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# THREE-DIMENSIONAL HYBRID FINITE ELEMENTS IN THE ANALYSIS OF REINFORCED PANELS MADE OF COMPOSITE MATERIALS

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***Abstract:*** *This work aims to apply three-dimensional hybrid finite elements (also called quasi-Trefftz) in the analysis of a typical aircraft structure (reinforced panel with several kinds of loading) made by composite material. The elements of Trefftz are hybrids finite elements in which the stresses are approximated in the field of element through hierarchical harmonic and orthogonal polynomials derivates from potential of Papkovitch-Neuber, which must satisfy locally the differential equations of Navier, and displacements are approximated in the static border through polynomials. The element quasi-Trefftz is an element that use stress approximation functions which are, primarily, solution of Navier equation for isotropic materials, later, adapted for composite materials. For this analysis, a local model representing the interface between the skin and the stiffener of the panel has been developed. Through the results, the efficiency of quasi-Trefftz element was verified. In a typical aircraft structure, the proposed element generated good results when compared with a conventional three-dimensional finite element, even using less refined meshes. The element also presented low sensitivity to the mesh distortion, allowing their use in complex geometries or in case with presence of singularities, such as cracks or holes.*

***Keywords:*** *Composite materials, Finite Elements, Hybrid Finite Elements*

## 1. INTRODUTION

This paper presents the application of three-dimensional hybrid finite element, also known as quasi-Trefftz, in the analysis of a typical aircraft structure, which is composed of a reinforced panel (skin and stringers) made of composite material.

To check the efficiency of elements in question, a comparison between the results of quasi-Trefftz elements and a conventional three-dimensional finite element is done.

### 1.1. Composite materials in aerospace industry

Although composite materials are known in several forms for many years, the story of what they call modern composites probably began in 1937 when an American salesman started selling fiberglass for companies in the United States. The fiberglass was created almost by accident when an engineer was intrigued to see a fiber that was formed in the process of writing the bottles of milk. The first products created with this fiber were intended for thermal and acoustic isolation, but the application in structural products was quickly discovered and developed.

Companies realized that the aerospace industry was a potential customer of this fiber due to the fact that this sector has constant need for new technologies, particularly in the area of materials. In the beginning, the fiberglass and, consequently, others composite materials, were used only for tooling, but the development of this material was increased during the Second World War. They started to use this kind of material in structural and semi-structural parts of the aircrafts. As an example there is the application in naceles of the Douglas A-20.

Nowadays the use of composite materials in the aerospace industry, both in the tooling and the manufacture of parts (structural or not), has become common due to the need to obtain better mechanical properties combined with low weight. A great example is the increase in the percentage of use of composites in modern aircraft by major manufacturers like the Brazilian Embraer, the European Airbus and the North American Boeing.

### 1.2. Hybrid finite elements

The Finite Element Method (FEM) has its origin in the '40s, but its adoption is driven more strongly only in the last 20 or 30 years due to technological advances in computer equipment. Due its flexibility, simplicity and numerical stability, the FEM can be implemented as a computer system in a consistently and systematically way. This fact become the method so popular today, being used in several areas of engineering, such as design and analysis of structures, analysis of fluid flow, temperature distribution and design of electromechanical equipment (machines, transformers and contactors). Since the ‘50s, the method has been widely used in aerospace industry, especially in the analysis of complex structures, generating a big boost to the method development and improvement.

The conventional MEF, used on the majority of computer systems, approaches directly only in the fields of displacements. Then, the strains and stresses are obtained subsequently. This application has some limitations, especially with regard to their sensitivity to mesh generation and definition. Extremely complex and refined meshes may be necessary, spending a big time for creation and processing the problem.

The three-dimensional hybrid finite element approaches directly not only in the fields of displacements as well as in the fields of stress. Element of Trefftz is hybrid finite element where stresses are approximate in the element through hierarchical harmonic and orthogonal polynomials derivatives from the potential of Papkovitch-Neuber, which must satisfy locally the differential equations of Navier and the displacements are approximated at the static boundary by polynomials.

In this work, the use of three-dimensional hybrid finite element, also called quasi-Trefftz, will be presented. The quasi-Trefftz is an element that use stress approximation functions which are, primarily, solution of Navier equation for isotropic materials, later, adapted for composite materials. These elements present good behavior in relation to the stresses and displacements, resulting in solutions with very good accuracy even with coarse meshes (with few elements). They also have high rates of convergence, even for approximations of stresses with degrees higher than six, where the basis is not complete. The elements also have low sensitivity to mesh distortion, in other words, the three-dimensional elements can be created with a size much larger than the other, without loss quality in the solution.

The quasi-Trefftz formulation allows two types of refinements to the problem in question. A refinement is obtained from the number of elements in the mesh, and the other refinement is the range of degrees used in the approach functions. Therefore, the element will be characterized by the notation HTS (ns, nu), where:

- ns is the degree of stress approach function in the element.
- nu is the degree of displacement approach function in the boundary.

The following elements were used in this work:

- HTS (7,3);
- HTS (9,3);
- HTS (9,4);
- HTS (10,3);
- HTS (10,4).

The element type used in this work is the eight-node hexahedron.

The motivation of this work is to verify the efficiency and applicability of the three-dimensional hybrid finite element quasi-Trefftz in the analysis of a typically aircraft structure. It is proposed a practical application through a typical structure, consisting of a reinforced panel (skin and stringer) made of composite material.

## 2. DEVELOPMENT

### 2.1. The skin-stringer assembly

The configuration of the analyzed structure consists, basically, of four components made of composite material: skin, pad and stringer (flange and web). The configuration of the skin-stringer assembly is sketched in Fig (1) below.

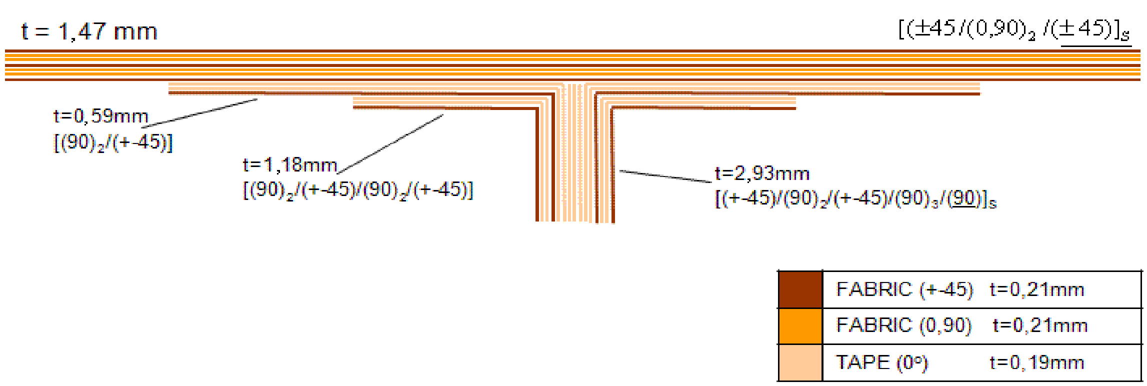


**Figure 1. Skin-stringer assembly.**

The structure dimensions are shown in Fig (2). The length of the assembly in the longitudinal direction (direction perpendicular to the plane shown in Fig (2)), is 200 mm.

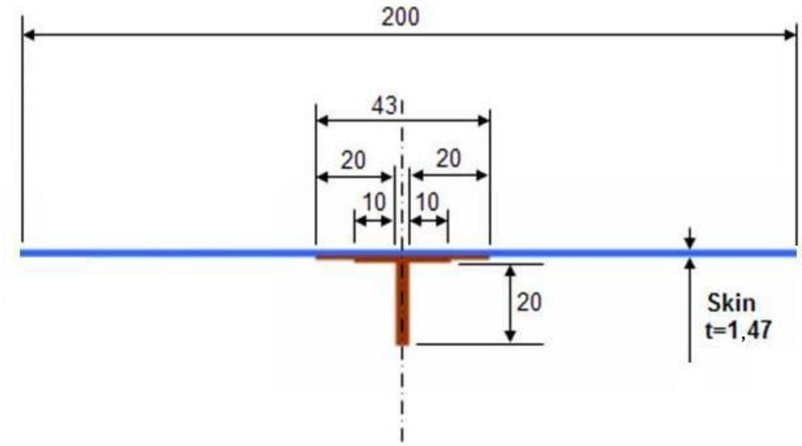



**Figure 2. Skin-stringer assembly dimensions (mm).**

The mechanical properties of the both materials used in this work are shown in Tab. (1), where Ei is the Young modulus in the i direction, NUij is the Poisson coefficient in the ij plane and Gij is the shear modulus in the ij plane.

**Table 1. Mechanical properties of composite materials.**

| Material | Thickness [mm] | E1 [MPa] | E2 [MPa] | NU 12 | G 12 [MPa] |
|---|---|---|---|---|---|
| *Carbono Epoxy Fabric* | 0.21 | 60800 | 58250 | 0.07 | 4550 |
| *Carbono Epoxy Tape* | 0.19 | 125450 | 9450 | 0.32 | 4700 |

From the configuration of the structure, in other words, from the knowledge of material properties and fiber orientation, the apparent properties for each element can be obtained by typical software. This software uses the classic laminate theory to obtain the apparent properties. The results are shown in Tab. (2) below.

**Table 2. Apparent properties.**

| Element | E1 [MPa] | E2 [MPa] | G12 [MPa] | NU12 |
|---|---|---|---|---|
| Skin | 45836 | 45836 | 14538 | 0.283 |
| Pad | 12835 | 60581 | 7659 | 0.131 |
| Flange | 16343 | 81444 | 11400 | 0.123 |
| Web | 16258 | 94816 | 11338 | 0.100 |

### 2.2. Nastran Model

In order to obtain the reference results for analysis using quasi-Trefftz element, a model was developed in the software MSC.Nastran ® 2004, the company's product "The MacNeal-Schwendler Corp.", since this is a higher performance tool to make calculations of engineering problems, with easy interface and very known by the professionals.

For the Nastran model, it was used the three-dimensional solid orthotropic element (MAT9). The constants were the properties have already been obtained as shown in Tab. (2).

In order to test the model convergence, it was modeled six different refinements, in other words, six different meshes.

A parameter for checking the quality of the mesh is the aspect ratio, which is obtained through the ratio between the highest and lowest side of the element. For conventional three-dimensional elements, it is better to keep this rate less than 100 to ensure good results. The Tab. (3) shows the number of elements, the maximum aspect ratio and the number of degrees of freedom for each Nastran model.

**Table 3. Nastran model – number of elements, maximum aspect ratio and degrees of freedom.**

| Mesh | Number of elements | Maximum aspect ratio | Degrees of freedom |
|---|---|---|---|
| **Mesh 1** | 90 | 68 | 480 |
| **Mesh 2** | 220 | 34 | 1200 |
| **Mesh 3** | 620 | 17 | 3480 |
| **Mesh 4** | 2360 | 8,5 | 12960 |
| **Mesh 5** | 3900 | 6,8 | 21000 |
| **Mesh 6** | 9040 | 5,1 | 48960 |

### 2.3. Three-dimensional hybrid model

Aiming to verify the applicability and to confirm the characteristics of the three-dimensional hybrid quasi-Trefftz element in the analysis of reinforced panel made of composite material, it was create some models through the software developed by Bussamra and Raimundo Jr, 2004.

In the same way that Nastran model, the constants were the properties that have already been obtained as shown in Tab. (2).

In order to analyze the element, it was modeled four different refinements, in other word, four different meshes. For the three-dimensional hybrid element, the convergence will be also analyzed according to the variation of the degrees used in the approach functions as already described.

The Tab. (4) shows the number of elements, the maximum aspect ratio and the number of degrees of freedom for each three-dimensional hybrid model.

**Table 4. Three-dimensional hybrid model – number of elements, maximum aspect ratio and degrees of freedom.**

| Mesh | Element type | Number of elements | Maximum aspect ratio | Degrees of freedom |
|---|---|---|---|---|
| Mesh 1 | HTS (7,3) | 22 | 339 | 6582 |
| | HTS (9,3) | | | 8606 |
| | HTS (9,4) | | | 9851 |
| | HTS (10,3) | | | 9662 |
| | HTS (10,4) | | | 10907 |
| Mesh 2 | HTS (7,3) | 44 | 169 | 13164 |
| | HTS (9,3) | | | 17212 |
| | HTS (9,4) | | | 19702 |
| | HTS (10,3) | | | 19324 |
| | HTS (10,4) | | | 21814 |
| Mesh 3 | HTS (7,3) | 66 | 113 | 19746 |
| | HTS (9,3) | | | 25818 |
| | HTS (9,4) | | | 29553 |
| | HTS (10,3) | | | 28986 |
| Mesh 4 | HTS (9,3) | 88 | 85 | 34424 |
| | HTS (10,3) | | | 38648 |
| | HTS (10,4) | | | 43628 |

### 2.4. Boundary condition and loading

As boundary condition, it was chosen the fixed condition, in other words, the restriction of the six degrees of freedom in the faces at y = 200 mm.

The loading was implemented by loads applied in the faces, i.e. applied stress (force per unit area). The kinds of loading were: torsion, compression and shear, all of them applied in the skin

The Fig. (3) shows these boundary and loading conditions.

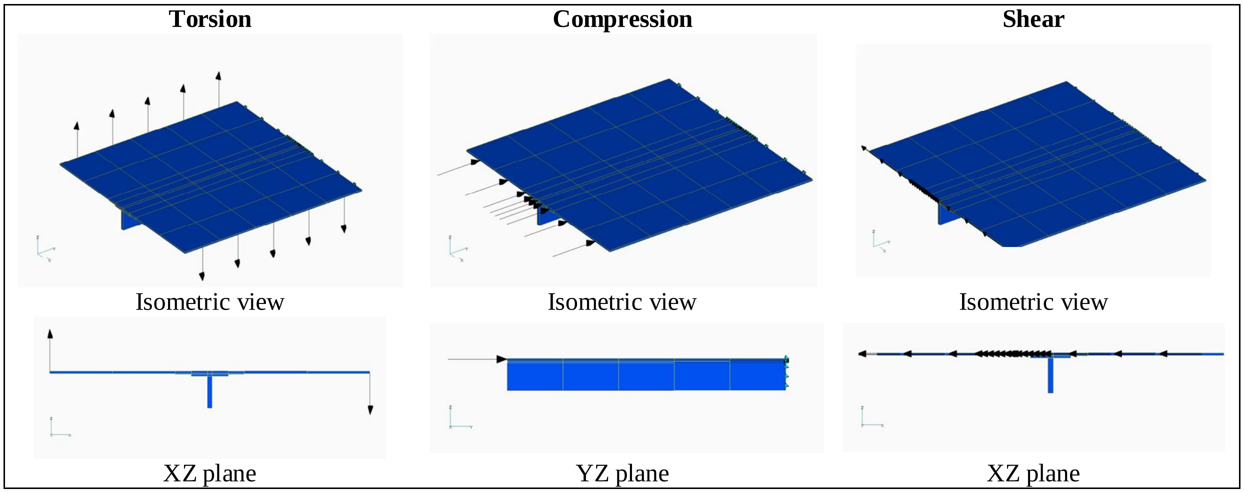



**Figure 3. Load cases.**

## 3. RESULTS

This section presents the results for the stresses and displacements using the Nastran model and the three-dimensional hybrid quasi-Trefftz model.

First, the convergence of the shear stress in xy plane (Sxy) was analyzed for the Nastran model. It was used a curve with the mesh refinement is in the x axis and the Sxy stresses is in the y axis.

Then, the Nastran model was taken as a reference and the results of the three-dimensional hybrid model was analyzed with curves where each line is related to the approach functions degrees. The refinement is in the x axis and the ratio S/Sref is in the y axis, where S is the result for Sxy obtained by the three-dimensional hybrid model, and Sref is the result obtained by the Nastran model.

From Fig. (4) to Fig. (6), it was presented the convergence curves for the Nastran model in each loading.

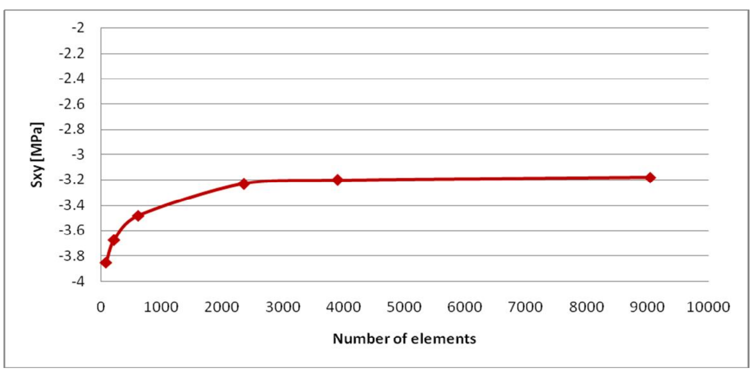



**Figure 4. Convergence of Sxy stress for Nastran model – Torsion.**

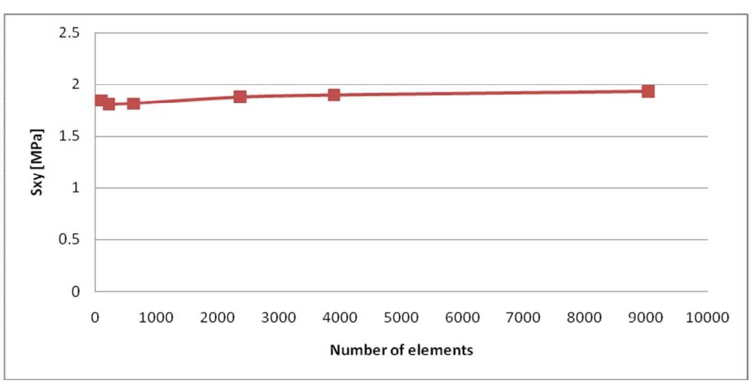


**Figure 5. Convergence of Sxy stress for Nastran model – Compression.**

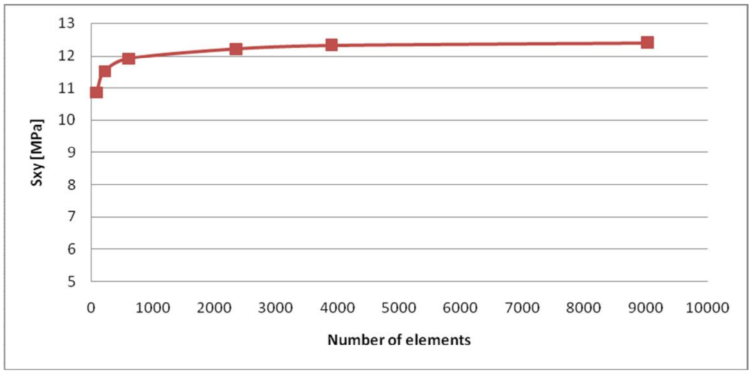



**Figure 6. Convergence of Sxy stress for Nastran model – Shear.**

The evaluation of the presented curves shows that for the three loading cases, the Nastran model presented convergence from the fourth point, which corresponds to mesh 4, composed by 2380 elements with maximum aspect ratio of 8,5.

The curves presented from Fig (9) to Fig (11) present the analyses of the results for the three-dimensional hybrid model by comparing with the Nastran model as already described.

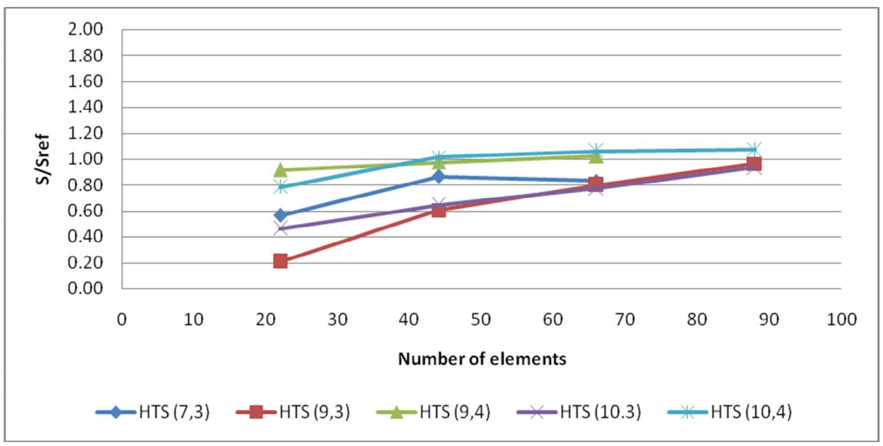



**Figure 7. Convergence of Sxy stress for Hybrid three-dimensional model – Torsion.**

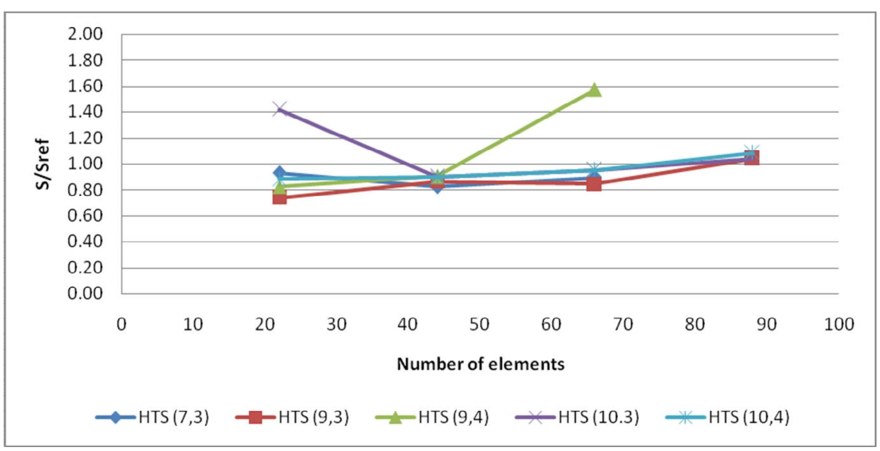



**Figure 8. Convergence of Sxy stress for Hybrid three-dimensional model – Compression.**

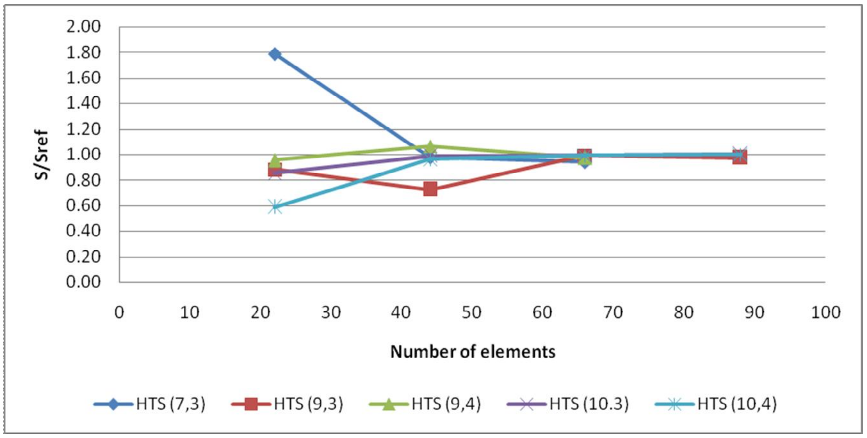



**Figure 9. Convergence of Sxy stress for Hybrid three-dimensional model – Shear.**

The evaluation of the curves shows that the results of tested model are very close to the Nastran model results from fourth point, which corresponds to the mesh 4, composed by 88 elements with maximum aspect ratio of 85. These results were obtained for the elements HTS (9.3), HTS (10.3) and HTS (10.4).

The Tab. (5) shows the values of Sxy stress for each developed model. The Tab. (6) shows the values of maximum displacement for the Nastran model with 2360 elements and the three-dimensional hybrid model with 88 elements HTS (10.4).

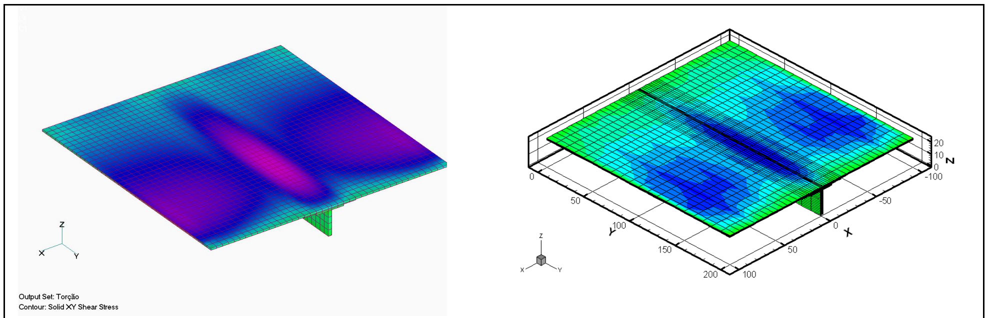
**Table 5. Results of shear stress Sxy.**

| Model | Number of elements | Torsion - Sxy Point(1,5;160;23,24) [MPa] | Compresssion - Sxy Point(21,5;160;21,77) [MPa] | Shear - Sxy Point(21,5;160;21,77) [MPa] |
|---|---|---|---|---|
| Nastran | 2360 | -3.23 | 1,93 | 12,20 |
| Three-dimensional hybrid HTS (7,3) | 22 | -1.79 | 1,75 | 21,85 |
| | 44 | -2.75 | 1,55 | 11,95 |
| | 66 | -2.65 | 1,68 | 11,48 |
| Three-dimensional hybrid HTS (9,3) | 22 | -0.66 | 1,39 | 10,79 |
| | 44 | -1.93 | 1,62 | 8,88 |
| | 66 | -2.54 | 1,59 | 12,12 |
| | 88 | -3.06 | 1,95 | 11,88 |
| Three-dimensional hybrid HTS (9,4) | 22 | -2.91 | 1,55 | 11,65 |
| | 44 | -3.09 | 1,70 | 13,02 |
| | 66 | -3.25 | 2,96 | 11,91 |
| Three-dimensional hybrid HTS (10,3) | 22 | -1.47 | 2,66 | 10,43 |
| | 44 | -2.06 | 1,70 | 12,08 |
| | 66 | -2.46 | 1,79 | 12,13 |
| | 88 | -2.98 | 1,95 | 12,24 |
| Three-dimensional hybrid HTS (10,4) | 22 | -2.49 | 1,66 | 7,15 |
| | 44 | -3.23 | 1,69 | 11,75 |
| | 66 | -3.38 | 1,79 | 12,13 |
| | 88 | -3.41 | 2,04 | 12,21 |

**Table 6. Results of maximum displacements.**

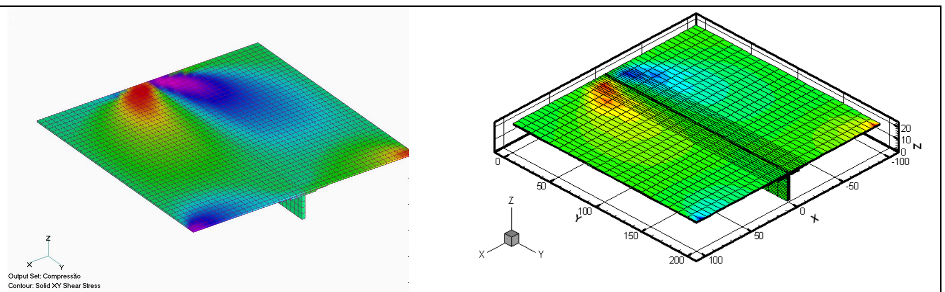
| Model | Number of elements | Torsion – Maximum displacement [mm] | Compression – Maximum displacement [mm] | Shear – Maximum displacement [mm] |
|---|---|---|---|---|
| Nastran | 2360 | 14,86 | 2,13 | 15,75 |
| Three-dimensional hybrid HTS (10, 4) | 88 | 15,88 | 2,11 | 15,20 |

From Fig. (10) to Fig. (12), it is presented the comparison of the shear stress Sxy distributions for Nastran model and the three-dimensional hybrid model, using the refinement in which the convergence was obtained.



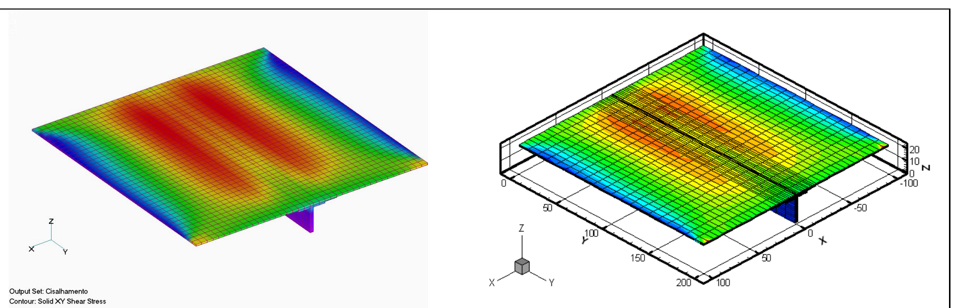
Nastran Model Three-dimensional hybrid model

**Figure 10. Comparison of shear stress Sxy distribution – torsion.**



Nastran Model Three-dimensional hybrid model

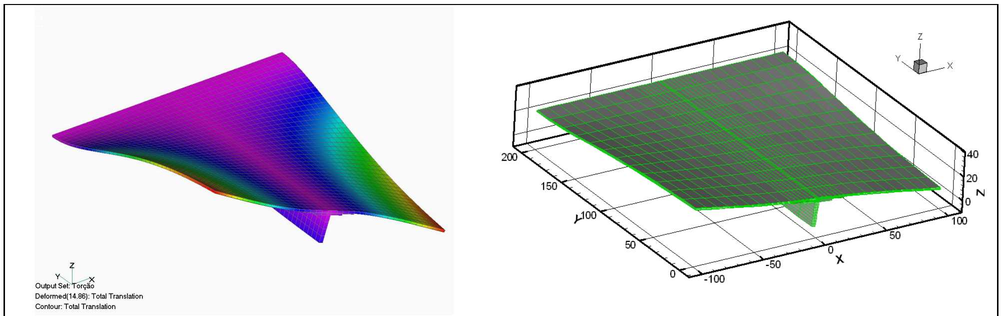
**Figure 11. Comparison of shear stress Sxy distribution – compression.**



Nastran Model Three-dimensional hybrid model

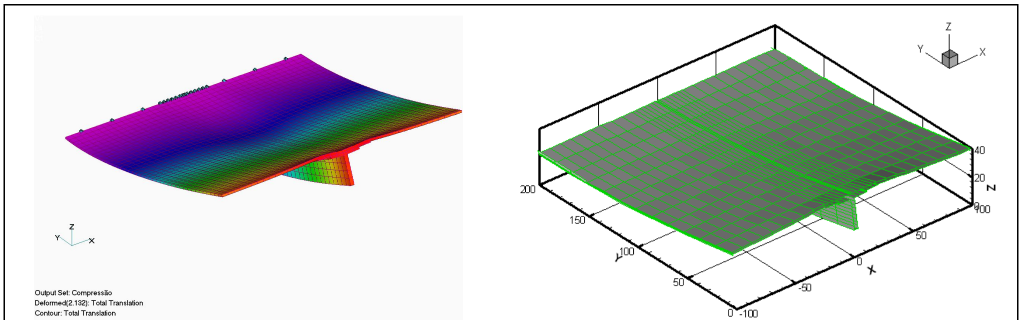
**Figure 12. Comparison of shear stress Sxy distribution – shear.**

From Fig (13) to Fig (15), it is presented the comparison of the displacements obtained for the Nastran model and the three-dimensional hybrid model, using the refinement in which the convergence was obtained.



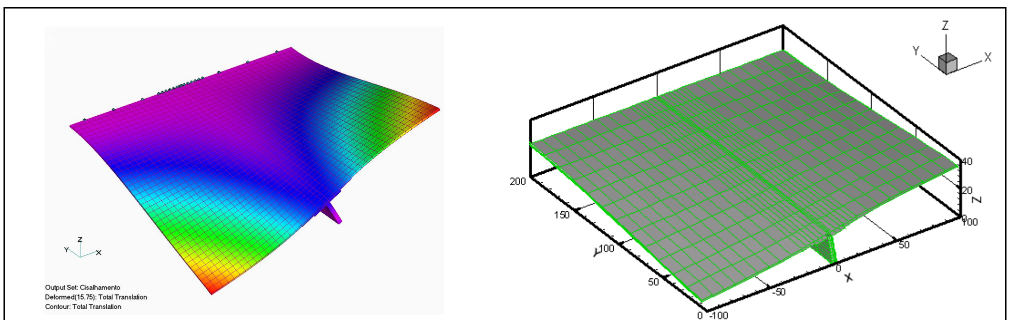

Nastran Model Three-dimensional hybrid model - Scale 1:1

**Figure 13. Comparison of displacements - torsion.**




Nastran Model Three-dimensional hybrid model - Scale 10:1

**Figure 14. Comparison of displacements - compression.**




Nastran Model Three-dimensional hybrid model - Scale 1:1

**Figure 15. Comparison of displacements - shear.**

## 4. CONCLUSION

The results, in terms of convergence and comparison between models, were similar for the three loadings. The Nastran model converges in the mesh with 2360 elements. For the three-dimensional hybrid quasi-Trefftz model, the results were very similar to Nastran from the mesh with 88 elements and using the elements HTS (9.3), HTS (10.3) and HTS (10.4). These results confirm the fact that the hybrid three-dimensional finite element presents a much faster convergence in terms of mesh when compared to other conventional element. The mesh has presented about 30 times less refined than the Nastran model.

Regarding the number of degrees of freedom, the quasi-Trefftz model, in the mesh with 88 elements, presented 34424 degrees for HTS (9.3), 38648 degrees for HTS (10.3) and 43628 degrees for HTS (10.4), while the conventional element in Nastran model presented 12960 degrees for the mesh with 2360 elements. Although the results indicate a smaller number of degrees of freedom for the Nastran model, the three-dimensional hybrid model takes advantage of its large sparse (about 99%) to solve the problem.

Displacements were also obtained for the two models as a means of verifying the reliability of the quasi-Trefftz model, and the results presented were very close in both models.

Thus, the conclusion is that three-dimensional hybrid finite element is able to produce good results for elastic analysis of structures typically of aircrafts made of composite material. The element, by using functions with high degrees, has a good performance in relation to the stresses and displacements, once the results showed good accuracy even when using relatively coarse meshes. At this configuration, the coarse meshes generated three-dimensional elements with high aspect ratio. The maximum aspect ratio was between 85 and 339 for quasi-Trefftz model, while for the conventional three-dimensional elements in the Nastran model, the maximum were between 5.1 and 68, with the convergence happening with a maximum of 8.5.

Then, the use of the three-dimensional hybrid finite quasi-Trefftz element make easier the creation of meshes in more complexes geometries or in those with singularities, because it requires less refinement of the mesh, but still obtaining a detailed distribution of stresses and displacements.

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