Estimative of the Boiling Zone in Liquid Fuel Combustion inside Porous Media using a Flamelet Analysis

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Abstract.*Combustion inside porous media has several advantages on non confined combustion, ranging from extended flammability limits to low pollutant emissions. Most of the analysis on in-situ combustion treat the gaseous premixed combustion. In this analysis, we study the case of diffusion flames confined in porous media. To be used in reacting, porous media flow simulation with the fuel in the liquid phase, a flamelet model is developed. The flow configuration is imposed by an oxidizer stream impinging on the fuel liquid surface. The flame is established above the interface liquid-gas, where the mass flux are in stoichiometric proportion. The fuel vaporization is controlled by the heat transferred from the flame to the liquid. The region where vaporization process takes place is thin and is close to the liquid-gas interface. In the present study we consider the interphase heat exchange to be high, and estimate the thickness of the boiling zone. The result is a flamelet model which is a tool to understand and simulate in-situ combustion in many physical conditions, such as the propagation of diffusion flame fronts inside oil well.*

Keywords: *porous media combustion, flamelet, diffusion flames*

1. INTRODUCTION

Combustion inside porous media has been extensively studied in the last decades (Howell *et al.* (1996); Jugjai and Phothiya (2007); Kayal and Chakravarty (2005)) due to it's technological potential. Porous media combustors are less pollutants and more compacts than usual combustors. The main characteristic of combustion inside porous media is the existence of heat recirculation due to the interaction of the high temperature gas phase with the solid matrix (Barra and Ellzey (2004)). Such characteristic provides a way of extending flame stability and possibility of burning lean fuel mixtures (Howell *et al.* (1996)).

The majority of the studies of combustion in porous media concern about premixed flame, few works exist concerning liquid fuel combustion inside porous media (Jugjai and Phothiya (2007); Kayal and Chakravarty (2005)). Nevertheless, such configuration is important in technological development, such as in-situ combustion (heavy oil recovery method) (Akkutlu and Yortsos (2003)), industrial waste combustors, etc.

The problem of heat and mass transfer inside a porous medium using a diffusion flame configurations has been recently analyzed (Raju and T'ien (2007)), but not much attention has been paid to the boiling zone and also has been considered the establishment of the flame outside the porous wick. In the present study we use the flamelet analyzes to obtain an estimative for the thickness of the boiling zone in the liquid fuel region. It's considered a counter-flow configuration of impinging oxidant flow against a liquid fuel pool, both inside a porous inert media.

2. MATHEMATICAL FORMULATION

Since in this work the liquid phase in the porous media combustion problem is analyzed, we focus on the conservation equations concerning such phase. It's assumed a single-component fuel and the case of multi-component fuels will be treated in future works. We consider the existence of the flame in the gas phase region, so that in the present problem the flame is only considered to be a heat source. All variables with a subscript 0 represent conditions at the interface, while subscript $+\infty$ and $-\infty$ correspond to initial oxidant and fuel flow conditions, respectively. Variables or parameters indexed with subscript l refer to liquid phase fuel, indexed with g represents gaseous phase, and subscript s refer to solid phase values.

The governing equations in the liquid phase are:

$$\rho_l \bar{v}_l = \bar{\bar{m}} \tag{1}$$

$$\varepsilon \rho_l \bar{v}_l c_l \frac{\partial T_l}{\partial \bar{z}} = \varepsilon \bar{\lambda}_l \frac{\partial^2 T_l}{\partial \bar{z}^2} + h_l(T_s - T_l) \tag{2}$$

In the portion of the solid phase filled with liquid, the energy conservation equation is

$$0 = (1 - \varepsilon)\bar{\lambda}_s \left(\frac{d^2 T_s}{d\bar{z}^2}\right) - h_l(T_s - T_l) \tag{3}$$

In which ε is the porosity of the medium, c_l is the constant specific heat of the fuel, $\bar{\lambda}$ is the thermal conductivity and h_l is the volumetric surface-convective coefficient which quantifies the heat exchange between the liquid and solid phases.

The boundary conditions are, at the liquid-gas interface, $\bar{z} = 0$:

$$\bar{v} - \bar{v}_0 = T_g - T_0 = Y_F - Y_{F0} = 0,$$

$$\rho\bar{D}_F \frac{\partial Y_F}{\partial\bar{z}}\bigg|_{\bar{z}=0} = -(1 - Y_{F0})\, \rho\bar{v}|_{\bar{z}=0^+}\,,$$

$$\bar{\lambda}_g \frac{\partial T_g}{\partial\bar{z}}\bigg|_{\bar{z}=0^+} = \rho\bar{v}|_{\bar{z}=0^+}\, L + \bar{\lambda}_l \frac{\partial T_l}{\partial\bar{z}}\bigg|_{\bar{z}=-\bar{z}_b} - h_l \int_{-\bar{z}_b}^{0^-} (T_s - T_l)d\bar{z}$$

The second condition above tell us that the fuel flux in the interface is equal to the flux of the amount of fuel that has evaporated. The third condition tell us where the heat delivered to the fuel from the flame goes. The first term in the right side corresponds to the heat used to evaporate the fuel, the second one represents how much of heat is used to pre-heat the fuel (note the appearing of L, the fuel latent heat of vaporization) and the third one is how much heat is used to heat the solid matrix. The energy conservation at the liquid-gas interface leads to the determination of a zone of thickness z_b with constant temperature which corresponds to that of the boiling fuel.

The velocity of the fuel in the gas phase at the liquid surface $\bar{v}_{0+}$ is related with the vaporization rate as

$$\rho.\bar{v}_{0+} = \rho_l.\bar{v}_{l0-} = \bar{\dot{m}} \tag{4}$$

The velocities $\bar{v}_{l0-}$ and $\bar{v}_{0+}$ represent the velocities of the liquid fuel and vapor fuel at the interface.

The boundary conditions in the inlet liquid fuel, $\bar{z} \to -\infty$, are

$$\bar{v}_l - \bar{v}_{l-\infty} = T_l - T_{-\infty} = Y_F - 1 = \rho_l.\bar{v}_{l-\infty} - \bar{\dot{m}} = 0$$

If we use non dimensional variables given by:

$$\varrho_l \equiv \rho_l/\rho_\infty, \quad z \equiv \bar{z}/l_c, \quad \theta_s \equiv T_s/T_\infty, \quad \theta_l \equiv T_l/T_\infty.$$

Where l_c is a characteristic length scale chosen in such way that:

$$l_c \equiv \alpha_s/\bar{v}_\infty = \bar{\lambda}_s/(\rho_\infty c_p \bar{v}_\infty)$$

In which c_p is the gas constant pressure specific heat.

Hence, one obtains the following set of equations for the liquid phase:

$$\varrho_l v_l = \frac{\bar{\dot{m}}}{\rho_\infty \bar{v}_\infty} \equiv \dot{m} \tag{5}$$

$$\varepsilon J \frac{d^2\theta_l}{dz^2} - \varepsilon M \frac{d\theta_l}{dz} = -N_l(\theta_s - \theta_l), \tag{6}$$

$$(1 - \varepsilon)\frac{d^2\theta_s}{dz^2} = N_l(\theta_s - \theta_l) \tag{7}$$

The parameters in Eqs. (6) and (7) are defined as

$$M \equiv \dot{m}(c_l/c_p), \quad N_l \equiv h_l\bar{\lambda}_s/(\rho_\infty\bar{v}_\infty c_p)^2$$

$$J \equiv (\bar{\lambda}_l/\bar{\lambda}_s)$$

The non dimensional boundary conditions at the interface are given by:

$$\theta_g - \theta_0 = \theta_s - \theta_{s0} = y_F - y_{F0} = 0, \quad \frac{1}{\Gamma}\frac{1}{L_F}\frac{dy_F}{d\eta}\bigg|_{\eta=0^+} = (1 - y_{F0})f_0,$$

$$\frac{1}{\Gamma}\frac{d\theta_g}{d\eta}\bigg|_{\eta=0^+} = -lf_0 + \frac{J}{a^{1/2}}\frac{d\theta_l}{dz}\bigg|_{z=-z_b} - \frac{N_l}{a^{1/2}}\int_{-z_b}^{0}(\theta_s - \theta_l)dz, \tag{8}$$

in which $l \equiv L/(c_p T_\infty)$ and $\Gamma \equiv \alpha_s/\alpha_g$, and a is the non-dimensional strain-rate given by $a \equiv (l_c/\bar{v}_\infty)d\bar{u}/d\bar{x}|_\infty$.

For the region above the interface we use a transformed $\bar{z}$ variable of the Howarth-Dorodnitsyn type (Williams (1965)) for a non-expansive gas:

$$\eta = a^{1/2}z$$

In which f is the non dimensional gas phase momentum given by (Krishnamurthy *et al.* (1976); Fachini (2007)):

$$v = -a^{1/2}f$$

3. RESULTS

In the following section, results are presented in the analysis of the equations in the liquid phase problem. It's shown analytical expressions for the two distinct regions: equilibrium and boiling zone.

3.1 Equilibrium region

We are considering that the interphase heat exchange is high, $N_l \gg 1$. So, in Eqs. (6) and (7) one must have:

$$(\theta_s - \theta_l) \approx 0 \tag{9}$$

Only in a small region near the interface (boiling zone) the above equation is not valid and one must consider the full Eqs. (6) and (7).

The region that satisfies Eq. (9) is the solid matrix filled with liquid, except in the boiling zone. In this region, solid and the liquid are in thermal equilibrium. The heat provided by the flame is convected by the solid matrix very quickly (more quickly than the heat conducted by the liquid fuel), but since the interphase heat exchange is high, both phases rapidly equalize temperature, providing us Eq. (9) that is valid for most part of the solid matrix filled with liquid fuel.

3.2 Boiling zone

In this region assumption (9) is not valid and one must use the full form of Eqs. (6) and (7). The thickness of this small region can be estimated using a variable change given by:

$$\tilde{z} = \beta z$$

So, we have:

$$\frac{d}{dz} = \beta\frac{d}{d\tilde{z}}, \quad \frac{d^2}{dz^2} = \beta^2\frac{d^2}{d\tilde{z}^2}$$

Since β is very large, analyzing the magnitude order of Eqs. (6) or (7) with the new variable $\tilde{z}$, one must have:

$$(1-\varepsilon)\beta^2\frac{d^2\theta_s}{d\tilde{z}^2} \sim N_l(\theta_s - \theta_l)$$

And one may estimate the thickness of the boiling zone as:

$$\boxed{\bar{z}_b = l_c\frac{\rho_\infty\bar{v}_\infty c_p}{(\bar{\lambda}_s h_l)^{1/2}}}$$

Remembering the expression for l_c, one may obtain:

$$\bar{z}_b = \left(\frac{\bar{\lambda}_s}{h_l}\right)^{1/2}$$

To show the dependence of the boiling zone thickness with the heat exchange coefficient we utilize a parameter b for $\bar{\lambda}_s^{1/2}$ and plot a graphic for different values of b:
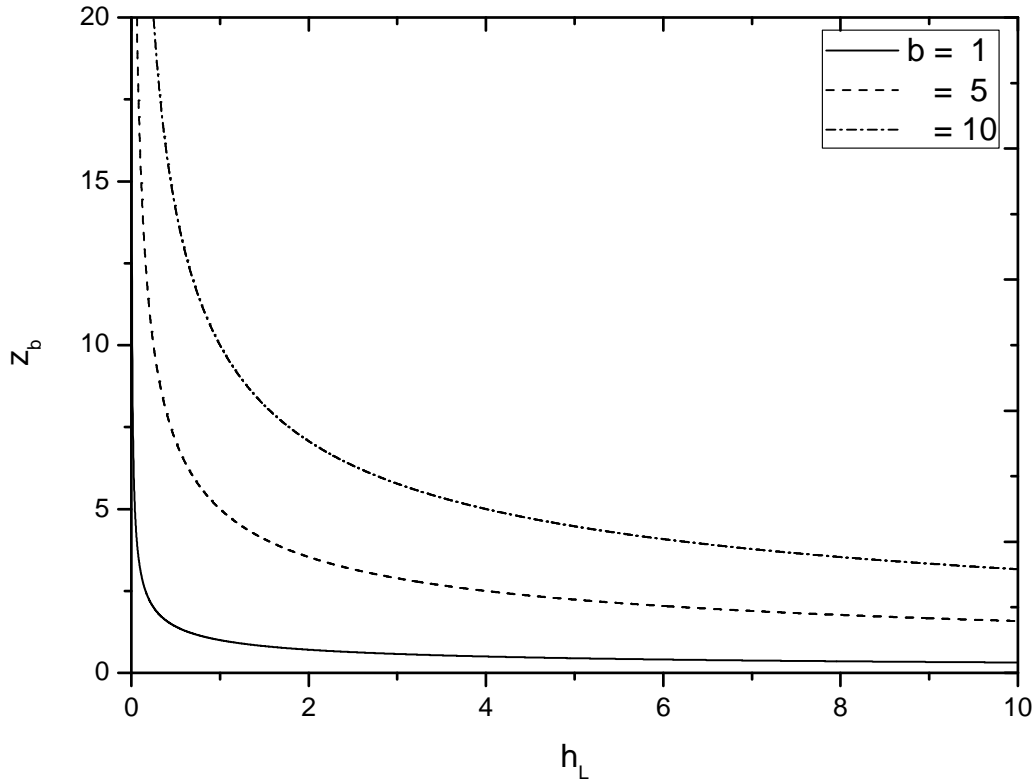
$$\bar{z}_b = b\frac{1}{h_l^{1/2}}$$

Figura 1. **Boiling zone thickness versus heat exchange coefficient.**

Note that the higher the value for the heat exchange parameter, smaller the boiling zone thickness. Both variables above are in a non-dimensional form.

If one wants to analyze this region, one must perform the coordinate transform $\tilde{z} = N_l^{1/2} z$ and the following expansions for the temperatures:

$$\theta_s = \theta_s^{(0)} + N_l^{-1/2}\theta_s^{(1)} + O(N_l^{-1/2})$$

$$\theta_l = \theta_{l0} + N_l^{-1/2}\theta_l^{(1)} + O(N_l^{-1/2})$$

$\theta_s^{(0)}$ and θ_{l0} are, respectively, the solid matrix temperature at the interface and the liquid fuel temperature in this boiling zone (equals to the fuel boiling temperature).

One also must couple the profiles obtained in this zone in the limit $\tilde{z} \to -\infty$ with the profile obtained for the equilibrium region. Not only the profiles must match, but the profiles must obey energy flux conservation between both regions, hence obtaining the full temperature profiles for the liquid phase in such problem.

4. Conclusion

A briefly analysis has been made concerning the liquid phase region in the problem of liquid fuel combustion inside porous inert media. The governing equations and boundary conditions have been presented in their non dimensional forms.

Two regions are recognized in the liquid phase problem. One denoted equilibrium region, where the liquid fuel and solid porous matrix are in thermal equilibrium due to the assumed high value for the interphase heat exchange coefficient, which brings both temperatures very rapidly to equilibrium. The other is called boiling zone. In this thin region the liquid reaches the boiling temperature and all heat received is used to evaporate the fuel to be used to sustain the flame. The solid matrix continues to raise it's temperature since it has no upper limit.

So, in this boiling zone the two temperatures are not in equilibrium anymore. This region is called non equilibrium and it's a very small thickness region near the liquid-gas interface. The determination of suck thickness has been estimated and, as expected, it's considerably small (goes as $N_l^{-1/2}$).

Further studies may obtain temperature profiles for both solid and liquid phases using asymptotic theory.

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