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ATTITUDE AND VIBRATION CONTROL OF A RIGID FLEXIBLE LINK USING LQR AND H-INFINITY METHODS

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Abstract: *Attitude Control System (ACS) for flexible satellites demands great reliability, autonomy and robustness, besides these flexible structures face low stiffness due to minimal mass weight requirements. Satellite ACS design based only on computer simulations without experimental verification can be instable and/or inefficient. Controller implementation can also be jeopardized by its dimension and complexity, parameters and model uncertainties. In that context, comparison and validation of new control techniques through simulation and/or prototypes is the way to increase control system confidence. Experimental set up also allows verifying a variety of control techniques dealing with stabilization, identification, attitude control and robustness in order to improve ACS performance. In this paper one investigates the robustness and performance of two different multivariable methodologies to design the ACS for a rigid-flexible satellite. The first one is the traditional time domain LQR (Linear Quadratic Regulator) approach and the second one is the frequency domain H-Infinity approach. Although these control techniques have their particular characteristics, this investigation try to highlight the advantages and benefits of each technique for the control algorithm implementation. The satellite ACS design is performed by computer simulation environment, using Quanser rotary flexible link model. This preliminary investigation has shown that the LQR method has good performance and robust properties only if all the states are available. On the other hand, the H-infinity loop-shaping method is time consuming, since it depends on finding all the relative weights to achieve the desired performance and robustness. For both methods a small parameters variation can change all the system response. Besides, even to this simple problem the desired loop-shape is strongly affected by the choice of the weight matrix, once the design involves conflicting restrictions. As for the control algorithm implementation in the satellite onboard computer, the LQR has some advantages over the H-infinity, since the fist controller is simpler and has small dimension than the second one. The next step of this investigation is the physical implementation of the designed controllers in the Quanser flexible link equipments, in order to verify the controllers' feasibility and experimental comparison of the methods.*

Keywords: *Satellite attitude control; Flexible structure*

1. Introduction

Recently many researches and applications like modeling and robust control in Popescu et al (2008), vision based control in Xu et al (2009), time-optimal trajectory control in Chen et al (2008), position control in Boomsma et al (2004), control of robotic arm using sliding modes in Etxebarria et al (2005), space large structures robust control in Bodineau (2004), optimal vibration control in Sethi et al (2005), vibration suppression in Ahmad et al in Ahmad et al (2008), flexible structures control in Barbosa et al (2008), have been made using Quanser equipments to study and simulate a wide variety of real problems in different areas.

Position control of system with vibration has always received considerable attention from researchers due it great importance on performance of structures and equipments. In satellite launching and its space operation this importance is bigger due regarding low energy consumption operation and repair if there is any fault of the equipment. So it is necessary to eliminate or reduce rapidly any vibration after any correction maneuver with minimum energy consumption.

In this work we consider the angle position control and consequently the remaining vibration control (to damp the vibrations) of a flexible beam coupled to a rigid hub of servo system, namely the SRV02-Series equipped with a rotary flexible link supplied by Quanser. Our objective is to compare the angle position control performance between two robust control techniques, the classical LQR control and H infinity control, for such a system.

2. Mathematical Model

The mathematical model describes the rotation dynamics and the tip deflection. Using a strain gage we obtain the measurement of the flexible arm deflection. The motion equations consist of modeling the flexible link and the rotational base as rigid bodies. Table 1 depicts a list of the variables used to derive the state-space equations (equations (2.1)) of the system according to the SRV02-Series Quanser manual.

Table 1: System variables

Symbol	Description
L	Length of flexible Link
m	Mass of flexible Link
K_Gage	Strain Gage Calibration factor (1 Volt/Inch)
θ	Servo load gear angle (radians)
α	Arm Deflection (radians)
D	Link End-point Deflection (Arc Length)
ω_c	Link´s Damped Natural Frequency (Experimentally Calculated)
J_{Link}	Modeled Link Moment of Inertia
K_{stiff}	Modeled Stiffness (Estimation)
J_{eq}	Equivalent Inertia
η_m	Motor Efficiency
η_g	Gear Efficiency
K_t	Motor Voltage Constant
K_g	High Gear Ratio
B_{eq}	Equivalent Viscous Friction (Referred to the secondary gear)
R_m	Armature Resistance
J_{Arm}	Link´s Moment of Inertia

$$\begin{bmatrix}\dot\theta\\ \dot\alpha\\ \ddot\theta\\ \ddot\alpha\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{K_{Stiff}}{J_{eq}} & \frac{-\eta_m\eta_g K_t K_m K_g^2 + B_{eq}R_m}{J_{eq}R_m} & 0\\ 0 & \frac{-K_{Stiff}(J_{eq}+J_{Arm})}{J_{eq}} & \frac{\eta_m\eta_g K_t K_m K_g^2 + B_{eq}R_m}{J_{eq}R_m} & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot\theta\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{\eta_m\eta_g K_t K_g}{J_{eq}R_m}\\ \frac{-\eta_m\eta_g K_t K_g}{J_{eq}R_m}\end{bmatrix} V_m \tag{1}$$

3. Robust Controllers

3.1. Linear Quadratic Regulator (LQR):

The Linear Quadratic Regulator problem is to find the control input u(t) in

$$\dot x(t) = Ax(t) + Bu(t) \tag{2}$$

with $x(t_0) = x_0$ so as to minimize the quadratic cost criterion (performance index)

$$J = \int_{t_0}^{\infty}[x'(t)Qx(t) + u'(t)Ru(t)]dt \tag{3}$$

where $Q \geq 0$ e $R > 0$ are given matrices. Without loss of generality, the matrix Q can be expressed as

$$Q = M'M \tag{4}$$

The solution of the LQR problem is given by

$$u(t) = -R^{-1}B'Px(t) \triangleq -K\,x(t) \tag{5}$$

Where P is an $n \times n$ symmetric matrix which is the unique positive semidefinite solution of the algebraic Riccati equation

$$A'P + PA - PBR^{-1}B'P + Q = 0 \tag{6}$$

It can be shown that the unique $P \geq 0$ satisfying the algebraic Riccati equation, exists under the assumptions (A, B) stabilizable and (M, B) detectable.

Furthermore the matrix $A - BR^{-1}B'P$ i.e. $(A - BK)$ is a stable matrix.

In LQR Control, a disadvantage is that all the states have to be known. However, it is well known that a LQR-controlled system has the robustness property as advantage, if one chooses the weight R to be diagonal, the system will have a gain margin equal to infinity, a gain reduction margin equal to 0.5, and a (minimum) phase margin of 60^o in each plant input control channel. The drawback of this method is that if at the input and output of the plant we have complex transfer functions, in general it gives no guarantees of satisfactory robustness properties (stability margins).

3.2. H-Infinity (H_∞):

Consider the system (Fig. (1)):

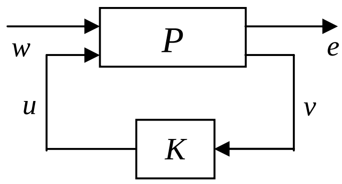


Figure 1: General control configuration

described by

$$\begin{bmatrix} e \\ v \end{bmatrix} = P(s)\begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix} \tag{7}$$

with the state realization of the generalized plant P given by

$$P \triangleq \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \tag{8}$$

where: u are the control variables, v are the measured variables, w are the exogenous signals as disturbances w_d and the commands r, and e are the error signals which are to be minimized to meet the control objectives. The closed-loop transfer function from w to e is given by the linear fractional transformation

$$e = F_l(P, K)w \tag{9}$$

where

$$F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{10}$$

Assuming that:

(A, B_2, C_2) is stabilizable and detectable.

D_{12} and D_{21} have full rank.

$\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω.

$\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all ω.
$D_{11} = 0$ and $D_{22} = 0$.
$D_{12} = \begin{bmatrix} 0 \\ I \end{bmatrix}$ and $D_{21} = [0 \quad I]$.
$D_{12}^T C_1 = 0$ and $B_1 D_{21}^T = 0$.
(A, B_1) is stabilizable and (A, C_1) is detectable.
The H_∞ optimal control problem is to find all stabilizing controllers K which minimize

$$\|F_1(P,K)\|_\infty = \max_\omega \bar{\sigma}(F_1(P,K)(j\omega)) \tag{11}$$

It also has a time domain interpretation as the induced two norm: Let $= F_1(P,K)\omega$. Then

$$\|F_1(P,K)\|_\infty = \max_{\omega(t)\neq 0} \frac{\|z(t)\|_2}{\|\omega(t)\|_2} \tag{12}$$

Where $\|z(t)\|_2 = \sqrt{\int_0^\infty \sum_i |z_i(t)|^2\, dt}$ is the 2-norm of the vector signal.

It is usually not necessary to obtain an optimal controller for the H_∞ problem. Let γ_{min} be the minimum value of $\|F_1(P,K)\|_\infty$ over all stabilizing controllers K, them one has a sub-optimal control

$$\|F_1(P,K)\|_\infty < \gamma \tag{13}$$

where $\gamma > \gamma_{min}$.

3.3. Mixed-sensitivity H_∞ control:

Mixed-sensitivity is the name given to a transfer function shaping problem in which the sensitivity function $S = (I + GK)^{-1}$ is shaped along with one or more other closed-loop transfer functions such as KS or the complementary sensitivity function $T = I - S$.

3.4. H_∞ Loop-shaping design:

The loop-shaping design used in this paper follows the design procedure presented in Skogestad et al (2005) which is based on robust H_∞ stabilization combined whit classical loop-shaping, where de open-loop plant is augmented by pre and post-compensators to give a desired shape to the singular values of the open-loop frequency response. Then the resulting shaped plant is robustly stabilized using H_∞ optimization.

4. Simulations and Results

4.1. LQR Controller Design:

The estimated state-space matrices are:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 591.9697 & -31.9744 & 0 \\ 0 & -947.2755 & 31.9744 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 56.2361 \\ -56.2361 \end{bmatrix}, C = [1 \quad 0 \quad 0 \quad 0], \quad D = [0]. \tag{14}$$

The LQR controller is designed to give the best performance. The R and Q matrices elements are chosen to allow a relative weighting of individual control inputs and individual state variables respectively. Starting with matrices Q and R as follows,

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = [1] \tag{15}$$

we change the values of the main diagonal of matrix Q to achieve, or get as close as possible to, the desired performance characteristics for the controlled system. Increasing equally the values of the main diagonal of matrix Q we do not have a good performance improvement for the controlled system.

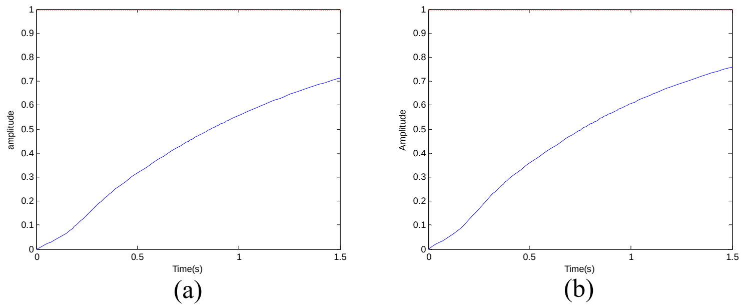


Figure 2: a) Step response of the LQR-controlled system for small values of matrix Q = diag [1 1 1 1]b) Step response of the LQR-controlled system for high values of matrix Q = diag [1000 1000 1000 1000].

Indeed if we consider assigning different values for each element of the main diagonal of matrix Q giving emphasis to the relative weight of each corresponding control variable (Fig. (3)), it is possible to achieve satisfactory performance characteristics by the combination of its different weights.

This implies that we need to give more emphasis (increase) to the relative weight of the first state variable (θ) and second state variable (α) once we need to control them simultaneously to reach the desired final link position. In Fig. (4) a) is presented the (over-damped) step response for the LQR-controlled system for the Q matrix weights

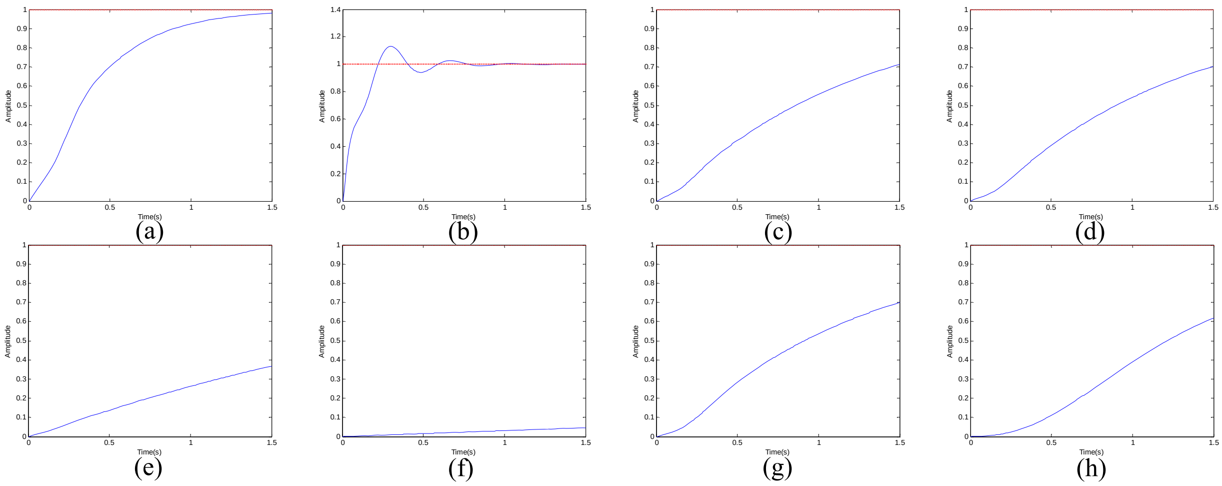


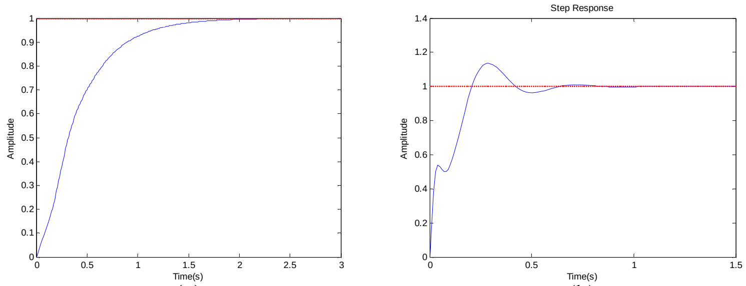
Figure 3: Step response of the LQR-controlled system for: a) Q = diag [10 1 1 1]; b) Q = diag [1000 1 1 1]; c) Q = diag [1 10 1 1]; d) Q = diag [1 1000 1 1]; e) Q = diag [1 1 10 1]; f) Q = diag [1 1 1000 1]; g) Q = diag [1 1 1 10]; h) Q = diag [1 1 1 1000].

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

For high relative weight values

$$Q = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

the LQR-controlled system behaves as in Fig. (4) b). However, it is necessary to adjust these relative weights to find an optimal response for the LQR-controlled system. A good choice for the matrices Q and R is:



(a) (b)

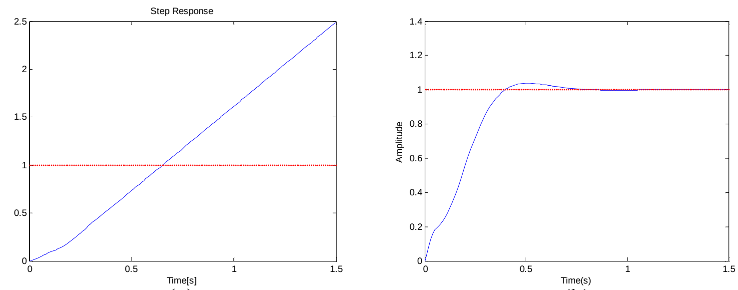
Figure 4: a) Step response of the LQR-controlled system for small relative weights; b) Step response of the LQR-controlled system for high relative weights.

$$Q = \begin{bmatrix} 255 & 0 & 0 & 0 \\ 0 & 3500 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = [1] \tag{18}$$

The optimal matrix K calculated, which minimizes the cost function J subject to the constraint defined by the plant is:

$$K = [15.9687 \quad -51.5602 \quad 2.1649 \quad 0.5598] \tag{19}$$

Figure (5) a) represents the closed loop step response of the plant and Fig. (5) b) represents the closed loop step response of the system composed by the plant and the controller



(a) (b)

Figure 5: a) Plant step response; b) Plant with controller step response.

4.2. LQR Simulink:

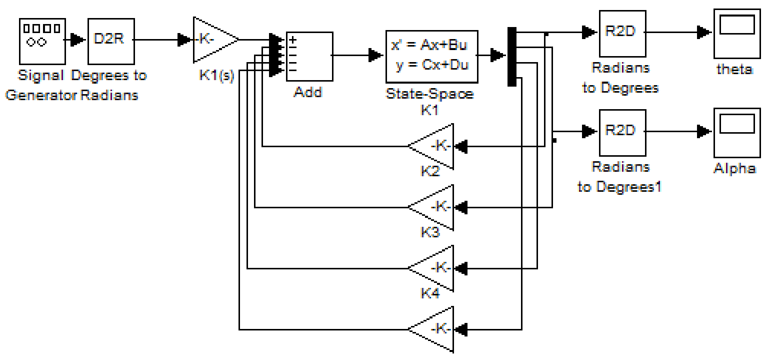
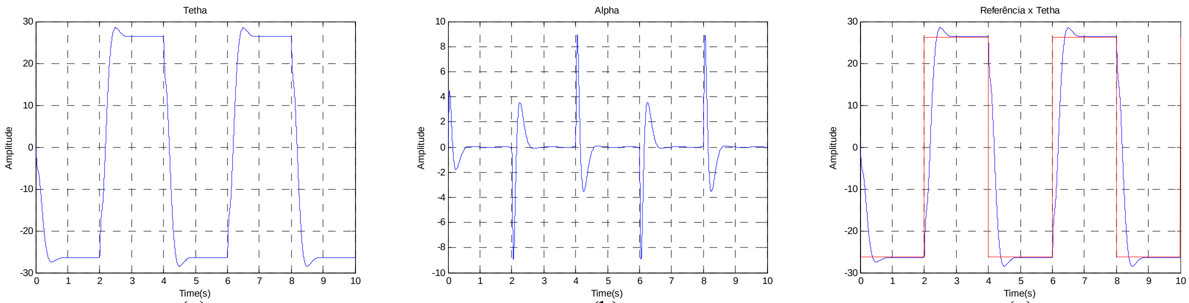


Figure 6: Simulink diagram for the compensated system with state feedback.

Once the optimal matrix K has been obtained one can use Simulink for the digital simulation. The simulation diagram constructed is represented in Fig. (6), where the blocks A, B, C and D are replaced by the values of the state-space matrices and the blocks K1, K2, K3 and K4 are replaced by the first to fourth values of matrix K. The signal generator generates a square input of amplitude 30 degrees and frequency 0.25 Hz. This condition is considered for a further implantation on real-time hardware, where the plant will be replaced by the real plant interface I/O´s.



(a) (b) (c)

Figure 7: a) Controlled state variable θ; b) Controlled state variable α; c) Comparison between the reference signal and the output (state variable θ).

As can be seen in the comparison of Fig. (7) c) the controlled system has a good response, once the position (state variable θ), which is to be controlled, tracks down the reference signal very well presenting a small overshoot, a small rise time and also a small settling time. As a consequence, the link vibration (state variable α) is also controlled (Fig. (7) b)).

4.3. H_∞ Controller Design:

The disturbance at the output is typically a low frequency signal, and it will be rejected if the maximum singular value of S is made small over the same low frequencies. In this way, we select a low-pass filter $w_1(s)$ (matrix in a general case) with a bandwidth equal to that of the disturbance, and find a stabilizing controller that minimizes$\|w_1 S\|_\infty$, where S is the sensitive function. Once this cost function focuses on just one closed-loop transfer function and for plants without right-half plane zeros the optimal controller has infinite gains. In the presence of a right-half plane zero, the stability will limit the controller gains, so we minimize $\left\|\begin{matrix} w_1 S \\ w_2 KS \end{matrix}\right\|_\infty$ where $w_2(s)$ is a high-pass filter with a crossover frequency approximately equal to that of the desired closed-loop bandwidth.

For the proposed tracking problem and noise attenuation we intend to find a stabilizing controller which minimizes$\left\|\begin{matrix} w_1 S \\ w_2 T \end{matrix}\right\|_\infty$. This controller is also important for robust stability with respect to multiple perturbations at the plant output. It is done to simplify the design, once the use of more than two functions becomes difficult (i.e. $\left\|\begin{matrix} w_1 S \\ w_2 T \\ w_3 KS \end{matrix}\right\|_\infty$) and the bandwidth requirements on each of this two functions are usually complementary and simple. Stable, low-pass and high-pass filters are sufficient to carry out the required shaping and trade-offs.

Once we are working in a matrix case it is necessary to have access to all the outputs (state variables variable θ and α) that will be controlled, so a change of the state-space matrices into the following form is necessary:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 591.9697 & -31.9744 & 0 \\ 0 & -947.2755 & 31.9744 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 56.2361 & 0 \\ -56.2361 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (20)$$

The H_∞ controller design, according to the loop shape technique, associates potentially conflicting specifications like performance, bandwidth and robustness. It is desirable to maximize the open loop gain, to obtain the better performance and it is also necessary to reduce this gain under 0dB to increase the controller robustness. Considering the bandwidth values, we have 4dB and 8dB corresponding to the lower and the upper values, once the crossover frequency occurs approximately in 2.4dB. Typically, to achieve good disturbance attenuation (performance) $w_1(s)$ is chosen to be small inside the desired control bandwidth w_b, and $w_3(s)$ is chosen to be small outside the control bandwidth, which ensures good stability margin (robustness), in this way

$$w_1(s) = \frac{s/M_s + w_s}{s + w_s * A_s} \quad (21)$$

where :
$w_s = 4$ rad/s (frequency inside the bandwidth);
$A_s = 0.001$ (desired disturbance attenuation inside the bandwidth);
$M_s = 2.0$ (desired bound on $\|S\|_\infty$and $\|T\|_\infty$)

$$w_3(s) = \frac{s+\frac{w_t}{M_t}}{A_t * s + w_t} \tag{22}$$

where :
$w_t = 8$ rad/s (frequency outside the bandwidth);
$A_t = 0.05$ (desired stability margin);
$M_t = 2.0$ (desired bound on $\|S\|_\infty$and $\|T\|_\infty$)
Calculating the H_∞ controller with a roll-off decay of -20 to -40dB/decade in high frequencies we obtain:

$$A_c = \begin{bmatrix} -0.004 & -1.483e-018 & 6.327e-017 & 2.41e-017 & -2e-006 & 6.829e-016 & 0 & 0 \\ -8.22e-026 & -0.004 & -2.543e-022 & 3e-019 & 1.137e-017 & -2e-006 & 0 & 0 \\ -9.309e-020 & -3.829e-029 & -160 & 1.955e-028 & 64 & -1.47e-026 & 0 & 0 \\ 0 & 0 & 0 & -160 & 0 & 64 & 0 & 0 \\ -2.327e-019 & -9.572e-029 & -2.838e-015 & 4.889e-028 & -5.821e-011 & -3.674e-026 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.821e-011 & 0 & 1 \\ 1.123e+005 & -0.0001407 & 1.46e+006 & -1.44e+006 & -6.624e+005 & 7.039e+005 & -6569 & -4916 \\ -1.123e+005 & 0.0001407 & -1.46e+006 & 1.46e+006 & 6.624e+005 & -7.039e+005 & 6569 & 4916 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0.06993 & 2.421e-017 \\ 3.975e-019 & 0.06993 \\ -1.628e-012 & -5.114e-028 \\ 0 & 0 \\ -4.071e-012 & -1.279e-027 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 5.712e+004 & -7.153e-005 & 7.424e+005 & -7.322e+005 & -3.369e+005 & 3.575e+005 & -3325 & -2500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{23}$$

Which are the augmented matrices of the H_∞ controller. Figure (8) shows the performance and robustness specifications according to the loop shaping technique while Fig. (9) a) shows the open-loop step response and Fig. (9) b) shows the step response of the system with the H_∞ controller.

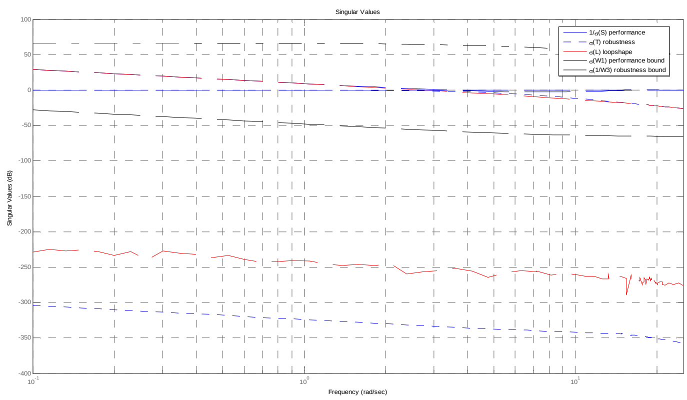
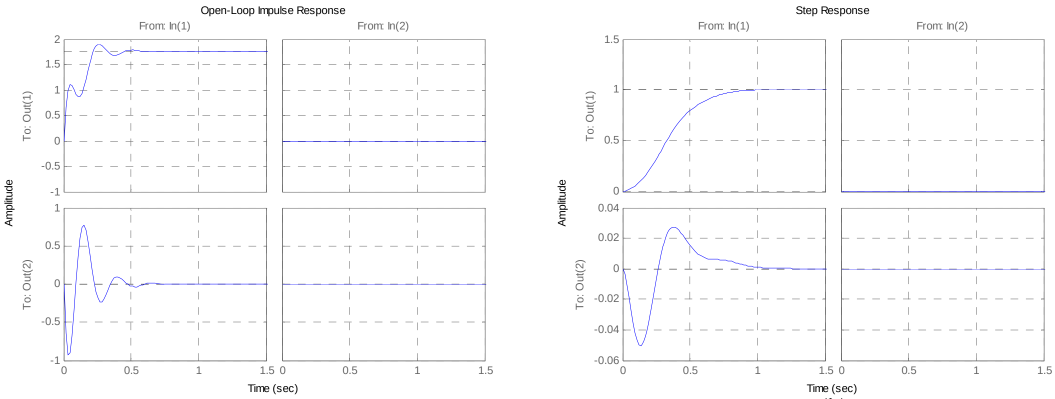


Figure 8: Performance and robustness specifications



(a) (b)

Figure 9: a) Open-loop step response b) Controlled system step response.

4.4. H_∞ Simulink:

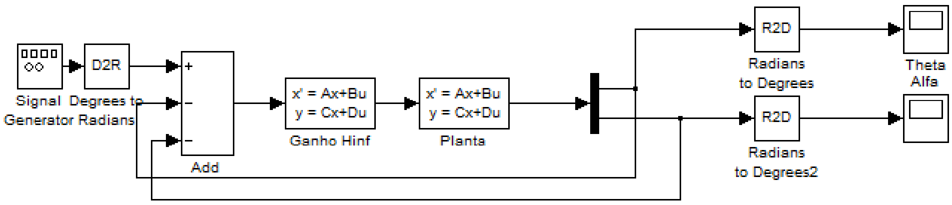
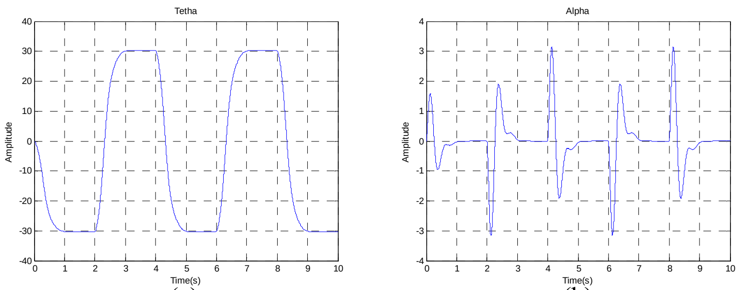


Figure 10: Simulink diagram for the system with state feedback and the H_∞ controller.

After the H_∞ controller has been calculated, Simulink (Fig. (10)) is used again for the digital simulation. But now besides replacing the plant matrices A, B, C and D by the state-space matrices we also replace the H_∞ controller´s matrices A, B, C and D by the augmented A_c, B_c, C_c and D_c respectively on the construction of the simulation diagram. Because of a further implantation on real-time hardware, as mentioned before, a square input of amplitude 30 degrees and frequency 0.25 Hz is also used as the reference signal.



(a) (b)

Figure 11: a) Controlled state variable θ; b) Controlled state variable α.

Concerning the results, controlled system (Fig. (12)) based on the loop-shape design presents a satisfactory tracking of the reference signal and a consequent control of the link vibration as well (Fig. (11) b)), but in this case a slower system response can be observed by a big rise time.

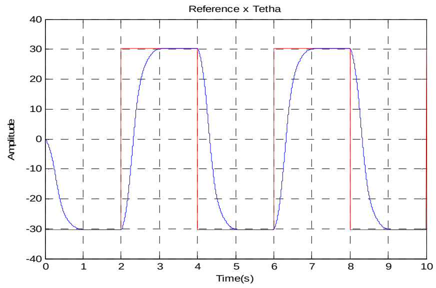


Figure 12: Comparison between the reference signal and the output (state variable θ).

5. Conclusions:

Both methods compared here are very good alternatives of robust control for the proposed problem. The LQR method, despite being easy to implement, can sometimes be more time consuming than the H_∞ loop-shaping method, since it depends on finding all the relative weights to achieve the desired performance because a small variation in a single parameter can change all the system response. Designer's knowledge and experience to choose properly these weights are necessary and in specifics cases the desired performance cannot be achieved. As could be seen the H_∞ loop-shaping method can also be used easily to stabilize robustly the feedback system, although it may not be easy to implement this method to stabilize systems with multiple gain crossover frequencies. Even to simple problems the desired loop-shape can be hard to design once we are working with conflicting restrictions, the chosen parameters for each weight matrix affects all the resulting system. A problem in the LQR method that we expect is the necessity to have all the states available, in the other hand an experimental problem in the H_∞ method is the controller complexity. However, the designer´s experience can overcome this entire problem in order to improve the system performance and try to achieve a higher level of robustness. With the results obtained it is possible now to follow to a physical implementation of the designed controllers, verify the controllers' feasibility, and a further real comparison of the methods.

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