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THE STRESS STIFFENING EFFECTS ON THE NATURAL FREQUECIES OF GAS TURBINE BLADES

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Abstract*. This paper presents an analytical model to predict the stress stiffening effects on the natural frequencies of turbine blades. The model formulation is based on the continuum dynamics and it incorporates the Von Karman nonlinear strain-displacement relationships accounting for flexural vibration modes only. A weak form formulation combined with the Rayleigh-Ritz method is proposed to solve the resultant differential equations of motion. The model predictions are compared with numerical results obtained using the Finite Element method. The predictions indicate that the natural frequencies are significantly affected by the turbine operating angular velocity.*

Keywords*: Gas turbines, vibrations, finite elements, stress stiffening*

1. INTRODUCTION

Turbines are being used for decades in power generation applications. They are classified according to the working fluid used in the power generation process, which can be wind, hydro, steam or gas. Great advances have been incorporated into the gas turbine in the last decades, perhaps beyond any other type of turbomachinery. Technological advances can be quoted as, for example, increased firing temperature, usage of advanced composite and alloyed materials, sophisticated cooling mechanism in components such as blade and nozzle, methods of reducing emissions, noise and thereby improving thermal efficiencies. Mainly due to these great technological advances gas turbines became the selected engine in various applications, including mechanical drive and power generation systems.

Gas turbines have unquestionably evolved in sophistication, but the approach used for vibration monitoring remained virtually unchanged. What has changed in recent years was the possibility of additional measures, such as combustion instability monitoring, hazardous gas detection, flame detection, exhaust gas temperature diagnostic, performance monitoring and overspeed detection (Maalouf, 2005).

The elastic behavior of structures may change considerably when they are in motion. For instance, when in rotation, the centrifugal force increases considerably the frequencies associated with flexural vibration modes of beams and blades, while the Coriolis coupling effects may result in different vibration mode shapes. Rotor blades are usually modeled as rotating beams in the open literature. This modeling approach allows getting some interesting characteristics related to the dynamic behavior of such structures.

Several researchers have been publishing different vibration analysis procedures for gas turbines and their components. Some research results were published by: Zirkelback e Ginsberg (2001); Boyce (2002); Chiang, Hsu, Tu (2004); HU et al. (2004); Lee, Lin e Wu (2004); Irwanto et al. (2004); Kaneko et al. (2006); Sipatov et al. (2007); Kar e Vance (2007); Lee e Sheu (2007); Gunda e Ganguli (2008); Seo e Lee (2008); Jafri e Vance (2008 a e b); Hylton (2008); Hashemi, Farhadia e Carra, (2009); Creci Filho, Menezes e Barbosa (2009); Turhan e Bulut (2009).

The aim of this study is to analyze the influence of the rotating shaft in the dynamic response of gas turbine rotor blades. Within this context, an analytical formulation of the problem using the Hamilton's principle was developed. The Rayleigh-Ritz method was used to solve the partial differential equations of motion. The solution of the differential equation was validated against numerical results obtained using the Finite Element method.

2. BLADE MODEL FORMULATION

The simplified model for a single blade is shown in Figure (1) and it consists of a cantilever beam clamped on a rigid disk subjected to angular velocity Ω .

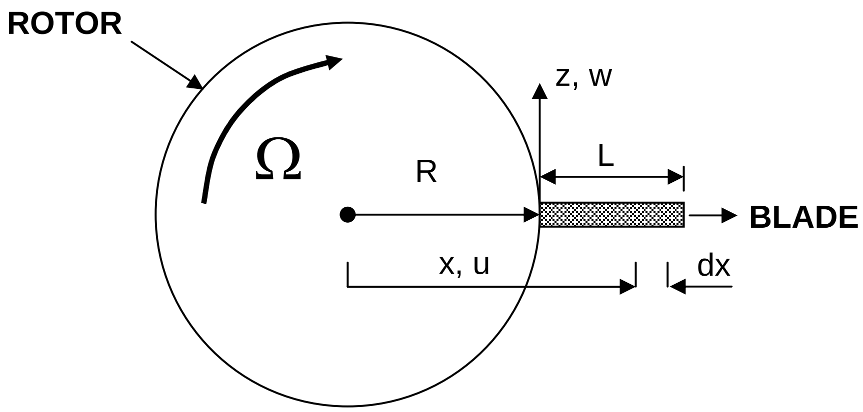

Figure 1. Simplified dynamic model of a gas turbine rotor

The differential equations of motion together with the natural and geometric boundary conditions are obtained from the Hamilton's Principle (Meirovitch, 1978) given by,

$$\delta\int_{t_1}^{t_2}\left(T-U+W_{nc}\right)dt=0 \tag{1}$$

where: T and U are the kinetic and strain energy, respectively and W_{nc} is the work done by the non-conservative forces. Considering only in-plane deflections the expressions for T , U and W_{nc} can be written as follows,

$$T=\frac{1}{2}\int_V \rho\left(\frac{\partial w}{\partial t}\right)^2 dV=\int_R^{R+L} m_L\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{2}$$

$$U=\frac{1}{2}\int_V \varepsilon\sigma dV \tag{3}$$

$$W_{nc}=\int_V \rho a_c dVu=\int_V \rho\Omega^2 xdVu=\int_R^{R+L} m_L\Omega^2 xdxu \tag{4}$$

where: ρ is the material density, $w(x,t)$ is the blade deflection, a_c is the centrifugal acceleration. ε , σ , m_L and u are the strain, stress, mass per unit of length and axial displacement, respectively. The kinematics relationships are given by,

$$\begin{aligned} u(x,y,z)&=u(x,y)-z\frac{\partial w(x,y)}{\partial x} \\ w(x,y,z)&=w(x,y) \end{aligned} \tag{5}$$

The strain-displacement is defined in terms of the Von Karman nonlinear relationship given as follows,

$$\varepsilon=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2=\varepsilon_L+\varepsilon_{NL} \tag{6}$$

Substituting, Eq. (5) into Eq. (6) and Eq. (6) into Eq. (3) and integrating in y and z directions we obtain,

$$U=\frac{1}{2}\int_R^{R+L} EA\left(\frac{\partial u}{\partial x}\right)^2 dx+\frac{1}{2}\int_R^{R+L} EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx+\frac{1}{2}\int_R^{R+L} N_x\left(\frac{\partial\ w}{\partial x}\right)^2 dx \tag{7}$$

where $N_x=EA\left(\partial u/\partial x\right)$. Substituting the equations listed above into the Hamilton's principle and taking the first variation on each term individually we obtain,

$$\delta T = \frac{1}{2}\int_R^{R+L} m_L \delta \left(\frac{\partial w}{\partial t} \right)^2 dx = \int_R^{R+L} m_L \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t} dx \tag{8}$$

$$\begin{aligned} \delta U &= \frac{1}{2}\int_R^{R+L} EA\delta\left(\frac{\partial u}{\partial x}\right)^2 dx + \frac{1}{2}\int_R^{R+L} EI\delta\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx + \frac{1}{2}\int_R^{R+L} N_x \delta\left(\frac{\partial\ w}{\partial x}\right)^2 dx = \\ &\int_R^{R+L} EA\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \delta u}{\partial x}\right)dx + \int_R^{R+L} EI\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 \delta w}{\partial x^2}\right)dx + \int_R^{R+L} N_x\left(\frac{\partial\ w}{\partial x}\right)\left(\frac{\partial\ \delta w}{\partial x}\right)dx \end{aligned} \tag{9}$$

$$\delta W_{nc} = \int_R^{R+L} m_L \Omega^2 x dx \delta u \tag{10}$$

which leads to the following expression for the Hamilton's principle,

$$\begin{aligned} &\int_{t_1}^{t_2}\left[\int_R^{R+L} m_L \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}dx - \int_R^{R+L} EA\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial \delta u}{\partial x}\right)dx - \int_R^{R+L} EI\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 \delta w}{\partial x^2}\right)dx\right. \\ &\left. -\int_R^{R+L} N_x\left(\frac{\partial\ w}{\partial x}\right)\left(\frac{\partial\ \delta w}{\partial x}\right)dx + \int_R^{R+L} m_L\Omega^2 x dx\delta u\right]dt = 0 \end{aligned} \tag{11}$$

Integrating by parts Eq. (11) we obtain,

$$\begin{aligned} &\int_{t_1}^{t_2}\left\{\int_R^{R+L}\left[-m_L\frac{\partial^2 w}{\partial t^2} - EI\frac{\partial^4 w}{\partial x^4} + N_x\frac{\partial^2 w}{\partial x^2}\right]\delta w dx + \int_R^{R+L}\left[EA\frac{\partial^2 u}{\partial x^2} + m_L\Omega^2 x\right]\delta u dx\right. \\ &\left. -EA\frac{\partial u}{\partial x}\delta u\Big|_R^{R+L} - EI\frac{\partial^2 w}{\partial x^2}\frac{\partial \delta w}{\partial x}\Big|_R^{R+L} + EI\frac{\partial^3 w}{\partial x^3}\delta w\Big|_R^{R+L} - N_x\frac{\partial w}{\partial x}\delta w\Big|_R^{R+L}\right\}dt = 0 \end{aligned} \tag{12}$$

which leads to the following differential equation for transverse displacement,

$$-m_L\frac{\partial^2 w}{\partial t^2} - EI\frac{\partial^4 w}{\partial x^4} + N_x\frac{\partial^2 w}{\partial x^2} = 0 \tag{13}$$

with natural and essential boundary conditions,

$$EI\frac{\partial^2 w}{\partial x^2}\frac{\partial \delta w}{\partial x}\Big|_R^{R+L} = EI\frac{\partial^3 w}{\partial x^3}\delta w\Big|_R^{R+L} = N_x\frac{\partial w}{\partial x}\delta w\Big|_R^{R+L} = 0 \tag{14}$$

For axial displacement the differential equation is given by,

$$EA\frac{\partial^2 u}{\partial x^2} + m_L\Omega^2 x = 0 \tag{15}$$

with the following natural and essential boundary conditions,

$$EA\frac{\partial u}{\partial x}\delta u\Big|_R^{R+L} = 0 \tag{16}$$

In order to solve this set of differential equations we need first to solve Eq. (15) in terms of $N_x = EA(\partial u / \partial x)$. After having N_x we then solve Eq. (13). The solution of Eq. (15) which satisfies the boundary conditions given by Eq. (16) is written as follows,

$$EA\frac{\partial u}{\partial x} = m_L\Omega^2\left[\frac{(R+L)^2}{2} - \frac{x^2}{2}\right] \tag{17}$$

3. RAYLEIGH-RITZ METHOD

The solution of Eq. (13) subjected to boundary conditions given in Eq. (14) is not trivial and it requires the application of numerical methods. For this purpose we have used the Rayleigh-Ritz method. The application of this method requires the weak form of Eq. (13) combined with a trial function which satisfies the essential or geometric boundary conditions given by Eq. (14). The first step consists of eliminating the time dependence of $w(x,t)$ in Eq. (13) by transforming the partial differential equation into a total differential equation dependent on space variable only. In order to do that lets assume a solution written in terms of variable separation given as follows,

$$w(x,t) = \phi(x)\eta(t) \tag{18}$$

where: $\phi(x)$ is the space function and $\eta(t) = \cos(\omega t)$ is the synchronous time dependent function. Substituting the general solution shown above into Eq. (13) leads to the following total differential equation,

$$EI\frac{d^4\phi}{dx^4} - N_x\frac{d^2\phi}{dx^2} - m_L\omega^2\phi = 0 \tag{19}$$

The weak form is obtained by multiplying Eq. (19) by a weight function W and integrating it twice in the x-domain,

$$\int_R^{R+L} W\left[EI\frac{d^4\phi}{dx^4} - N_x\frac{d^2\phi}{dx^2} - m_L\omega^2\phi\right]dx = 0 \tag{20}$$

which results in the following weak form,

$$\int_R^{R+L}\left[EI\frac{d^2\phi}{dx^2}\frac{d^2W}{dx^2} + N_x\frac{d\phi}{dx}\frac{dW}{dx} - m_L\omega^2\phi W\right]dx = 0 \tag{21}$$

Assuming $W = \phi = \sum_{i=1}^{n} c_i\phi_i$ we can rewrite Eq. (21) in the form,

$$\int_R^{R+L}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} c_ic_jEI\frac{d^2\phi_i}{dx^2}\frac{d^2\phi_j}{dx^2} + N_x\sum_{i=1}^{n}\sum_{j=1}^{n} c_ic_j\frac{d\phi_i}{dx}\frac{d\phi_j}{dx} - \omega^2\sum_{i=1}^{n}\sum_{j=1}^{n} c_ic_jm_L\phi_i\phi_j\right]dx = 0 \tag{22}$$

or in the matrix form,

$$\begin{aligned}&\int_R^{R+L} EI\{c\}^T\left[\frac{d^2\phi}{dx^2}\right]^T\left[\frac{d^2\phi}{dx^2}\right]\{c\}\,dx + \int_R^{R+L} N_x\{c\}^T\left[\frac{d\phi}{dx}\right]^T\left[\frac{d\phi}{dx}\right]\{c\}\,dx\\&-\omega^2\int_R^{R+L} m_L\{c\}^T[\phi]^T[\phi]\{c\}\,dx = 0\end{aligned} \tag{23}$$

Minimizing Eq (23) in respect to the coefficients vector $\{c\}$ we obtain the solution of Eq. (13) in the form of an eigenvalue problem given as follows,

$$\{[K] + [K_G] - \omega^2[M]\}\{c\} = \{0\} \tag{24}$$

with

$$[K] = \int_R^{R+L} EI\left[\frac{d^2\phi}{dx^2}\right]^T\left[\frac{d^2\phi}{dx^2}\right]dx \tag{25}$$

$$[K_G] = \int_R^{R+L} N_x \left[\frac{d\phi}{dx}\right]^T \left[\frac{d\phi}{dx}\right] dx \tag{26}$$

$$[M] = \int_R^{R+L} m_L [\phi]^T [\phi] dx = 0 \tag{27}$$

The idea now is to find a trial function $\phi(x) = \sum_{i=1}^{n} \phi_i(x) c_i = [\phi(x)]\{c\}$ that satisfies the essential or geometric boundary conditions and fulfils the C^1 continuity requirement imposed by the weak form given by Eq. (21). A candidate function proposed in this work which satisfies these conditions is given in the following form,

$$\phi_i(x) = \left(\frac{x-R}{L}\right)^{i+1} \tag{28}$$

where $\phi_i(R) = \phi_i^{'}(R) = 0$.

4. MODEL VALIDATION

In order to validate the model two distinct analyses were carried out: (i) Free vibration analysis for a stationary cantilever beam ($\Omega = 0$ rpm), (ii) Free vibration analysis for a rotating beam ($\Omega = 9550$ rpm). The beam was assumed to be made of conventional steel with a length of 0.840 m, 0.140 m wide and 0.02 m thick. The beam was assumed to be clamped on a rigid wall (*R* = 0) for both cases. Figure (2) shows the convergence rate for the four first natural frequencies associated with the flexural vibration modes with increasing number of terms in the solution of Eq. (22). The results indicate that the convergence for the *k-th* frequency, (k = $1\ldots\infty$), is reached using *2k* terms in the Ritz solution. Comparisons between the natural frequencies predicted using the proposed semi-analytical model and results obtained from finite element analyses (ABAQUS®, 2005) for both cases are shown in Tables 1 and 2. Figures (3) and (4) show the vibration modes obtained from FE analyses and using the proposed model, respectively. The results obtained using the proposed model correlate very well with results obtained from FE analyses.

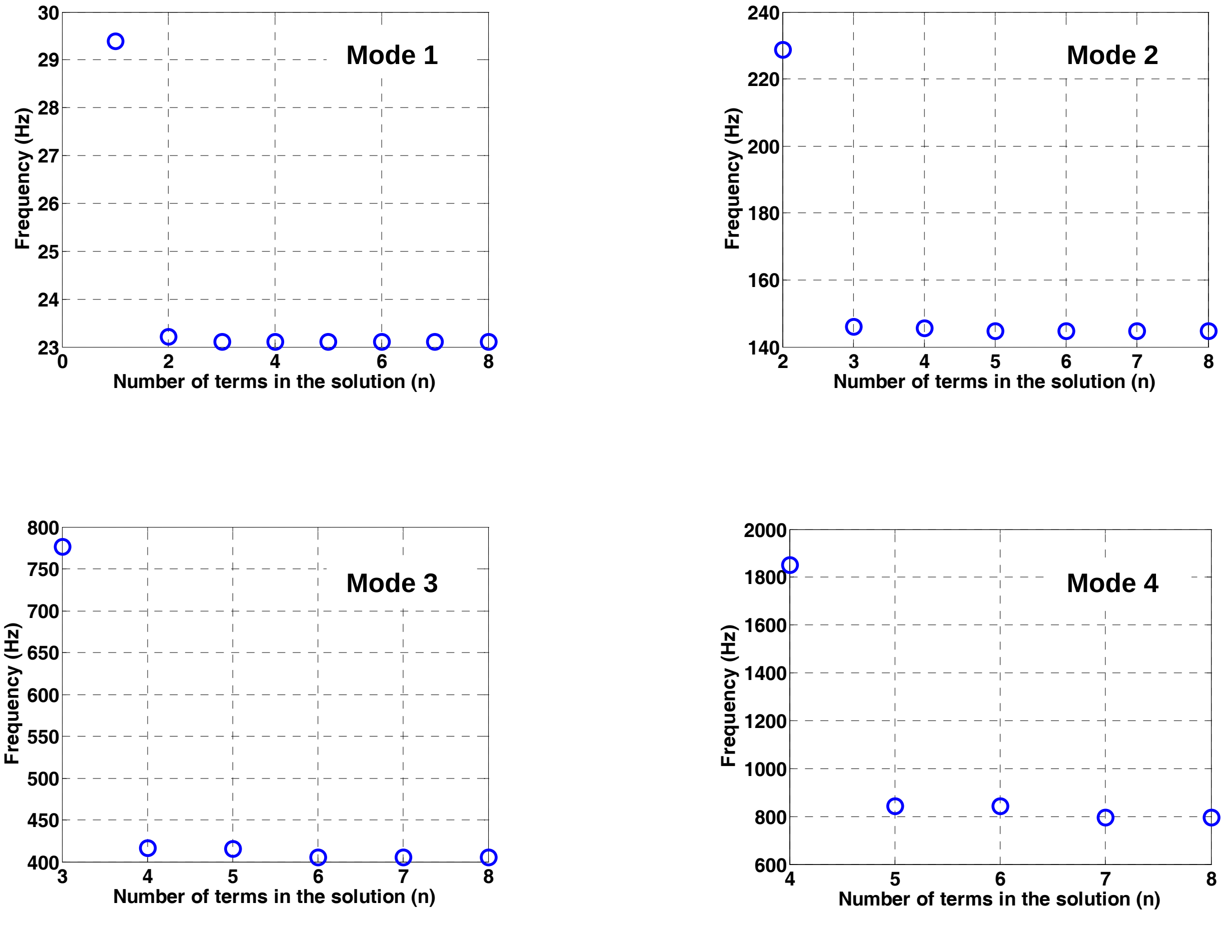

Figure 2. Convergence rate for natural frequencies

Table 1. Comparison between model predictions and FE analyses for $\Omega = 0$ rpm .

Flexural modes	Frequency (Hz)		Error (%)
	FE model	Proposed model	
1	23.30	23.11	0.82
2	145.64	144.83	0.55
3	406.82	405.55	0.31
4	794.82	796.13	0.16

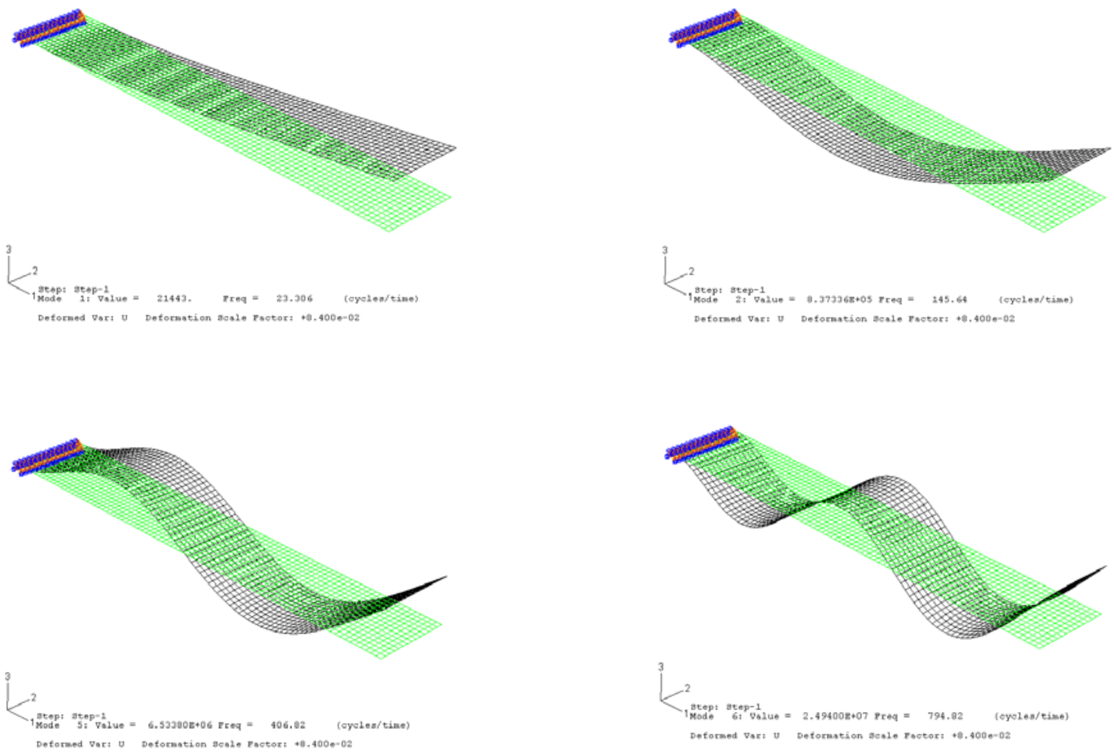

Figure 3. Vibration modes obtained from FE analyses ($\Omega = 0$ rpm), (ABAQUS®, 2005)

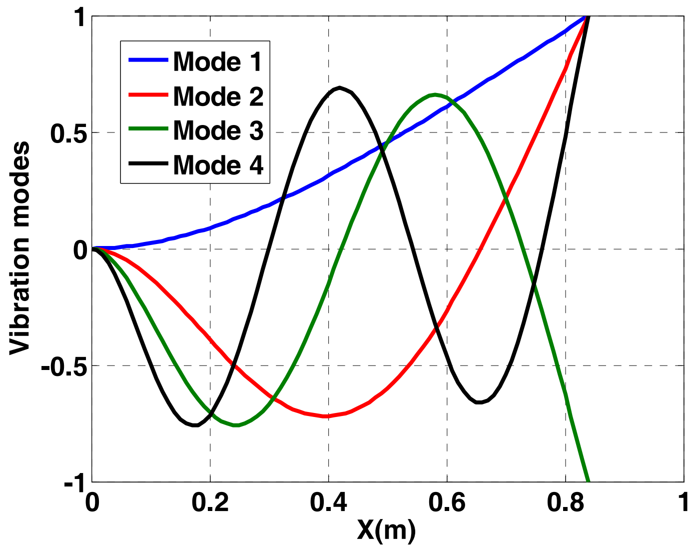

Figure 4. Vibration modes predicted using the proposed model ($\Omega = 0$ rpm**)**

Table 2. Comparison between model predictions and FE analyses for $\Omega = 9550$ rpm **.**

Flexural modes	Frequency (Hz)		Error (%)
	FE model	Proposed model	
1	166.42	167.31	0.53
2	426.24	431.80	1.29
3	772.22	794.39	2.79

The stress stiffening effects on the natural frequencies of the rotating beam are shown in Figure (5). For this case the rotor angular velocity was varied from 0 to 30,000 rpm. The results clearly indicate that the natural frequencies of the rotating beam are significantly affected by the rotor angular speed. It is also worth noticing that these effects are much more pronounced for the lowest frequency of the structure.

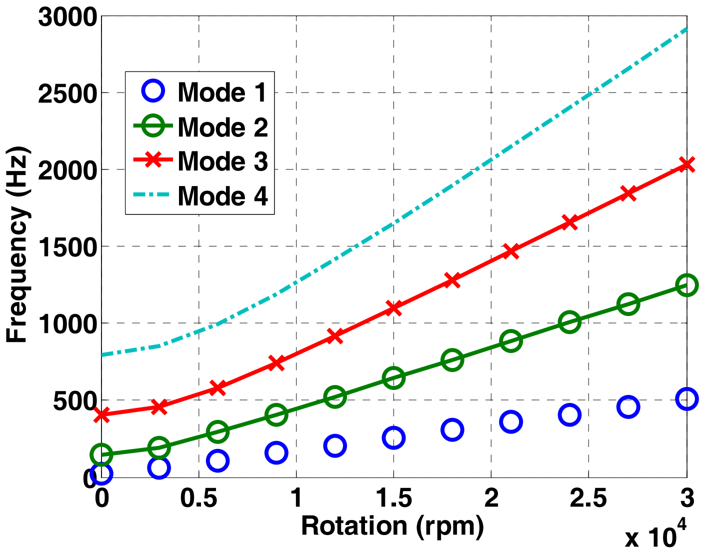

Figure 5. Influence of the rotation on the natural frequencies

5. VIBRATION ANALYSIS OF GAS TURBINE BLADES

This section presents the effects of the angular velocity on the natural frequencies of a typical gas turbine blade. The blade was assumed to be made of steel with length L = 0.05 m, width B = 0.02 m and thickness h = 0.003 m clamped on a rigid rotor disk of radius R = 0.2 m. Figure 6 shows the effects of the angular velocity on the first four natural frequencies. To better illustrate the stress stiffening effects, the natural frequencies of the blade were normalized in respect to the frequencies obtained for the case where $\Omega = 0$ rpm. Figure (6) clearly indicates that the lowest frequency associated with the first flexural vibration mode is the most affected frequency within the gas turbine operation angular velocity range studied in this work. Figure (7) shows the normal stress distribution with increasing angular velocities obtained from Eq. (17).

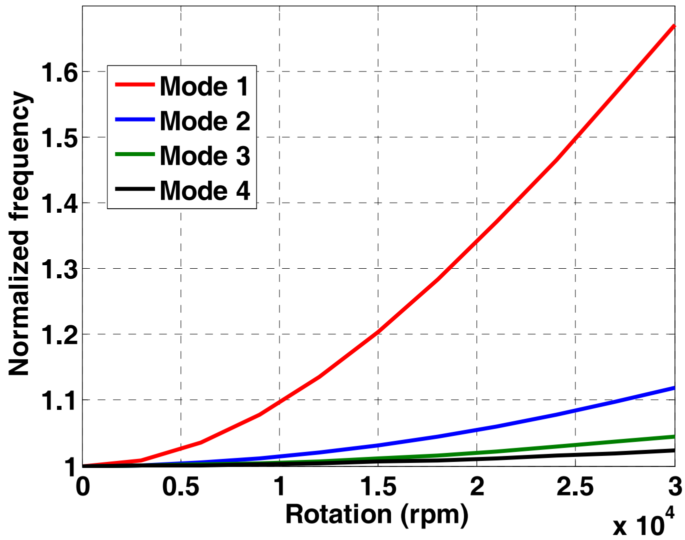

Figure 6. Influence of the rotation on the natural frequencies of a typical gas turbine blade

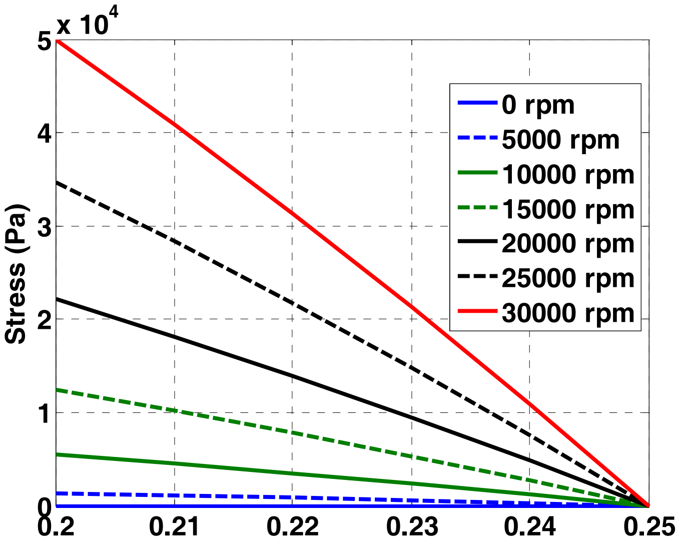

Figure 7. Stress distribution along the blade

6. CONCLUSIONS

This paper presented a detailed analytical formulation for rotating beams subjected to stress stiffening effects. The analytical model was validated against numerical results obtained from FE analyses. A very good agreement between results obtained from FE analyses and predictions obtained using the proposed model was obtained.

An application of the model for natural frequencies prediction of gas turbine blades was also presented and discussed. The predictions obtained using the proposed models clearly indicate that the natural frequencies of gas turbine blades are significantly affected by the rotor angular velocity. It was also demonstrated that the stress stiffening effects are much more pronounced for the lowest natural frequency of the structure.

It is worth mentioning that due to aerodynamic restrictions, the geometry of gas turbine blades is far more complex than the simplified beam geometry adopted in this paper. However, the simplified model proposed here gives some insight into the dynamic behavior of gas turbine structures. It is also a preliminary step towards the development of more elaborated models.

7. ACKNOWLEDGEMENTS

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