VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

AERONAUTICAL COMPOSITE PANEL MULTICRITERIA OPTIMIZATION USING COMPRESSIVE LOADS AND LAMINATION PARAMETERS

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Abstract*. This work presents the buckling and fundamental frequency simultaneous optimization of an aeronautical composite plate. The Powell's method is used aiming at getting the best design for the applied compressive loading. The design variables are the lamina orientation angles. However, buckling and fundamental frequency are not convex functions of these variables. Therefore, it is necessary to include lamination parameters in the optimization formulation to obtain a convex problem. The proposed procedure guarantees that the global maximum of the objective function is obtained.*

Keywords*: multicriteria optimization, buckling, fundamental frequency, lamination parameters.*

1. INTRODUCTION

The interest in laminated composite materials by aeronautical industry is due to their high modulus of elasticity and low mass density. Since these type of materials are composed of a certain number of laminas with arbitrary angular orientation, the optimization strategies for composite structures generally use as design variables the lamina thicknesses and/or the lamina angular orientations. Furthermore, a common optimization goal for aeronautical structures is to maximize independently either the buckling load (Faria, 2002) or the fundamental frequency (Faria and Almeida, 2006; Topal, 2009). The buckling and fundamental frequency simultaneous optimization makes possible to design robust structures statically and dynamically. Among the techniques used for multicriteria optimization are the minimax strategy (Dem'Yanov and Malozemov, 1974) and Pareto (Grierson, 2008).

The structure chosen for optimization in this work is a composite plate under edge compressive loads as represented in Fig. 1. In this type of structures, buckling and natural frequencies are closely correlated. That is, if the buckling load is optimized the fundamental frequency also gets better. In this special situation, Powell's method (Vanderplaats, 1984) can be used for the multicriteria optimization and by the end of the process it will be possible define which is the dominant criterion.

The design variables in this work are the lamina angular orientations. Since buckling and fundamental frequency are not convex functions of the laminas angular orientations, the present work uses a general approach developed by Foldager et al (1998) that makes possible to find the global optimum. Section 2 presents the buckling and fundamental frequency analytical solution for composite plates.

The optimization strategy includes Powell's method and Foldager et al (1998) approach. The complete optimization process is described in Section 3.

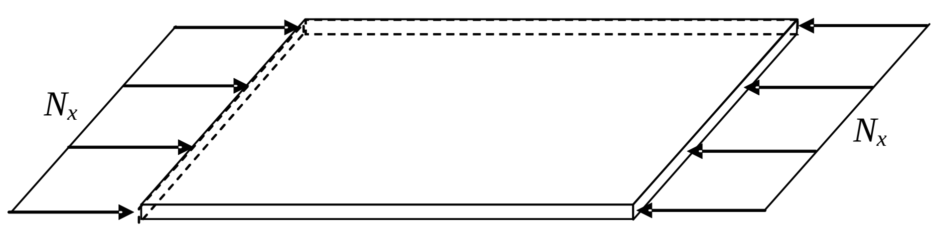

Figure 1 – Plate under compressive loads.

2. BUCKLING AND FUNDAMENTAL FREQUENCY ANALYTICAL SOLUTION

The buckling and fundamental frequency analytical solutions presented in Jones (1975) are given in Eq. (1) and (2), respectively. The fundamental frequency solution considering stress stiffness is given in Eq. (3). It was derived from Jones (1975):

$$\lambda = \pi^2 \left[D_{11} \left(\frac{m}{a}\right)^2 + 2\left(D_{12} + 2D_{66}\right)\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4 \left(\frac{a}{m}\right)^2 \right] \tag{1}$$

$$\varpi^2 = \frac{\pi^4}{\rho h}\left[D_{11}\left(\frac{m}{a}\right)^4 + 2\left(D_{12} + 2D_{66}\right)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4 \right] \tag{2}$$

$$\varpi^2 = \frac{\pi^4}{\rho h}\left[D_{11}\left(\frac{m}{a}\right)^4 + 2\left(D_{12} + 2D_{66}\right)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4 \right] - \frac{\lambda}{\rho h}\left(\frac{m\pi}{a}\right)^2 \tag{3}$$

where λ is the buckling load and ω is the fundamental frequency; a and b are the plate horizontal and vertical dimension; m and n are the number of buckle half wavelengths in the x- and y-directions, respectively. The smallest value of λ occurs when $n = 1$ and for small plate aspect ratios ($a/b < 2.5$) the plate buckles into a single half-wave in the x-direction. In this way it is considered that $m = n = 1$. D_{11}, D_{22}, D_{12}, D_{66} are the laminate stiffness.

The stiffness elements of matrices [A], [B] and [D] can be given in terms of the angular orientation in Eq. (4) or in terms of lamination parameters in Eq. (5). In the first case the stiffness matrix in structural coordinates $\left[\bar{Q}\right]$ is used and in the second it is used the matrix of invariants [U] and lamination parameters $\xi^A_{[1,2,3,4]}$, $\xi^B_{[1,2,3,4]}$ and $\xi^D_{[1,2,3,4]}$.

$$\begin{aligned} [A] &= \sum_{k=1}^{n}(z_k - z_{k-1})\left[\bar{Q}\right]_k \\ [B] &= \sum_{k=1}^{n}\left(\frac{z_k^2 - z_{k-1}^2}{2}\right)\left[\bar{Q}\right]_k \\ [D] &= \sum_{k=1}^{n}\left(\frac{z_k^3 - z_{k-1}^3}{3}\right)\left[\bar{Q}\right]_k \end{aligned} \tag{4}$$

where z_k is the lamina thickness, and:

$$\begin{aligned} A_{ij} &= t[U]\left\{1 \xi_1^A \xi_2^A \xi_3^A \xi_4^A\right\}^T, \quad i,j = 1,2,6 \\ B_{ij} &= t^2[U]\left\{1 \xi_1^B \xi_2^B \xi_3^B \xi_4^B\right\}^T, \quad i,j = 1,2,6 \\ D_{ij} &= \frac{t^3}{12}[U]\left\{1 \xi_1^D \xi_2^D \xi_3^D \xi_4^D\right\}^T, \quad i,j = 1,2,6 \end{aligned} \tag{5}$$

where t is the plate thickness.

$$\begin{aligned} \xi^A_{[1,2,3,4]} &= \frac{1}{t}\sum_{K=1}^{N}(z_k - z_{k+1})[\cos 2\theta_k \quad \sin 2\theta_k \quad \cos 4\theta_k \quad \sin 4\theta_k] \\ \xi^B_{[1,2,3,4]} &= \frac{2}{t^2}\sum_{K=1}^{N}(z_k^2 - z_{k+1}^2)[\cos 2\theta_k \quad \sin 2\theta_k \quad \cos 4\theta_k \quad \sin 4\theta_k] \\ \xi^D_{[1,2,3,4]} &= \frac{4}{t^3}\sum_{K=1}^{N}(z_k^3 - z_{k+1}^3)[\cos 2\theta_k \quad \sin 2\theta_k \quad \cos 4\theta_k \quad \sin 4\theta_k] \end{aligned} \tag{6}$$

3. OPTIMIZATION PROCESS

The optimization process includes the Powell's method and the Foldager et al (1998) approach. The Powell's method is used to find the lamina angular orientation that yields maximum buckling load and fundamental frequency. However, since buckling and fundamental frequency are not convex functions of the angular orientation, the maximum value found can be local. In order to assure that the obtained maximum is global, an optimization process that includes lamination parameters and an objective function proposed by Foldager et al (1998) is performed. This approach is based on the assumption that the optimization problem where the laminate stiffness matrices [*A*], [*B*], and [*D*] are expressed in terms of lamination parameters is convex (Foldager et al, 1998). The optimization process can be summarized in the following steps:

1) Define some initial design variables $\{\theta\}_{initial}$. In this work the design variables are the laminas angular orientations.

2) Using Powell's method, find a $\{\theta\}_{optm}$ set that maximizes the objective functions:

$$\max_{\{\theta\}} \begin{Bmatrix} \omega(\{\theta\}) \\ \lambda(\{\theta\}) \end{Bmatrix} = \max_{\{\theta\}} \{F\{\theta\}\} \tag{7}$$

In this step it is important to explain that the buckling and fundamental frequency values should be normalized so that a proper comparison can be done. In this work the buckling load is normalized by the applied load and the fundamental frequency is normalized by some minimum frequency design requirements. When the buckling normalized value is less than 1, it means that the applied load is greater than the buckling load. If this happens the structure lost its stability and the linearized fundamental frequency tends to 0.0 *Hz*.

3) Using the $\{\theta\}_{optm}$ in previous step and Eq. (6) compute the equivalent lamination parameters:

$$\{\theta\}_{optm} \rightarrow \{\xi\}^* \tag{8}$$

4) Using the Powell's method, minimize the Foldager et al (1998) internal objective function:

$$\min_{\{\theta\}} f\left(\{\xi\}^*, \xi(\{\theta\}), \frac{\partial F}{\partial\{\xi\}^*}\right) \tag{9}$$

$$f = 4 f_1\left(\{\xi\}^*, \xi(\{\theta\})\right) + f_2\left(\{\xi\}^*, \xi(\{\theta\}), \frac{\partial F}{\partial\{\xi\}^*}\right) \tag{10}$$

This internal objective function considers the lamination parameters corresponding to $\{\theta\}_{optm}$, that are defined by $\{\xi\}^*$, the lamination parameters corresponding to the internal Powell's iteration $\xi(\{\theta\})$, and the lamination parameters sensitivities $\partial F/\partial\{\xi\}^*$. By the end of this internal optimization process, it is found a $\{\theta\}_{new}$ set that minimizes the internal objective function. The equations of Foldager et al (1998) approach necessary to define the function *f* in Eq. (10) are repeated in this work in Eq. (11) to (15).

$$f_1 = \sum_{i=1}^{NLP} \left\{1 - \frac{\xi_i(\theta)}{\xi_i^*}\right\}^2 \tag{11}$$

$$f_2 = \sum_{i=1}^{NLP} \begin{Bmatrix} a_{11}\xi_i(\theta) + b_{11} | \xi_i(\theta) \le \xi_i^* \\ a_{12}\xi_i(\theta) + b_{12} | \xi_i(\theta) > \xi_i^* \end{Bmatrix} | \frac{\partial OF}{\partial \xi_i^*} \le 0 \\ \begin{Bmatrix} a_{21}\xi_i(\theta) + b_{21} | \xi_i(\theta) \le \xi_i^* \\ a_{22}\xi_i(\theta) + b_{22} | \xi_i(\theta) > \xi_i^* \end{Bmatrix} | \frac{\partial OF}{\partial \xi_i^*} > 0 \tag{12}$$

where NLP is the number of lamination parameters.

$$a_{11} = \frac{0.8h_i}{\xi_i^* - 0.25h_i\left(1-\xi_i^*\right)+1}$$
$$a_{12} = \frac{0.05h_i}{\xi_i^* - 0.25h_i\left(1-\xi_i^*\right)-1}$$
$$a_{21} = \frac{-0.05h_i}{\xi_i^* - 0.25h_i\left(1-\xi_i^*\right)+1} \tag{13}$$
$$a_{22} = \frac{-0.8h_i}{\xi_i^* - 0.25h_i\left(1-\xi_i^*\right)-1}$$

$$\begin{aligned} b_{11} &= -h_i + a_{11} \\ b_{12} &= -0.2h_i - a_{12} \\ b_{21} &= 0.2h_i + a_{21} \\ b_{22} &= h_i - a_{22} \end{aligned} \tag{14}$$

$$h_i = \frac{\partial F / \partial \xi_i^*}{\left|\partial F / \partial \xi^*\right|_{\max}} \tag{15}$$

The minimization of function f_1 implies minimization of the distance between the given set of lamination parameters $\{\xi\}^*$ and those $\xi(\{\theta\})$ calculated from the design variables $\{\theta\}$. The second function, f_2, forces the minimization process to favor lay-ups with a lower value of the objective function f. The internal objective minimization goal is to find a new set of design variables ($\{\theta\}_{new}$) that yields a better value of the objective function. This tends to take the optimization process away from a local maximum.

The sensitivity computation is an important stage on the optimization process and in this work it is done by chain rule combined with finite difference:

$$\frac{\partial F}{\partial\{\xi\}^*} = \frac{\partial F}{\partial\{\theta\}}\frac{\partial\{\theta\}}{\partial\{\xi\}^*} \tag{16}$$

5) Using the resulting $\{\theta\}_{new}$, compute $F(\{\theta\}_{new})$. If $F(\{\theta\}_{new}) > F(\{\theta\}_{optm})$, the process should return to step 1 with $\{\theta\}_{initial} = \{\theta\}_{new}$.

The process is repeated until a $F(\{\theta\}_{new}) > F(\{\theta\}_{optm})$ is not found. When this happens $\{\theta\}_{optm}$ is taken as the global optimum.

4. NUMERICAL RESULTS

The structure chosen for the optimization is the plate represented in Fig. 1. This structure and a similar optimization process was used in a previous work (Lariú, 2008), but only for buckling optimization. It is a crossply laminated rectangular plate with initial stacking sequence of $[(0/45/90)]_s$. The horizontal edge of the plate has dimension a = 0.3 m and vertical dimension b = 0.24 m. The lamina thicknesses are considered constant and equal to 2.54 mm. The material properties are given in Table 1.

Table 1 – Material properties.

Property	Value
Longitudinal modulus of elasticity, E_1 (GPa)	207
Transverse modulus of elasticity, E_2 (GPa)	20.7
In-plane and transverse shear modulus, G_{12} and G_{13} (GPa)	6.9
Transverse shear modulus, G_{23} (GPa)	6.9
In-plane Poisson's ratio, ν_{12}	0.3
Density, ρ (kg/m^3)	1580

The optimization process considers three different magnitudes of applied compressive loads and three different values of minimum frequencies design requirements. These values are also used as normalization values (ω_0) for the frequencies. The combination of the three load magnitudes with the three frequencies requirements results in nine different optimization analyses. The load magnitudes and minimum frequency requirements are given in Table 2:

Table 2 – Load magnitude and minimum frequency value requirement.

N_x (kN/m)	1200	7200	14400
	50	50	50
ω_0 (Hz)	100	100	100
	150	150	150

The optimization results are given in Tables 3, 4 and 5.

Table 3 – Optimization results for N_x = 1200 (kN/m).

	ω_0 = 50 Hz		ω_0 = 100 Hz		ω_0 = 150 Hz	
	Initial	Optimum	Initial	Optimum	Initial	Optimum
$[\theta_1/\theta_2/\theta_3]_s$	$[0/45/90]_s$	$[51.88/51.79/51.87]_s$	$[0/45/90]_s$	$[51.45/51.50/50.35]_s$	$[0/45/90]_s$	$[51.39/51.86/45.67]_s$
λ/N_x	11.88	20.37	11.88	20.37	11.88	20.35
ω/ω_0	24.54	32.75	12.27	16.37	8.18	10.91

Examining Table 3 it is possible to see that when N_x = 1200 kN/m and ω_0 = 50 Hz the dominant criterion is buckling. When ω_0 = 100 Hz the dominant criterion is buckling at the beginning and frequency by the end of the optimization process. When ω_0 = 150 *Hz* the dominant criterion is frequency. For ω_0 = 50 and 100 Hz the optimal point is practically the same. However, when ω_0 = 150 Hz, θ_3 orientation is different from the other minimum frequency requirement values. Furthermore, buckling was slightly decreased by optimization procedure when frequency is the dominant criterion.

Table 4 – Optimization results for N_x = 7200 (kN/m).

	ω_0 = 50 Hz		ω_0 = 100 Hz		ω_0 = 150 Hz	
	Initial	Optimum	Initial	Optimum	Initial	Optimum
$[\theta_1/\theta_2/\theta_3]_s$	$[0/45/90]_s$	$[51.90/51.92/51.78]_s$	$[0/45/90]_s$	$[51.90/51.92/51.78]_s$	$[0/45/90]_s$	$[51.90/51.92/51.78]_s$
λ/N_x	1.98	3.39	1.98	3.39	1.98	3.39
ω/ω_0	18.04	28.21	9.02	14.10	6.01	9.40

Examining Table 4 it is possible to see that when N_x = 7200 kN/m and ω_0 = 50, 100 and 150 Hz the dominant criterion is buckling and the optimal angular orientation is exactly the same. In these cases buckling is strongly dominant and the frequency optimization is just a consequence of the optimization process.

Table 5 – Optimization results for N_x = 14400 (kN/m).

	ω_0 = 50 Hz		ω_0 = 100 Hz		ω_0 = 150 Hz	
	Initial	Optimum	Initial	Optimum	Initial	Optimum
$[\theta_1/\theta_2/\theta_3]_s$	$[0/45/90]_s$	$[53.27/50.37/46.52]_s$	$[0/45/90]_s$	$[53.27/50.37/46.52]_s$	$[0/45/90]_s$	$[53.24/50.37/46.49]_s$
λ/λ_0	0.99	1.69	0.99	1.69	0.99	1.69
ω/ω_0	0	21.49	0	10.75	0	7.16

Examining Table 5 it is possible to see that the applied load (N_x = 14400 kN/m) results in a buckled initial structure. However, the optimal structure supports loading 69% greater than the one applied. When ω_0 = 50 and 100 Hz the optimal structure is the same. When ω_0 = 150 Hz the optimal angular orientation is slightly different. This indicates that the extreme condition of loading and high minimum frequency value requirement impose some difficulties for the optimization process that did not found the exact optimal point but a very close one.

5. CONCLUSION

The present work shows the buckling and fundamental frequency optimization of a composite plate under compressive loads. In all cases presented the optimization process has succeed, even when the initial structure was buckled. The optimization process is simplified by the fact that, for this structure under this type of loading, buckling and fundamental frequency have similar behaviors. It means that if the buckling load is improved, the fundamental frequency also is. However, it should be emphasized that this is applicable only for this particular case. In future works more realistic loading representation will be included. Also, standard techniques for multicriteria optimization such as minimax strategy and geometrically more complex structures shall also be analyzed.

6. ACKNOWLEDGEMENTS

This work is financed by the Brazilian Agency Fapesp (Grant. No. 06/60929-0).

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