frac{M_a + M_f - Fat \cdot H_{CG} + (N_{NLG} \cdot A - N_{MLG} \cdot B) \cdot cos(\gamma)}{I_{yy}} \quad (10)$$

$$\dot{\alpha} = q - \dot{\gamma} \quad (11)$$

$$\dot{H} = V \cdot sin(\gamma) \quad (12)$$

$$\dot{x} = V \cdot \cos(\gamma) \quad (13)$$

Where:
F engine thrust
L lift force
D drag force
Fat friction force
m aircraft mass
g gravity acceleration
α_f angle of the propulsion force in body frame
Ma aerodynamic moment
Mf engine thrust moment
I_{yy} inertia moment
N_{NLG} normal force applied by nose landing gear
N_{MLG} normal force applied by main landing gear
H_{CG} distance from ground to CG position
A distance from nose tire contact point to CG position
B distance from main tire contact point to CG position

3.2. Coupled Equation

Until this point it was presented the equations for each independent body and component involved in ground rolling dynamics. These differential equations are coupled by aircraft motion and in order to allow the integration of them, they must be coupled somehow. The physical coupling between then are the aircraft attitude, pitch rate and acceleration and CG vertical position.

Observing the Fig. (6), it can be seen what is the cinematic relation above exposed:

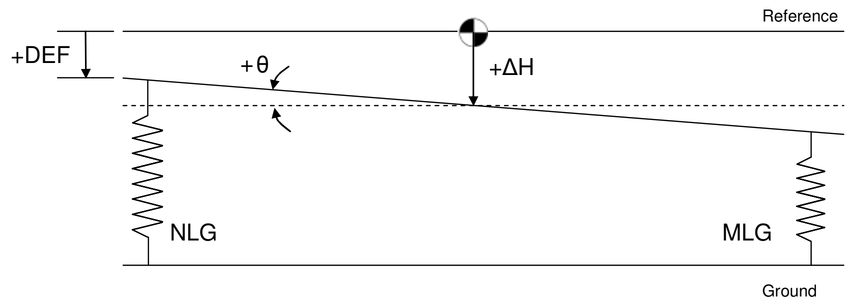

Figure 6: Relation between aircraft attitude, CG and shock absorbers deflection

Resuming the shock absorber equations shown in 2.1, the shock absorber displacement (DEF) and can be expressed for nose and main landing gears as function of parameters from Fig. (6):

$$DEF_{NLG} = -sin(\theta) \cdot A + \Delta H \quad (14)$$
$$DEF_{MLG} = sin(\theta) \cdot B + \Delta H \quad (15)$$

The determination of shock absorber compression velocity and acceleration is given by calculating the first and second derived of the Eq (14) and Eq. (15).

Nose Landing Gear:

$$D\dot{E}F_{NLG} = -q \cdot cos(\theta) \cdot A + \Delta\dot{H} \quad (16)$$
$$D\ddot{E}F_{NLG} = (-\dot{q} \cdot cos(\theta) + q^2 \cdot \sin(\theta)) \cdot A + \Delta\ddot{H} \quad (17)$$

Main Landing Gear:

$$D\dot{E}F_{MLG} = q \cdot cos(\theta) \cdot B + \Delta\dot{H} \quad (18)$$

$$DE\ddot{F}_{MLG} = (\dot{q} \cdot cos(\theta) - q^2 \cdot \sin(\theta)\,) \cdot B + \Delta\ddot{H} \tag{19}$$

Thus, the final coupled equations for nose and main landing gear reaction forces is obtained by replacing the terms for position, velocity and acceleration above calculated in Eq. (4).

4. ANALYTICAL CALCULATION

In this section the analytical calculation of the take-off runway length is presented. The principle of the following equations is based on the 2^{nd} Newton's law for the longitudinal movement. Recalling to Eq. (8), where the sum of forces in the inertial X direction is performed, and applying the simplification hypothesis that $\gamma = 0$, it results in $\alpha = \theta$. Moreover, considering $\alpha_f = 0$, the resulting simplified equation is shown in Eq. (20):

$$\dot{V} \cdot m = F \cdot \cos(\alpha) - D - m \cdot g \cdot sin(\theta) - Fat \tag{20}$$

Replacing the friction force by μ.N, and rewriting the reaction normal force (N) in terms of sum of forces in Z direction, Eq. (21) and Eq. (22) are obtained.

$$\dot{V} \cdot m = F \cdot \cos(\alpha) - D - m \cdot g \cdot sin(\theta) - \mu . N \tag{21}$$
$$\mathrm{N} = \mathrm{m} \cdot \mathrm{g} \cdot \cos(\theta) - L - \mathrm{F} \cdot \cos(\alpha) \tag{22}$$

Replacing Eq. (22) in Eq. (21):

$$\dot{V} \cdot m = F \cdot [\cos(\alpha) + \mu \cdot \sin(\alpha)] - D + \mu \cdot \mathrm{L} - \mu \cdot m \cdot g \cdot sin(\theta) \tag{23}$$

The traction force from a typical turbofan engine is equated as a parabolic law, as shown in Eq. (24).

$$\mathrm{F} = \mathrm{F}_0(1 + \mathrm{c}_1 \cdot \mathrm{V} + \mathrm{c}_2 \cdot \mathrm{V}^2) \tag{24}$$

Where F_0 is the maximum static traction force, V is the inlet air speed and c_1 and c_2 are motor constants.
Replacing Eq. (24) in Eq. (23), it becomes a parabolic equation, which can be written in terms of coefficients:

$$\dot{\mathrm{V}} = \mathrm{A} + \mathrm{B} \cdot \mathrm{V} + \mathrm{C} \cdot \mathrm{V}^2 \tag{25}$$

In order to find the total runway length (Δx), this equation must be integrated as follows:

$$\Delta\mathrm{x} = \int_0^{\mathrm{V_{LOF}}} \mathrm{V} \cdot (\mathrm{A} + \mathrm{B} \cdot \mathrm{V} + \mathrm{C} \cdot \mathrm{V}^2)^{-1}\, \mathrm{dV} \tag{26}$$

The analytical solution of the Eq. (26) depends on the factoring possibility of the denominators. This development is not the purpose of this study, so that the final equation for the runway length is presented in Eq. (27) (Paglione, 2009):

$$\Delta\mathrm{x} = \frac{\mathrm{V_d^2}}{2\mathrm{g}\left(\mathrm{F}/\mathrm{mg} - \mu\right)} \tag{27}$$

5. RESULTS

The simulation of the aircraft ground rolling dynamics was performed in the software Simuink®, in two different steps. The first one intended to determine the initial condition of the state variables, based on a given contour condition. The second step is the simulation itself, which calculates the time history of the state variables in response to a perturbation of the stead state condition.

The result of the analytical calculation describe in section 4 is also presented in this section, as function of friction coefficient, since this is a variable value in the numerical simulation.

5.1. Simulation

5.1.1. Initial Condition

The simulation proposed by this study was done to analyze the behavior of the aircraft take-off running. Since the running start by the aircraft at the stop condition, the state variables that must be defined prior to running the simulating is the shock absorbers compression, which imply in the aircraft attitude and the contour condition for this case is the aircraft velocity. However, the 0.0 m/s velocity cannot be applied directly because of the division by zero error that would occur when the calculation of the Wheel Slip is performed.

To solve this issue, some additional conditional calculation routines have been added to the model. In case of the speed is zero, the Wheel Slip is automatically set to zero. Additionally, the friction coefficient turns from dynamic to static value.

5.1.2. Results

In this section, the results of the take-off running are shown. The time history of the landing gear compression variables, aircraft pitch angle and runway length are plotted. The take-off run starts by applying full throttle followed by elevator deflection to produce pitch moment for rolling take-off. The simulation runs until both main and nose landing gears deflection is zero, which mean the aircraft is no longer in ground condition.

In Fig. (7) the aircraft running is presented, showing the aircraft general data, such as attitude angle (θ), angle of attack (α), runway length an altitude:

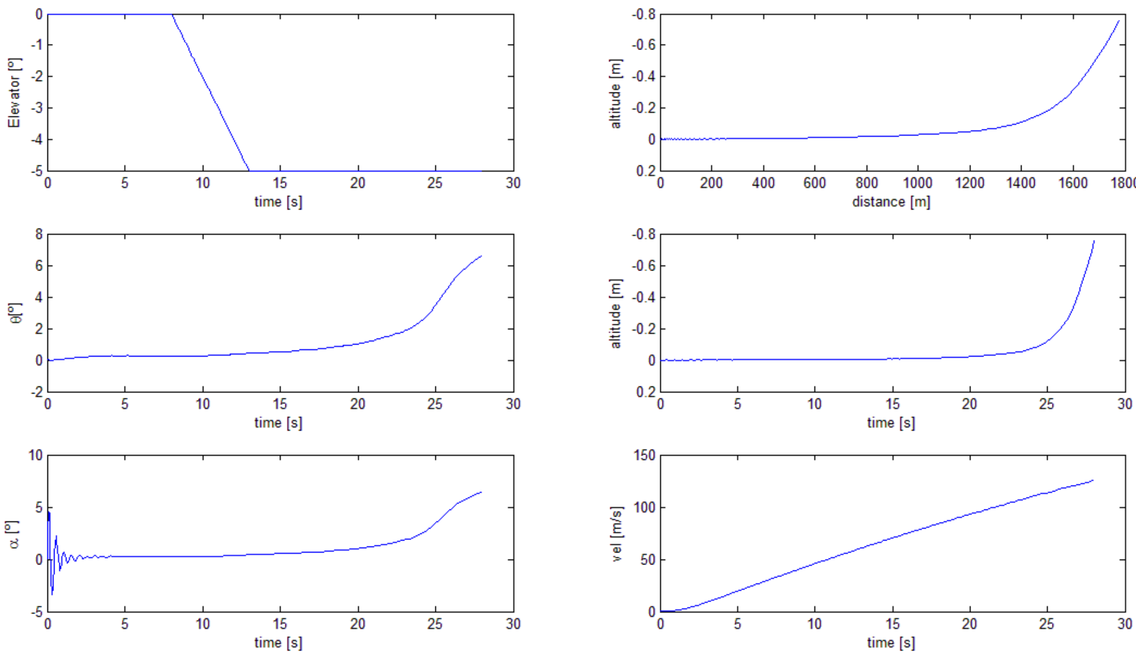

Figure 7: Take-off Running

In Fig. (8) it is plotted the runway length as function of aircraft speed, to be compared with the analytical solution.

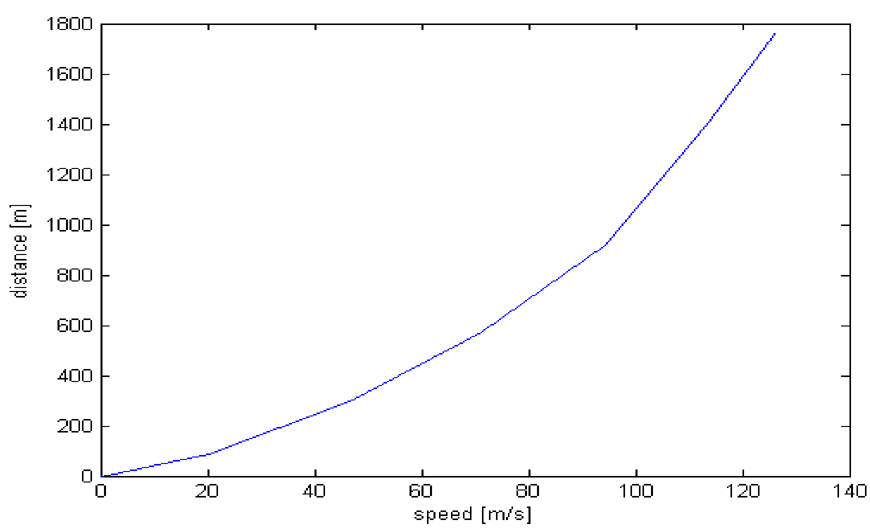

Figure 8: Take-off Running Runway length

Figure (9) shows the nose landing gear state variables time history. Compression, velocity and acceleration of compression/expansion are shown, as well as the force resultant due to each one of these states. The Y axis for the compression, velocity and acceleration are inverted, being positive downwards to reflect the signal convention adopted for the equations.

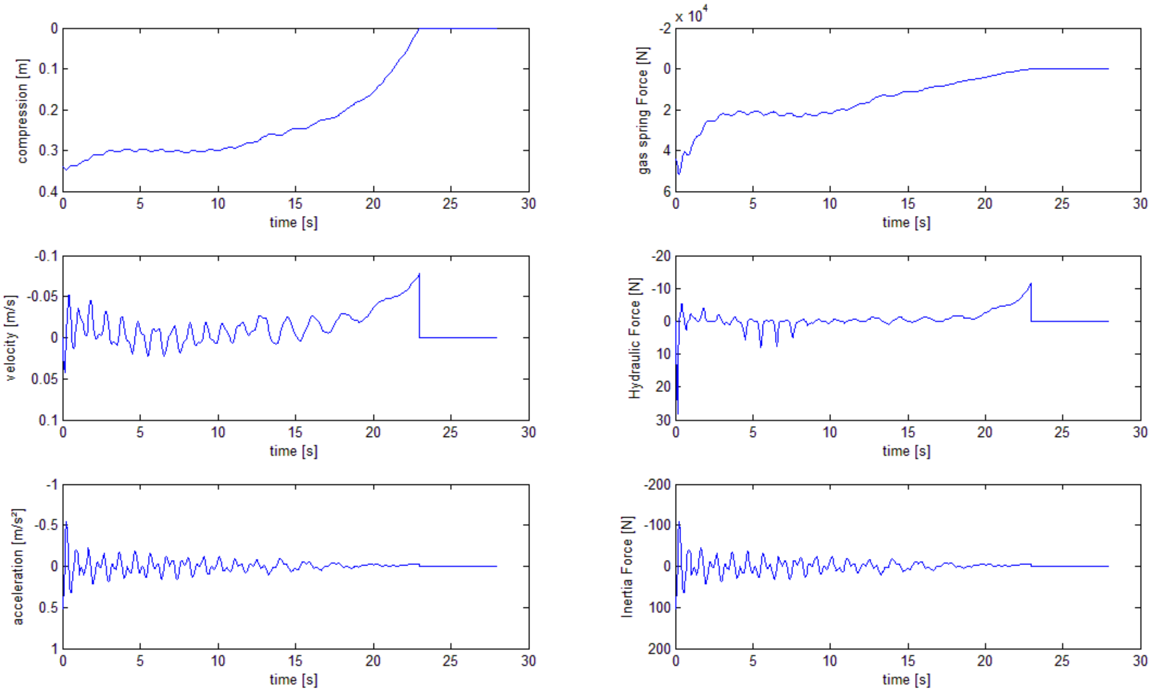

Figure 9: Nose Landing Gear Time History

Figure (10) shows the same data presented if Fig. (9), but to the Main Landing Gear.

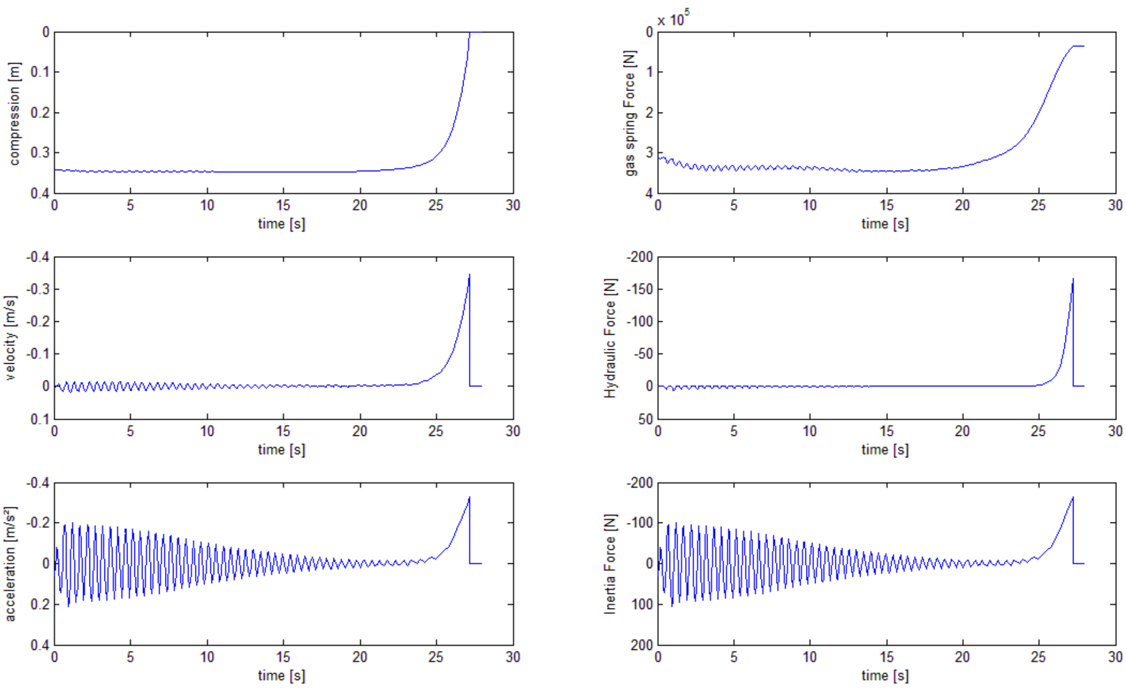

Figure 10: Main Landing Gear Time History

5.2. Analytical

Applying the Eq. (27) for a range of μ from 0.25 to 0.50 in steps of 0.05, the results as function on the velocity are shown in Fig. (11):

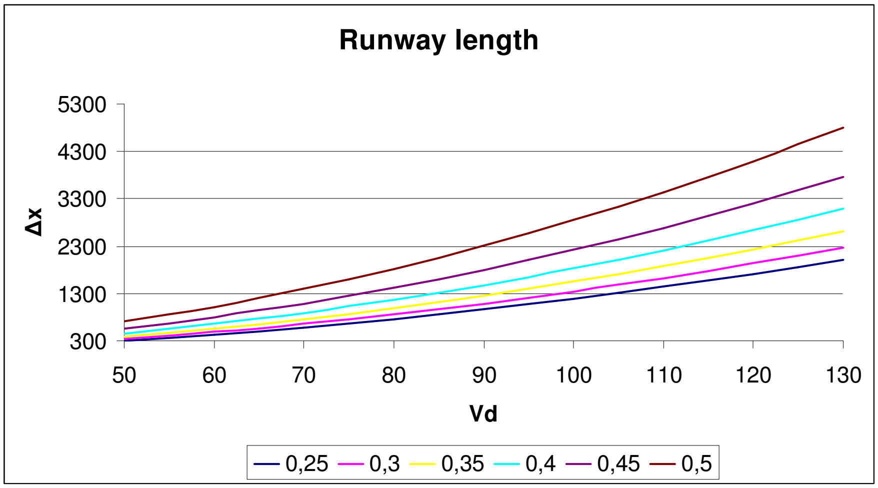

Figure 11: Runway length analytical results

6. CONCLUSION

The work presented herein showed the development of a model to simulate and aircraft in the ground rolling condition. The methodology of model each independent component followed by integrating their equations seemed to be suitable for the required complexity level.

The results obtained by the calculation of the static aircraft' attitude and shock absorbers compression are consistent with the values found in practice, where shock absorbers compression are approximately 80% compressed with the aircraft at the maximum take-off weight.

As described in section 5.1.2, the simulation started by the application of full throttle on both engines. The sudden increase in traction force and consequently pitch moment excites the shock absorbers, specially the nose landing due to bigger moment arm. This excitation decreases during the running as expected by the damping function. The main landing gear is also excited, however the amplitude of its movement is substantially smaller than the movements present in the nose landing gear. The traction pitch moment changes the level of compression of the main landing gear which stays in a range until the elevator deflection occurs.

At the moment that the elevator is deflected, the aerodynamics resultant moment increases, rotating the aircraft around the CG. The rotation is maintained by the aerodynamic forces, so that no additional load is imposed to the main landing gears, which does not present significant changes is its compression condition, as shown in Fig. (10). The nose landing gear, however, presents a different behavior of the main gear. During the rotation, as the pitch angle (θ) increases, the compression of the shock absorber decreases, presenting the normal expected behavior. The forces from shock absorbers compression, plotted in Fig. (9) and Fig. (10), decrease as the shock absorbers expand.

The resulting force from hydraulic source is almost inconsiderable when compared to the gas spring force. In fact, the hydraulic force is dimensioned to comply with requirements for landing absorption energy, thus, as ground operation loads are extremely lower than landing loads, it is expected that the contribution of the hydraulic force be low.

At the moment where the lift force increases and the aircraft gains altitude quickly it is possible to see that the nose landing gear is already full depleted and the main landing gear expands at the same speedy of the vertical velocity but such expansion speed may not be found physically. As explained in section 3.2, the landing gear compression depends on the aircraft attitude and CG variation, however, when the aircraft suddenly lift from the ground, the gas expansion effect associated with hydraulic restriction may not be enough to follow such a quick displacement. In this case, the aircraft would be in the air and the landing gear shock absorbers still would be extending.

This result shows that the approach herein described does not completely fits the needs for take-off simulation. It would be necessary to include shock absorbers expansion saturation and compare these values with the vertical CG variation rate.

Comparing the results obtained by the analytical solution and the simulation, it is possible to verify that the runway length in the analytical solution is greater than in the numerical simulation. One significant difference between those approaches is that in the analytical solution the friction coefficient is considered constant, while in the numerical simulation, it depends on the wheel slip. Furthermore, in the numerical solution, the attitude oscillation implies in the increase of lift force, reducing the friction force, while in the analytical solution these affects are neglected.

Summarizing, the modeling presented by this study is suitable for study the aircraft characteristics and performance on ground, but additional consideration shall be performed in order to enlarge its application to others purposes.

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