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LOW COST ALTERNATIVES FOR VISCOUS DAMPING COEFFICIENT EVALUATION

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Abstract: *An experimental assembly of an one degree of freedom mass/damper/spring system is used, with logarithmic decrement approach, to evaluate viscous damping coefficient of a damper. A displacement transducer and an acceletrometer were used to measure the experimental output. A comparison between the performances of these transducers was done, clarifying the strong and weak points of each approach. In special the difficulties associated with the numeric double integration approach for the accelerometer data were accessed. Actions as use a low-pass filters, select a high data aaquisition sample rate and the removal of the initial value of all acceleration data were implemented and the results commented.*

Keywords: *viscous damping coefficient, experimental approach, logarithmic decrement*

1. INTRODUCTION

In this work a one degree of freedom system, that already used in former work (Kenedi et al., 2006), is utilized to obtain experimental underdamped outputs. The evaluation of viscous damping coefficient is done by utilization of two different types of transducers: a displacement transducer, the same of former work, and an accelerometer. The performances of each transducer are analyzed and its advantages and disadvantages are commented.

2. ANALYTIC MODEL

An one degree of freedom system is schematically shown at Fig.1.a and has a mass, a damper and a spring. Figure 1.b and 1.c shows this system at two different moments. At Fig.1.b the system is at a static equilibrium position, where δ_{est} is the displacement caused by acceleration of gravity. At Fig.1.c the system is at a non-equilibrium position imposed by x_0 displacement. At Fig. 1.d a free body diagram of Fig. 1.c is shown.

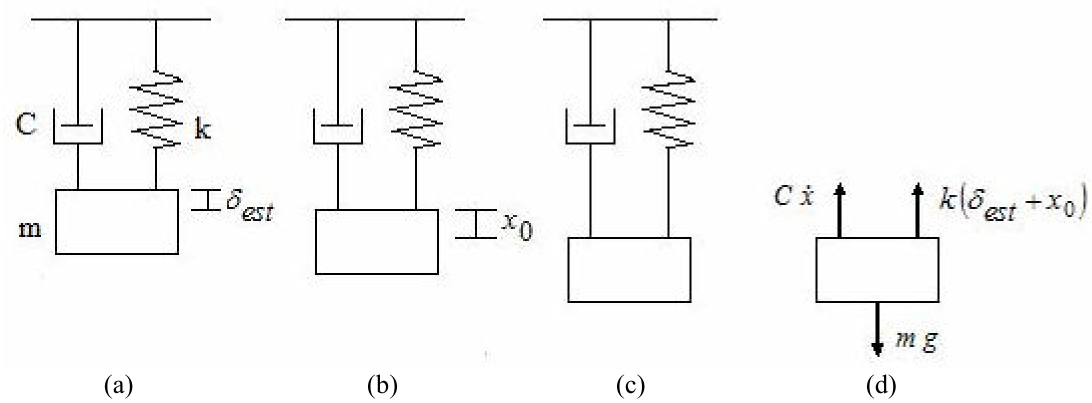


Figure 1. (a) mass/damper/spring system, (b) static equilibrium displacement, (c) adding initial non-equilibrium displacement and (d) free body diagram of last position.

The governing expressions of one degree of freedom mass/damper/spring system for free vibration are cast from dynamic equilibrium of Fig. 1.d (Steidel, 1989):

$$\ddot{x} + \frac{C}{m} \cdot \dot{x} + \omega_n^2 \cdot x = 0 \tag{1}$$

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2}$$

Where, x is the displacement (x_0 is the maximum amplitude), $\dot{x}$ is the velocity and $\ddot{x}$ is the acceleration, ω_n is the natural circular frequency, C is the viscous damping coefficient, k is the spring constant and m is the mass of the system.

Another way of writing (1) and (2) is (Steidel, 1989):

$$\ddot{x} + (2 \cdot \zeta \cdot \omega_n) \cdot \dot{x} + \omega_n^2 \cdot x = 0 \tag{3}$$

$$\zeta = \frac{C}{2 \cdot m \cdot \omega_n} \tag{4}$$

Where, ζ is the damping ratio. The analyzed system is said to be underdamped ($0 < \zeta < 1$). Solving (3) and (4) results in (Steidel, 1989):

$$x = e^{-\zeta \cdot \omega_n \cdot t} \cdot x_0 \cdot \left(\frac{(\zeta \cdot \omega_n)}{\omega_d} \cdot sin(\omega_d \cdot t) + \cos(\omega_d \cdot t) \right) \tag{5}$$

$$\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2} \tag{6}$$

where, t is time and ω_d is the damped natural frequency. By differentiating (5), the velocity expression (7) can obtained. Differentiating again the acceleration expressions (8) can be also obtained (Steidel, 1989):

$$\dot{x} = -e^{-\zeta \cdot \omega_n \cdot t} \cdot x_0 \cdot \omega_d \cdot \left[\left(1 + \left(\frac{\zeta \cdot \omega_n}{\omega_d} \right)^2 \right) \cdot \sin(\omega_d \cdot t) \right] \tag{7}$$

$$\ddot{x} = e^{-\varsigma \cdot \omega_n \cdot t} \cdot x_0 \cdot \omega_d^2 \cdot \left[\left(\frac{\zeta \cdot \omega_n}{\omega_d} \right) \cdot \left(1 + \left(\frac{\zeta \cdot \omega_n}{\omega_d} \right)^2 \right) \cdot \sin(\omega_d \cdot t) - \left(1 + \left(\frac{\zeta \cdot \omega_n}{\omega_d} \right)^2 \right) \cdot \cos(\omega_d \cdot t) \right] \tag{8}$$

The expression (9), obtained in (Kenedi et al., 2006), shows that is possible to estimate the viscous damping coefficient C by experimental acquisition of two sequential amplitudes of decreasing oscillatory pattern of a one degree mass/damper/spring system and by knowing the mass m and spring constant k values.

$$C = 2 \cdot \sqrt{\frac{k \cdot m}{\left[\frac{2 \cdot \pi \cdot n}{\ln\left(\frac{x_0}{x_n} \right)} \right]^2 + 1}} \tag{9}$$

For the first amplitude $x = x_0$ and $t = t_0 = 0$. For nth amplitude, $x = x_n$ and $t = t_n$, where n is the number of cycles between amplitude x_0 and x_n.

Mass and spring constant values are readily obtained, the first using a balance and the second imposing a force and measuring the resultant displacement in a material testing machine. The amplitudes x_0 and x_n are obtained experimentally using, for instance, the output signal of the experimental apparatus shown at Fig.2.

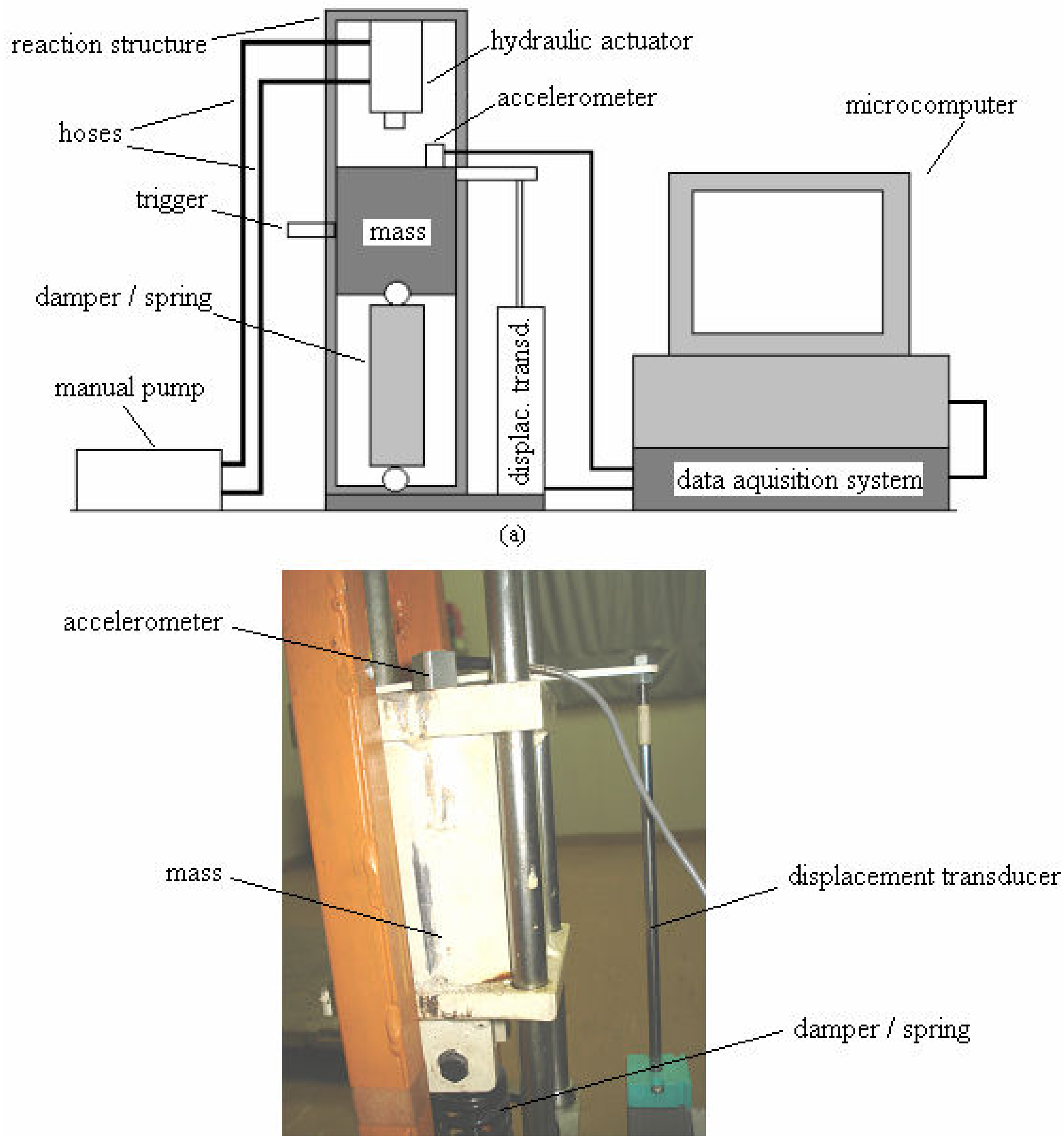
Two types of transducer are used in this experiment: a displacement transducer with potenciometric principle and an accelerometer with extensometric principle. Both displacement transducer and accelerometer generates analog voltage signals, the first proportional to displacement and the second proportional to acceleration.

3. EXPERIMENTAL APPROACH

An experimental apparatus, shown at Fig. 2, was constructed to impose an initial displacement x_0 to the mass/damper/spring system. The actuation of a hydraulic cylinder imposes x_0 that is maintained by a trigger until the beginning of experiment. The two transducers, for displacement and for acceleration, are firmly connected to the system as shown as Fig, 2.b.

3.1. Experimental Device

Figures 2.a and 2.b shows the apparatus utilized to impose an initial non-equilibrium displacement x_0 to a mass/damper/spring system. As the mass/damper/spring system is released by the trigger, the system is submitted to a decreasing oscillatory pattern that is monitored through the utilization of displacement and acceleration transducers



(b)

Figure 2. (a) Schematic view of experiment and (b) detailed view of experiment.

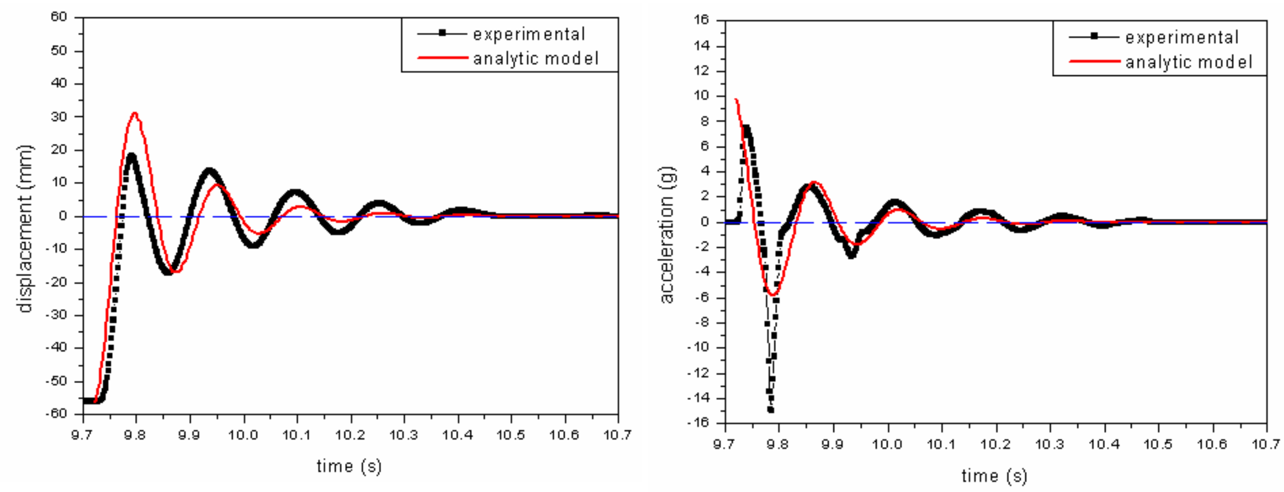
Figure 2.a shows a schematic view of the experiment. The apparatus uses the hydraulic actuator, commanded by a manual hydraulic pump, to positioning the mass/damper/spring system to a non-equilibrium position (like Fig. 1.c). A trigger is activated and the hydraulic actuator is disconnected. The system rests in this position until the trigger is deactivated, causing oscillations of the mass/damper/spring system.

The acquisition system was a Spider 8 system (HBM, 2006), which provides excitation to transducers and amplification / filtering for signals from transducers. The A/D conversor has 16 bit resolution and for this experimental test two sample rates were selected: 100 S/s and 1200 S/s. At the accelerometer channel was used a 20 Hz Bessel filter (low pass filter) to prevent the low frequency signal of accelerometer being affected by spurious higher frequencies. The system works in combination with a microcomputer, where is installed a professional software of data acquisition, the Catman software (HBM, 2006).

A detailed explanation of components and operation of experimental apparatus can be obtained at (Kenedi et al., 2006).

3.2. Experimental Results

At Fig. 3.a a comparison between analytic curve (5) and the displacement signal is shown and at fig. 3.b a comparison between analytic curve (8) and the acceleration signal is shown. A motorcycle damper/spring was utilized and has k = 15.9 kN/m and m = 9.3 kg. Note that the C = 142 $\frac{N \cdot s}{m}$ was estimated using expression (9) with amplitudes x_0 and x_n obtained by experimental displacements results (black points of Fig.3.a).



(a) (b)

Figure 3. Graphical comparison of experimental signals of (a) displacement and (b) acceleration.

Note at Fig. 3.a the displacement signal is relatively close to analytic curve, while the acceleration signal, at Fig.3.b, is not so close to respective analytic curve at the beginning of signal transient.

4. TREATEMENT OF RESULTS

The data obtained by the displacement transducer not need further treatment and was used as experimental reference. On the other hand the accelerometer data need to be twice integrated to be transformed in displacement data.

4.1 Integration of Acceleration Data

The accelerometer data obtained by data acquisition system has to be integrated twice to generate displacement data. A numerical integration, based on Trapezoidal Rule (Borato, 1984), was utilized. The expressions used to estimate velocity v_i and displacement d_i are:

$$v_i = \left[\Delta t\left(\frac{a_i + a_{i-1}}{2}\right)\right] + v_{i-1} \tag{10}$$

$$d_i = \left[\Delta t\left(\frac{v_i + v_{i-1}}{2}\right)\right] + d_{i-1} \tag{11}$$

Where, the underscore *i* means the i-th element of a list and Δt is the time interval or 1/(sample rate).

The double integration of acceleration can produce rather erroneous displacements results. In order to minimize such errors is important to:

a) Select a low-pass filter during data acquisition of accelerometer signal to prevent spurious high frequency signals,

b) Remove the initial value from all acceleration data:

$$a_i = a_i^* - a_0^* \tag{12}$$

where, a_0^* is the initial value of acceleration data and * means the measured acceleration data,

c) Select a small value for Δt .

To show the influence of items a), b) and c) in the integration of accelerometer data, three figures were generated. Figure 4 shows a displacement versus time graph were items a) and c) were implemented but not item b) (it has 1 g initial acceleration value):

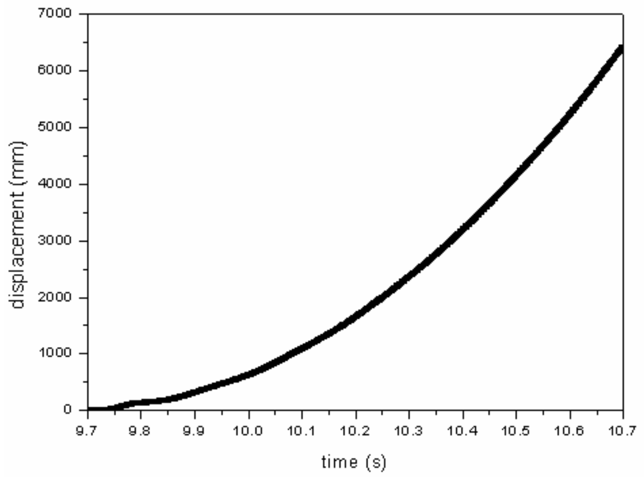
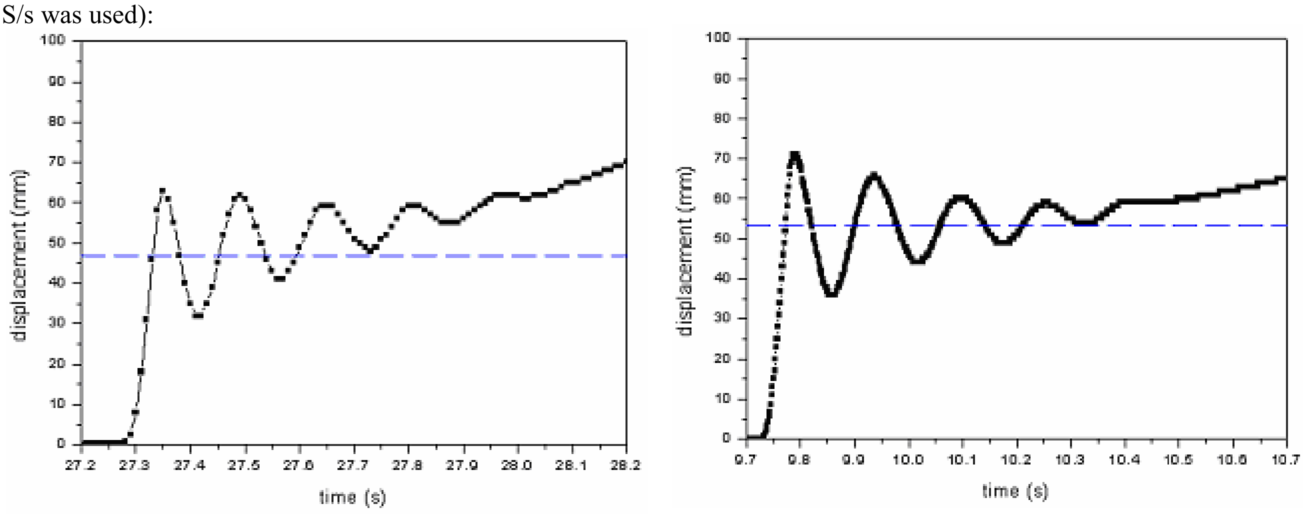


Figure 4. Incorrect displacement result, after double integration, using items a) and c).

Figures 5.a and 5.b shows displacement versus time graphs. At Fig. 5.a items a) and b) were implemented but not item c) (a sample rate of 100 S/s was used) and at fig. 5.b items a), b) and c) were implemented (a sample rate of 1200 S/s was used):



(a) (b)

Figure 5. Displacement results, after double integration, using: (a) items a) and b) and (b) all three items.

The not implementation of item b) at fig.4 conducted to completely incorrect results by an excessive propagation of errors in double integration of acceleration data.

Figures 5.a and 5.b shows, graphically, reasonable results for double integration of acceleration data with lesser influence of propagation of errors. Note that in both figures as the time pass the displacement results tend to lose contact with the equilibrium position represented by the blue dashed line, with lesser deviation for Fig.5.b which has a smaller Δt .

Discarding results of Fig. 4 and using Figures 5.a and 5.b to do the calculations of *C* results, respectively, 130 $\frac{N \cdot s}{m}$ and 132 $\frac{N \cdot s}{m}$, both with errors less than 10% in comparison to *C* estimated by results of displacements transducer experiment (used as reference). Note that the implementation of all three items, shown in fig. 5.b, generates the best result from accelerometer data.

5. CONCLUSION

Two transducers were used to estimate the viscous damping coefficient *C*, a displacement transducer and an accelerometer transducer from a mass/damper/spring system, which generated an underdamped output. The difficulties associated with the numeric double integration approach for the accelerometer data were accessed and the results commented. Although the estimative of *C* obtained by accelerometer data, after post processing, was reasonable, the use of displacement transducer seems to be straightforward, no needing further processing of obtained data.

6. REFERENCES

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