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NONLINEAR DYNAMICS OF THE DAISYWORLD

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Abstract: *Global warming is the observed increase of the average temperature of the Earths´ atmosphere and oceans. The primary cause of this phenomenon is the greenhouse gases released by burning of fossil fuels, land cleaning, agriculture, among others, leading to the increase of the so-called greenhouse effect. The consequences of this warming is unpredictable. The mathematical modeling of ecological phenomena has an increasing importance in recent years. These models may describe time evolution and spatial distribution and may explain some important characteristics of these systems. Although there are many difficulties related to the system description, their modeling may define at least a system caricature, which may be useful for different goals. This contribution deals with the modeling of the global warming in a dynamical point of view. Mathematical modeling is based on the daisyworld that is able to describe the global regulation that can emerge from the interaction between life and environment. This idea became famous as the Gaia theory of the Earth that establishes self-regulation of the planetary system. In brief, daisyworld represents life by daisy populations while the environment is represented by temperature. Here, two daisy populations are of concern, black and white daisies, and an extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Besides, energy equation is considered in order to investigate transients phenomena related to temperature variation. Numerical simulations are carried out presenting a qualitative description of the phenomenon.*

Key-words: *Global warming, daisyworld, nonlinear dynamics.*

1. INTRODUCTION

The mechanism of the Earth's heating is related to the energy balance where the main aspects are the radiation energy from the sun and the thermal radiation from the Earth that is radiated out to space. The atmosphere plays an essential role in this process and the presence of greenhouse gases tends to break this balance since they are transparent to the sun short wave radiation, however, they absorb some of the longer infrared radiation emitted from the Earth. Therefore, the increase amounts of these gases makes the Earth cool more difficult increasing the Earth's surface temperature.

Global warming is a specific case of the more general term climate change that is induced either by natural processes or by human activities. In general, it is important to establish a difference between climate change and climate variability and it is possible to identify numerous researches trying to make a proper distinction of these phenomena. In brief, global warming is the observed increase of the average temperature of the Earth's atmosphere and oceans. The primary cause of this phenomenon is the release of greenhouse gases by burning of fossil fuels and large-scale deforestation, leading to the increase of the so-called greenhouse effect that arises as a consequence of the presence of greenhouse gases in the atmosphere. Among others, the main greenhouse gases are the water vapor, the carbon dioxide, the methane and the nitrous oxide. (Houghton, 2005).

Based on Intergovernmental Panel on Climate Change (IPCC, 2007) data, the amount of greenhouse gases in the atmosphere has significantly increased since the industrial revolution. Moreover, during the 20th century, the Earth's surface average temperature has increased approximately 0.4 to 0.8°C. The consequences of the global warming are unpredictable, however, many authors points the climate sensitivity and other changes related to the frequency and intensity of extreme weather events.

The mathematical modeling of ecological phenomena has an increasing importance in recent years (Savi, 2005, 2006). These models may describe time evolution and spatial distribution and may explain some important characteristics of these systems. The mathematical analysis is exploiting the possibility that many of these phenomena may have their roots in some underlying dynamical effect. Although there are many difficulties related to the system description, their modeling may define at least a system caricature, which may be useful for different goals.

This contribution deals with the modeling of the global warming in a dynamical point of view. A mathematical model based on the daisyworld (Lenton & Lovelock, 2000, 2001; Lovelock, 1992) is proposed. The daisyworld is an archetypal of the Earth and is able to describe the global regulation that can emerge from the interaction between life and environment. This idea became famous as the Gaia theory of the Earth that establishes self-regulation of the planetary system. In brief, daisyworld represents life by daisy populations while the environment is represented by temperature. Here, two daisy populations are of concern, black and white daisies, and an extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Besides, energy equation is considered in order to investigate transients phenomena related to temperature variation. Numerical simulations are carried out in order to present a qualitative description of the global warming.

2. DAISYWORLD MODEL

Climate system has an inherent complexity due to different kinds of phenomena involved. The equilibrium of this system is a consequence of different aspects related to the atmosphere, oceans, biosphere and many others, and the sun activity provides the driving force for this system. The Earth's heating mechanism may be understood as the balance between the radiation energy from the sun and the thermal radiation from the Earth and the atmosphere that is radiated out to space. The presence of greenhouse gases tends to break this balance since they are transparent to the sun short wave radiation, however, they absorb some of the longer infrared radiation emitted from the Earth. Therefore, the increase amounts of these gases makes the Earth cool more difficult increasing the Earth's surface temperature.

Lovelock (1983a, b) proposed a model to demonstrate that global regulation can emerge from the interaction between life and environment. This behavior was represented by an archetypal model called daisyworld that represents an imaginary planet populated by organisms in coexistence. The daisyworld is basically composed by the environment, represented by the temperature, and by populations of daisies representing life. In brief, it is assumed that daisyworld is like the Earth but with less oceans and with the whole surface being fertile. The original daisyworld includes only two populations of daisies but further investigations include herbivores and carnivores as well as daisies.

The first step of the daisyworld modeling is the definition of life, represented by daisies, which evolution is described by the following general equation where $\alpha_i \, (i = 1,2,...N)$ represents the area coverage by daisy populations:

$$\dot{\alpha}_i = \alpha_i \left[\alpha_g \beta(T_i) - \gamma \right] \tag{1}$$

where dot represents time derivative, β is variable growth rate that is temperature dependent and γ is the death rate. Daisy colors define the amount of energy absorption and the balance between daisy populations can control the planet temperature. A first approach to this archetypal model is to consider only two daisy populations: black, α_b, and white, α_w. Black daisies absorb more energy while white daisies absorb less energy.

In order to incorporate the greenhouse gases in the daisyworld, a new population is included into the model. The idea is to represent the albedo increase due to the effect of the greenhouse gases. In this regard, the inclusion of the greenhouse variable G has an effect similar to black daisies. Therefore, the model is written as in the classical way, but now there is a function that establishes the greenhouse gases time history:

$$G = G(t) \tag{2}$$

The variable α_g is the fractional area coverage of the planet represented by:

$$\alpha_g = p - \sum_{i=1}^{N} \alpha_i - G \tag{3}$$

Here, p represents the proportion of land suitable for the growth of daisies and N represents the biodiversity related to the number of populations involved in the system.

The mean planetary albedo of the daisyworld, A, can be estimated from the individual albedo of each population (a_i for daisies, a_g for the bare ground and a_G due to greenhouse gases):

$$A = \alpha_g a_g + \sum_{i=1}^{N} \alpha_i a_i + G a_G \tag{4}$$

Afterwards, the local temperature of each population is defined as follows:

$$T_i^4 = q(A - a_i) + T^4 \tag{5}$$
$$T_g^4 = q(A - a_g) + T^4 \tag{6}$$
$$T_G^4 = q(A - a_G) + T^4 \tag{7}$$

where T is the globally-averaged temperature of daisyworld, and q is a constant used to calculate local temperature as a function of albedo. Finally, it is important to establish the thermal balance of the daisyworld (Foong, 2006), and therefore, the absorbed energy is given by (Nevison *et al.*, 1999):

$$\dot{T} = (1/c)(SL(1 - A) - \sigma T^4) \tag{8}$$

L is the solar luminosity and S is the solar constant that establishes the average solar energy, SL; σ is the Stefan-Boltzmann constant; $\boldsymbol{c}$ is a measure of the average heat capacity or thermal inertia of the planet.

The functional form for β_i is usually assumed to be a symmetric single-peaked function as follows:

$$\beta_i(T) = \begin{cases} B\left[1 - \left(\dfrac{T_{opt} - T_i}{k}\right)^2\right] & |T_{opt} - T_i| < k \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where T_{opt} is the optimal temperature usually assumed to be T_{opt} = 295 K = 22.5°C. The parabolic width k is chosen in order to establish proper life conditions as for example, between 5°C and 40°C (De Gregorio *et al.*, 1992), which is related to $k = 17.5$. In the same way, B alters these values in order to represent different environmental characteristics.

The daisyworld model can be simulated using classical procedures for numerical integration. Here, the fourth order Runge-Kutta method is employed. In general, the following parameters are assumed: q=2.06×10^9 K^4, σ = 5.67×10^{-8} W/m^2 K^4, S = 917 W/m^2. Other parameters are varied in order to analyze different situations. Moreover, it is important to highlight that only black and white daisy populations are considered.

3. CLASSICAL DAISYWORLD: NUMERICAL SIMULATIONS

This section investigates the dynamics of the daisyworld in order to establish a proper comprehension of the self-regulation of the Earth. Greenhouse gases are not considered in this section. Parameters used in this simulation are a_w = 0.75, a_b = 0.25, a_g = 0.5, γ = 0.3, B = 1 and the initial conditions $\alpha_w = \alpha_b$ = 0.01. All simulation employs time steps less than 0.01.

Initially, constant luminosity is of concern assuming the classical situation with c = 0. The daisyworld has self-regulation due to the interaction between life and environment, represented respectively by daisy populations and the planet temperature. Therefore, the planetary system tends to maintain a constant temperature due to the interaction between black and white daisy populations. The increase of the black daisies tends to increase the planet temperature since they absorb more energy, and the opposite occurs concerning white daisies. Hence, the population growth is in such a way that temperature remains constant in a favorable value as shown in Figure 1.

Afterwards, the influence of thermal inertia is investigated. Basically, three different values are of concern: c = 300, 1000 and 3000 J/m^{2}K s. Figures 2-4 show the system response for these situations. Note that there is an oscillatory behavior from daisies populations that causes temperature oscillation.

A linear increase of the solar luminosity (0.75 < L < 1.7) is now in focus, representing a more realistic representation of the sun activity. Under this condition, the planet temperature would tend to increase linearly, following the luminosity increase. Nevertheless, the self-regulation of the daisyworld tends to maintain a constant temperature due to the interaction between black and white daisy populations. Once again, the increase of the black daisies tends to increase the planet temperature since they absorb more energy. This occurs when the solar luminosity has small values. The increase in solar luminosity causes the decrease of the black daisies population and the increase of the white daises. This balance is represented by a tendency of constant values of temperature. Figure 5 shows the evolution of the daisy populations and the temperature for c = 0. It is clear that, when the luminosity is small, black daisies are preponderant. The more luminosity increases, the more white daisies increase. Figure 5b also establishes a comparison between temperature evolution for dead planet (without life, or daisies) and the planet with life (with daisies).

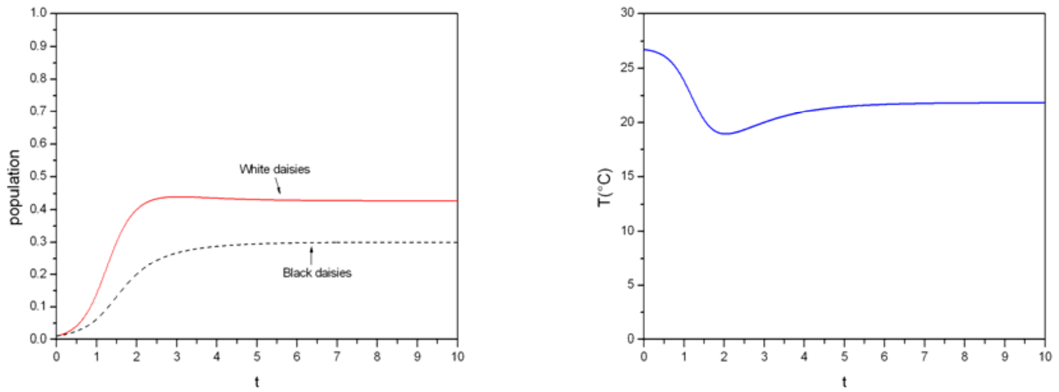

Figure 1. Daisyworld response with constant solar luminosity (L = 1) and c = 0. Daisy population (left) and the temperature (right).

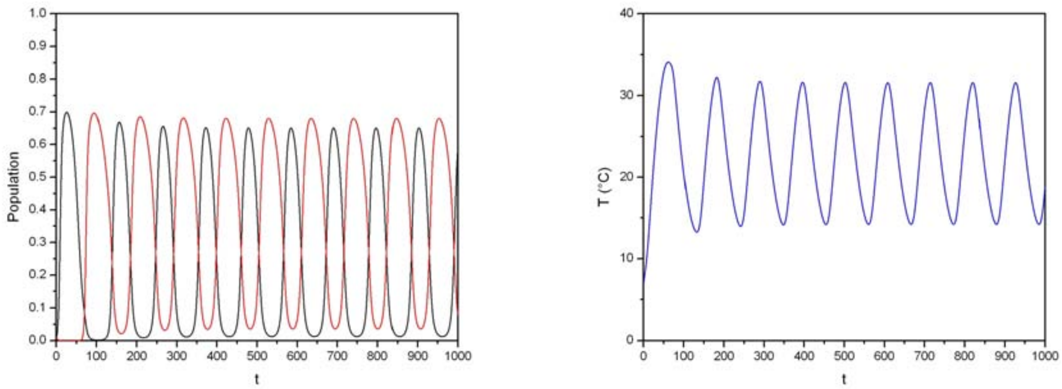

Figure 2. Daisyworld response with constant solar luminosity (L = 1) and c = 300 J/m^{2}K s.

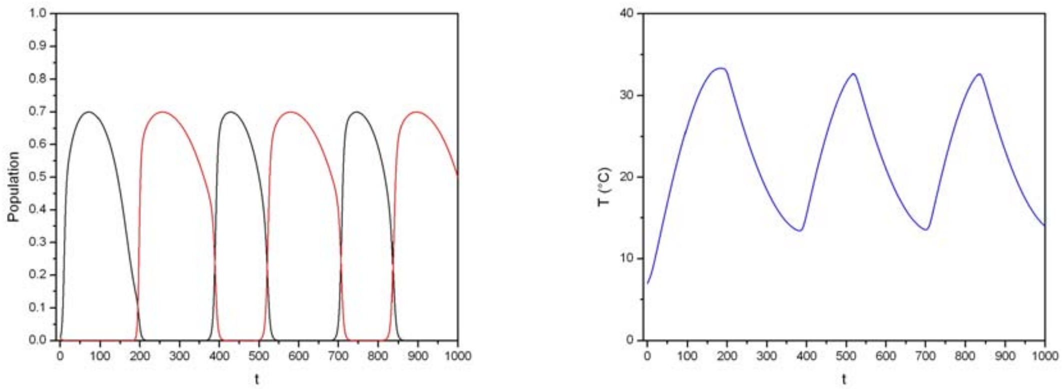

Figure 3. Daisyworld response with constant solar luminosity (L = 1) and c = 1000 J/m^{2}K s.

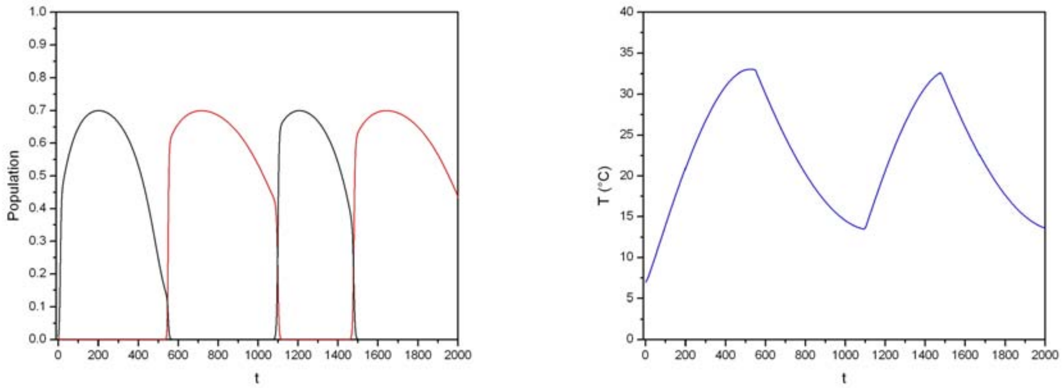

Figure 4. Daisyworld response with constant solar luminosity (*L* = 1) and *c* = 3000 J/m^{2}K s.

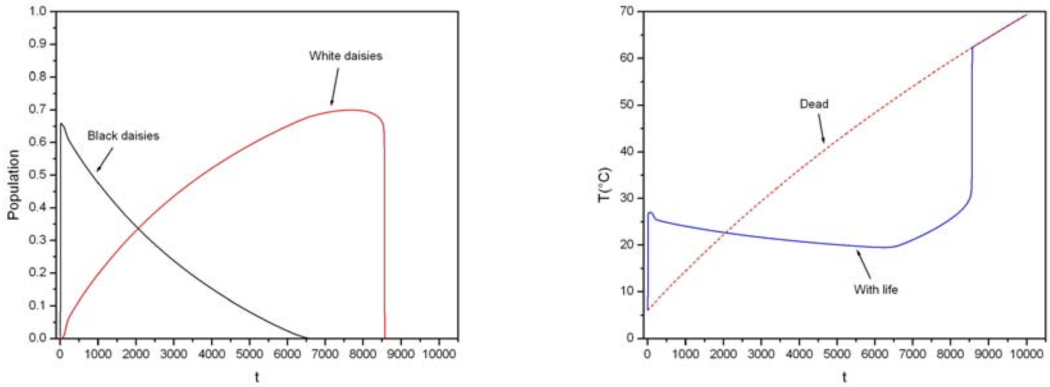

Figure 5. Daisyworld response with linear increase solar luminosity (0.75 < *L* < 1.7) and *c* = 0.

The influence of planet thermal inertia is now in focus by assuming three different values of *c*: 300 (Figure 6), 1000 (Figure 7) and 3000 (Figure 8) J/m^{2}K s. Once again, the increase of thermal planet inertia tends to promote oscillatory variations of all involved variables and it should be highlighted a proper balance between both populations and the temperature.

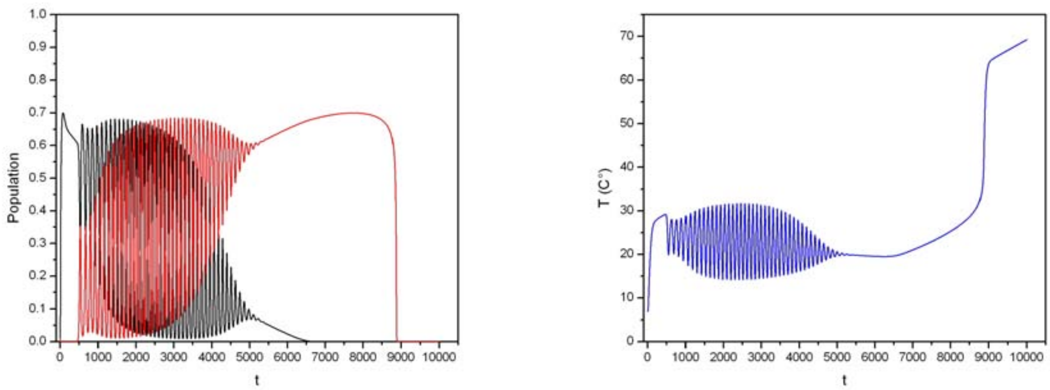

Figure 6. Daisyworld response with linear increase solar luminosity (0.75 < *L* < 1.7) and *c* = 300 J/m^{2}K s..

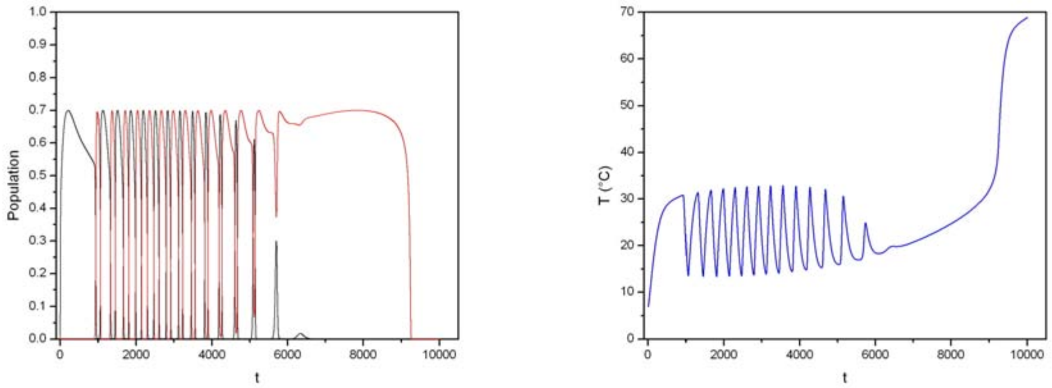

Figure 7. Daisyworld response with linear increase solar luminosity (0.75 < *L* < 1.7) and *c* = 1000 J/m^{2}K s.

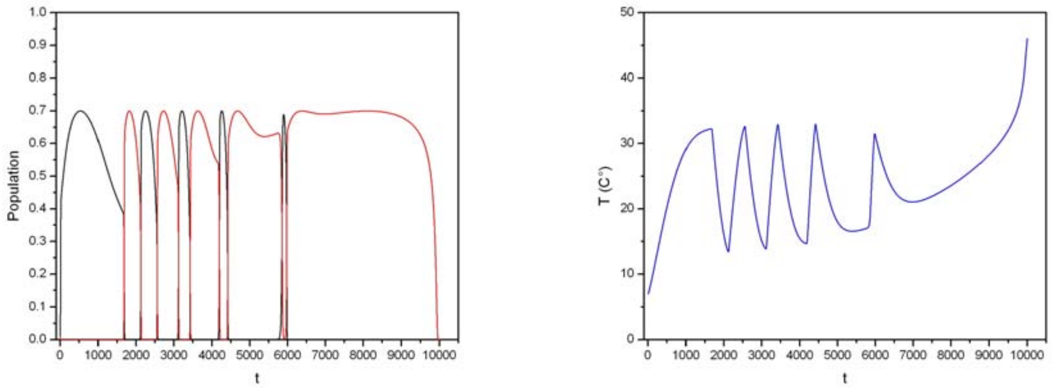

Figure 8. Daisyworld response with linear increase solar luminosity (0.75 < *L* < 1.7) and *c* = 3000 J/m^{2}K s.

4. DAISYWORLD WITH GREENHOUSE GASES: NUMERICAL SIMULATIONS

This section considers greenhouse gases in the daisyworld. Basically, it is assumed that these gases are known being related to a function or a time series. Experimental values are used as a reference to characterize the general tendency of these gases. In this regard, CO_2 emissions from 1958 to 2009 are used (NOAA, 2009) and Figure 9 presents the average of the annual emission. The definition of *G* values is estimated from this general tendency. It should be observed that there is a linear increase in CO_2 emissions, and this information is used in numerical simulations that assume time steps of 0.01. The basic idea of this section is to establish a comparison with results of the preceding section that do not consider greenhouse gases.

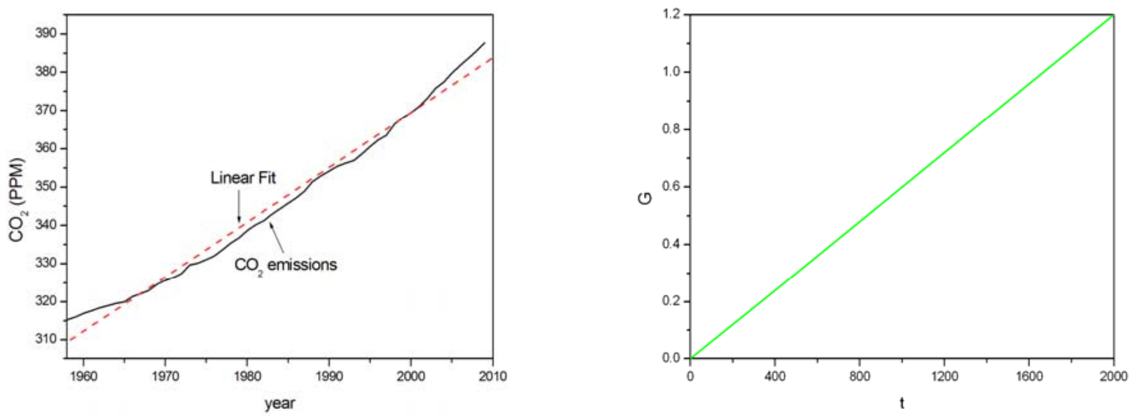

Figure 9. Annual average of the CO_2 emissions.

Initially, a linear increase luminosity is treated (0.75 < L < 1.7) and the greenhouse gases are incorporated in the model. Parameters used in this simulation are the same from the previous simulations. Figure 10 presents results of the daisyworld response showing the same behavior of the previous case, without greenhouse gases. Nevertheless, it is important to observe that the increase of the planet temperature promoted by greenhouse gases tends to cause the death of daisy populations earlier when compared to the planet without gases. This Figure also presents a comparison between the dead and the live planets showing how life interaction promotes the self-regulation of the planet.

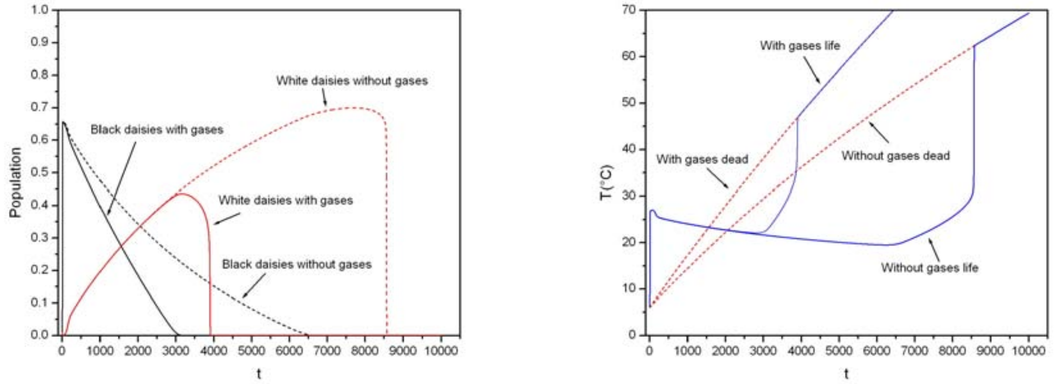

Figure 10. Daisyworld response with linear increase solar luminosity (0.75 < L < 1.7), greenhouse gases (0.0 < G < 0.8) and c = 0.

The forthcoming analysis considers the influence of the planet thermal inertia by assuming linear increase of the solar luminosity (0.75 < L < 1.7). Figures 11-13 shows the system behavior for different values of c: 300 (Figure 11), 1000 (Figure 12) and 3000 (Figure 13) J/m^{2}K s. Note that there is a proper balance between both populations and the temperature tends to be constant, however, thermal inertia tends to cause an oscillatory response.

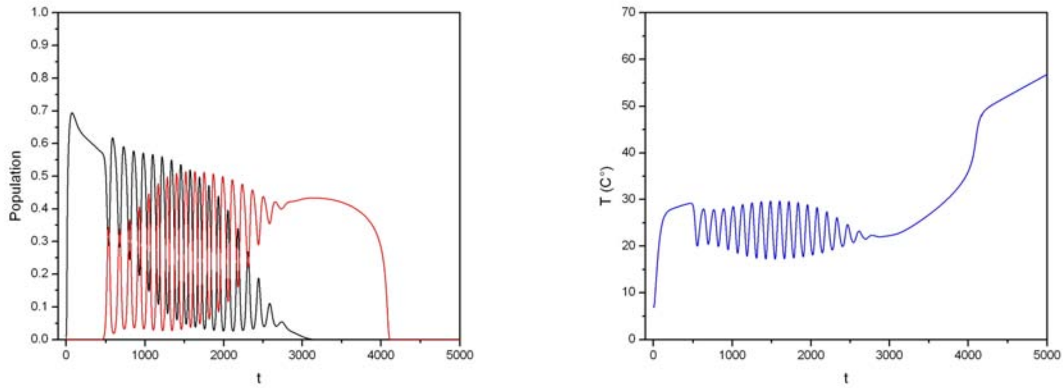

Figure 11. Daisyworld response with linear increase solar luminosity (0.75 < L < 1.7), greenhouse gases (0.0 < G < 0.8) and c = 300 J/m^{2}K s.

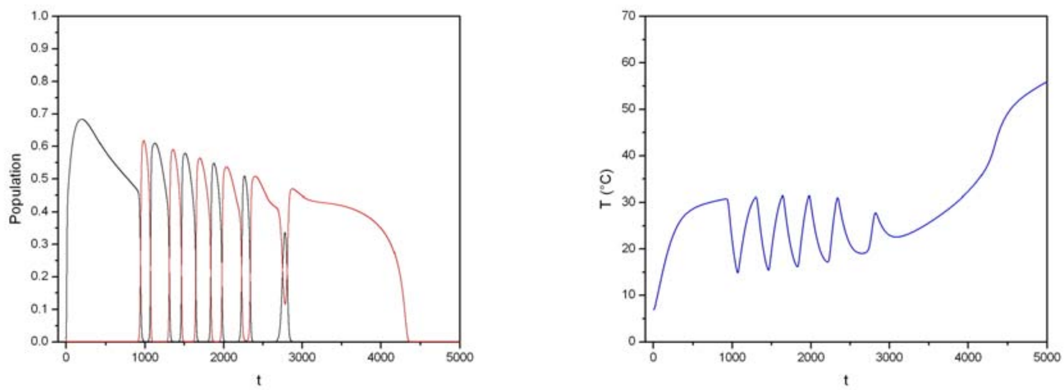

Figure 12. Daisyworld response with linear increase solar luminosity (0.75 < L < 1.7), greenhouse gases (0.0 < G < 0.8) and c = 1000 J/m^{2}K s.

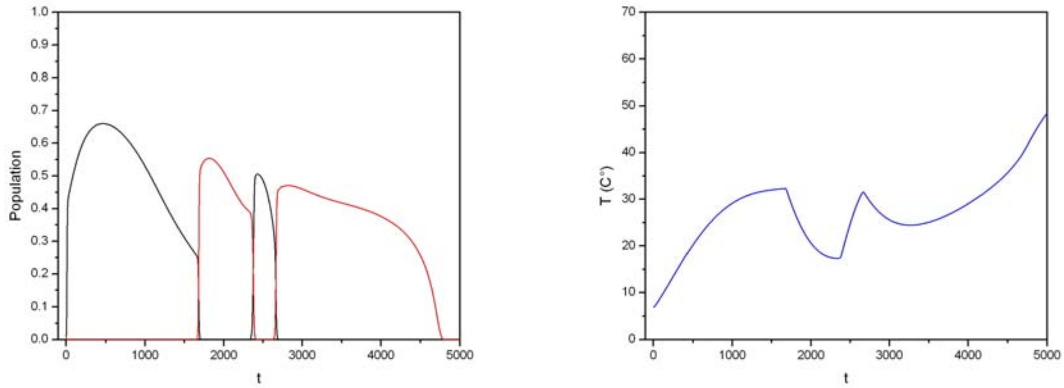

Figure 13. Daisyworld response with linear increase solar luminosity (0.75 < L < 1.7), greenhouse gases (0.0 < G < 0.8) and c = 3000 J/m^{2}K s.

Global warming is the observed increase in the average temperature of the Earth and the primary cause of this phenomenon is the greenhouse effect related to greenhouse gases. An important question related to the global warming concerns with its reversibility. In other words, it is important to know if the decrease of the greenhouse gases emissions is enough to make the planet to reach temperatures of the past. In order to observe this kind of behavior, we make a simulation establishing a loading-unloading of the greenhouse gases depicted in Figure 14 that considers a linear increase of luminosity. Under this condition, a different kind of response occurs being representative of a more realistic situation. The thermal inertia (c = 1000 J/m^{2}K s) tends to promote an oscillatory response of the system.

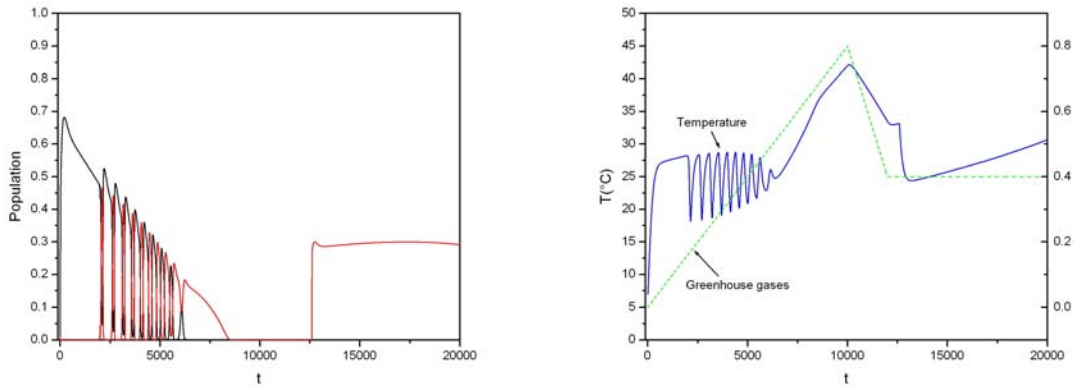

Figure 14. Evolution of daisy population (left) and temperature (right) with variations in greenhouse gases emissions, considering luminosity with linear increase and c = 1000 J/m^{2}K s.

Nowadays, there is an important distinction that should be establishes between climate change and climate variability. A key difference between both terms is the persistence of anomalous conditions. In order to investigate the effect of climate variability in the daisyworld, it is assumed a sun luminosity with a sinusoidal variation represented as follows:

$$L = \overline{L} + \left(L_0 \sin(\omega t)\right) \tag{10}$$

The forthcoming simulations consider linear increase of luminosity ($0.75 \le \overline{L} \le 1.7$) and a sinusoidal variation with L_0 = 0.5 and ω = 0.05. Moreover, it is assumed c = 1000 J/m^{2}K s. Two different situations are carried out: without greenhouse gases (Figure 15) and with gases (Figure 16). It should be observed that the system has an irregular behavior. Chaotic behavior of the daisyworld was addressed in different references. Zeng *et al.* (1990) used logistic map to represent the continuous system but actually, this representation cannot be reproduced from the continuous system. On the other hand, Flynn (1993) used an excitation with time delay. Our approach seems to be more realistic and needs to be further investigated.

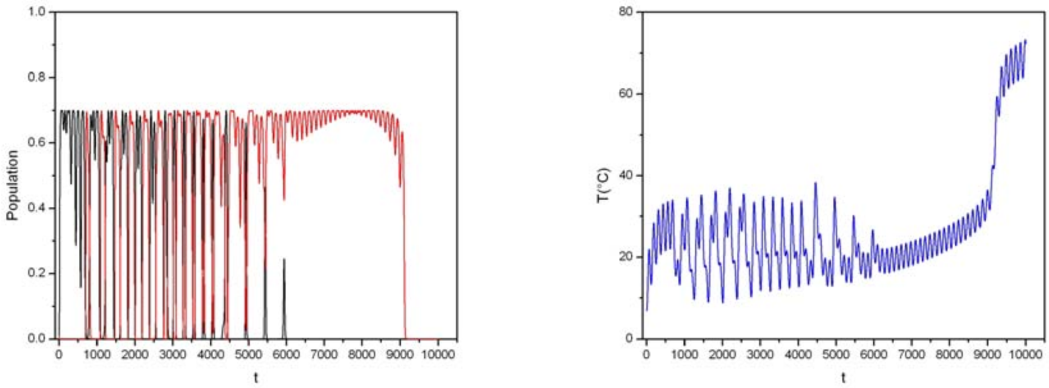

Figure 15. Evolution of daisy population (left) and temperature (right), considering luminosity with linear increase, without greenhouse gases and c = 1000 J/m^{2}K s.

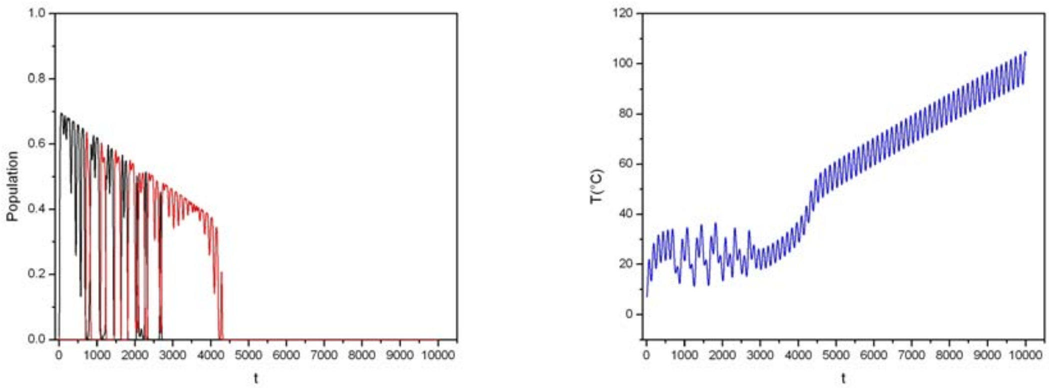

Figure 16. Evolution of daisy population (left) and temperature (right), with variations in greenhouse gases emissions, considering luminosity with linear increase and c = 1000 J/m^{2}K s.

5. CONCLUSIONS

The daisyworld is an archetypal of the Earth and is able to describe the global regulation that can emerge from the interaction between life and environment. In brief, daisyworld represents life by black and white daisy populations while the environment is represented by temperature. An extra variable related to greenhouse gases is incorporated in the model allowing the analysis of the global warming. Besides, energy equation is considered in order to investigate transients phenomena related to temperature variation. A general analysis of the daisyworld is carried out analyzing constant and linear increase of the solar luminosity. Afterwards, the influence of greenhouse gases in the daisyworld dynamics is treated establishing a comparison with the classical model. In general, these gases tend to increase the planet temperature, accelerating the death of populations and decreasing the capacity of global regulation. Thermal inertia of the planet is also of concern showing its influence in the system response. Climate variability is investigated by assuming a sinusoidal variation of the sun luminosity. Results show irregular pattern that need to be further investigated. The authors believe that the proposed model can be used for a qualitative description of the global warming phenomenon that is an essential problem of this century.

6. ACKNOWLEDGEMENTS

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