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# A NUMERICAL STUDY ON THE POSTBUCKLING BEHAVIOUR OF SHALLOW SINGLY-CURVED PANELS

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Abstract: A numerical investigation on the postbuckling behaviour of singly-curved panels loaded in compression is presented in this paper. The analysis take into account geometrical and material nonlinearities as well as the geometrical imperfection effects. Firstly a linear buckling analysis was carried out to determine the buckling loads/modes. Subsequently a postbuckling analysis was performed using the dynamic relaxation algorithm. The geometrical imperfections superimposed to the panels initial geometry were defined in terms of the buckling modes obtained from the linear buckling analyses. Results for typical simply-supported (SS) and clamped (CL) aeronautical panels are presented and discussed.

Keywords: Postbuckling, geometric imperfection, nonlinear finite elements

### 1. INTRODUCTION

Flat and curved panel elements constitute a major portion of the structure of aerospace vehicles. They are found in the aircraft components as primary load carrying structures (such as wing surfaces), horizontal and vertical stabilizers, and fuselage sections as well as in spacecraft and missile structural applications. With the advent of the new joining techniques such as Friction Stir Welding (FSW) and their incorporation in the flight vehicle structures there is an imperative need for a better understanding of their behavior. To this end, more adequate methods of analysis, taking into account their specific features, have to be developed and implemented. One of the problems deserving special attention is the study of the postbuckling of flat/curved panels subjected to compressive edge loads. This is due to the well-known fact that the panels used in components of flight vehicle structures exhibit a considerable load bearing capacity in the postbuckling range. An essential requirement towards full employment of this strength lies in a better understanding of their postbuckling behavior. These structures exhibit unavoidable initial geometric imperfections. Their presence may result in big differences and sometimes even in drastic changes in their postbuckling behavior compared with their perfect counterparts.

The present paper addresses the problem of the postbuckling behavior of singly-curved panels. The analyses take into account geometrical and material nonlinearities as well as the geometrical imperfection effects. Firstly a linear buckling analysis was carried out to determine the buckling loads/modes. Subsequently a postbuckling analysis was performed using the dynamic relaxation algorithm. The geometrical imperfections superimposed to the panels initial geometry were defined in terms of the buckling modes obtained from the linear buckling analyses. Results for typical simply-supported (SS) and clamped (CL) aeronautical panels are presented and discussed.

# 2. STUDY CASES

The dimensions of the panels investigated in this work are representative of an aircraft fuselage sub-panel and they are indicated in Fig. 1 (a). The panel is 1.1 mm thick, 415 mm long with a arc length and curvature radius of 164 mm and 1100 mm, respectively. The material properties of the panel together with the Ramberg-Osgood material model parameters are listed in Table 1. The analyses were performed for Simply-Supported (SS) and Clamped (CL) panels loaded in compression in the longitudinal direction. The panel was discretized using S4R shell elements available in ABAQUS [1]. The finite element model of the panel including boundary conditions and loads is depicted in Fig. 1 (b).

E (GPa)	n	$F_{cv}$ (MPa)	F <sub>07</sub> (MPa)	Ve
73.80	12.00	255.00	246.00	0.33

#### Table 1. Mechanical properties and material model parameters.



Figure 1. (a): Panel dimensions, (b) Finite element model.

# 3. LINEAR BUCKLING ANALYSIS

The linear buckling analyses were carried out using single-integration point bilinear shell elements available in ABAQUS FE code. The buckling loads and buckling modes were obtained from the standard eigenvalue problem given by Eq. (1). The determination of the eigenvalues and eigenvectors was numerically performed using the iteration by subspace algorithm available in ABAQUS (ABAQUS, 2005).

$$\left\{ \begin{bmatrix} K \end{bmatrix} - \lambda \begin{bmatrix} K_G \end{bmatrix} \right\} \left\{ \phi \right\} = \left\{ 0 \right\} \tag{1}$$

## 4. NONLINEAR POSTBUCKLING ANALYSIS

The postbuckling analyses were carried out under displacement control using the dynamic relaxation method. The method consists of loading the structure quasi-statically in a way that dynamic effects are minimized. By using this method the general expression for the residual force vector is defined in terms of displacements, velocities and nodal accelerations as follows,

$$\left\{g(d, \dot{d}, \ddot{d}, t)\right\} = \left\{F_i(d, \dot{d}, \ddot{d}, t)\right\} - \left\{F_e(t)\right\}$$
(2)

where  $\{g(d, \dot{d}, \ddot{d}, t)\}$  is the residual force vector,  $\{F_i(d, \dot{d}, \ddot{d}, t)\}$  is the nonlinear internal force vector which includes structural damping and inertia effects and  $\{F_e(t)\}$  is the external load vector. To minimize dynamic oscillations a proportional Rayleigh damping has been defined as a linear combination between tangent stiffness and mass matrices,

$$[C] = \alpha[M] + \beta[K_t]$$
<sup>(3)</sup>

where the proportionality factors  $\alpha$  and  $\beta$  are obtained by solving the linear system of equations shown below,

$$\begin{cases} \alpha \\ \beta \end{cases} = 2 \begin{bmatrix} 1 & \omega_{high}^2 \\ 1 & \omega_{low}^2 \end{bmatrix}^{-1} \begin{cases} \xi_{high} \omega_{high}^2 \\ \xi_{low} \omega_{low}^2 \end{cases}$$
 (4)

where  $[K_i]$ , [M] and [C] are the tangent stiffness matrix, mass matrix and damping matrix, respectively.  $\omega_{high}$  and  $\omega_{low}$  are the highest and lowest frequencies defining the frequency range to be damped.  $\xi_{high}$  and  $\xi_{low}$  are the damping factors associated with the highest and lowest frequencies within the frequency range to be damped, respectively.

The postbuckling analyses involve introducing the imperfection into the structure. A single mode or a combination of modes may be used to construct the imperfection. In this work the geometric imperfection has been defined in terms of the first buckling mode only. To compare the results obtained with different imperfections, the imperfection size must be fixed. The measure of the imperfection size used in this problem is the out-of-roundness of the shell, which is computed as the radial distance from the axis of the cylinder to the perturbed node minus the radius of the perfect structure. The scale factor associated with the eigenmode used to seed the imperfection is computed with a *FORTRAN* program. The program reads the results file produced by the linear analysis and determines the scale factors so that the out-of-roundness of the shell structure is equal to a specified value. This value is taken as a fraction of the shell thickness.

#### 5. NUMERICAL RESULTS

#### 5.1. Linear buckling analyses

The results in terms of buckling stress for the simply-supported and clamped panels are summarized in Tables 1 and 2 respectively. These tables also show a comparison between numerical predictions and closed form solution for the first buckling load only.

Buckling mode	Buckling stress (MPa)	Analytical solution (Rizzi, 2009)
1	26.00	25.10
2	30.70	
3	34.90	
4	38.85	

Table 2. Comparison between numerical predictions and analytical solutions for the simply-supported panel

Table 3. Comparison between numeric	al predictions and analytic	cal solutions for the c	lamped panel
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Buckling mode	Buckling stress (MPa)	Analytical solution (Rizzi, 2009)
1	35.00	32.00
2	44.90	
3	45.25	
4	46.82	

The buckling modes for the simply-supported and clamped panels are illustrated in Figs. 2 and 3, respectively.



Figure 2. Buckling modes for the simply-supported panel



Figure 3. Buckling modes for the clamped panel

#### 5.2. Postbuckling analyses

Figures 4 and 5 show the effects of the geometric imperfection amplitude on the postbuckling behaviour of the panels. The amplitude of the geometric imperfection was varied from ten to one-hundred percent of the panel total thickness for each case. Additionally, the imperfection direction effects were also investigated. The study case named positive imperfection refers to the case where the geometric imperfection is defined in terms of the positive amplitude of the first buckling mode in the radial direction of the panel. On the other hand, the study case named negative imperfection refers to the case where the geometric imperfection is defined in terms of the negative amplitude of the first buckling mode in the radial direction of the panel.

The results clearly indicated that the structural behaviour of the panels is significantly affected by the geometric imperfection. The predictions also indicate that the postbuckling behaviour of simply-supported panels is more affected by the magnitude and direction of the geometric imperfection when compared to the fully clamped panels as shown in Figs. 4 and 5. The effects of the direction of the imperfection on the postbuckling behaviour of the panels for both boundary conditions are shown in Fig. 6 for an imperfection amplitude equals to 50% of the panel thickness. This figure clearly indicates that the boundary condition plays an important role in the postbuckling behaviour of simply-curved panels. It also confirms that the structural integrity of simply-supported panels is more affected by the direction of the geometric imperfection.



Figure 4. Effects of the positive imperfection on the postbuckling behaviour



Figure 5. Effects of the negative imperfection on the postbuckling behaviour



Figure 6. The effects of the direction of the geometric imperfection on the postbuckling behaviour

Figures 7 and 8 depict a typical postbuckled shape for the simply-supported and clamped panels, respectively.



(A): Positive imperfection



(B): Negative imperfection

Figure 7. Postbuckled shape of the simply-supported panel



(A): Positive imperfection



(B): Negative imperfection

Figure 8. Postbuckled shape of the clamped panel

#### 6. CONCLUSIONS

This paper presented a numerical procedure to investigate the structural behaviour of shallow singly-curved panels loaded in compression in the postbuckling regime. The proposed procedure is fully nonlinear and it was based on the dynamic relaxation method. The numerical predictions indicate that the structural behaviour of the panels studied in this work is significantly affected by the geometric imperfection effects. It was also demonstrated that the postbuckling behaviour of simply-supported curved panels is more affected by the magnitude and direction of the geometric imperfection when compared to the fully clamped panels. From the practical point of view, the results presented here may give some insight into the selection of the skin/stiffener joining technology to be used to manufacture stiffened panels in the aeronautical industry, where stiffer the bonding interface better is the structural behaviour of the panel in the postbuckling regime.

#### 7. ACKNOWLEDGEMENTS

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