ANALYSING THE PERFORMANCE OF AN AEROSPACIAL VEHICLE CONTROLLER CONSIDERING UNCERTAINTIES AT THE STATES

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Abstract. *This work proposes designing a controller for an aerospacial vehicle, in order to track a well-defined trajectory, based on the theory of Optimal Control. The model of the vehicle is non-linear and simplified, i.e., it does not consider all the effects the vehicle is subject to. While tracking this trajectory, the aerospacial vehicle indirectly measures the position and velocity of a second vehicle that sail at the ocean.*
We consider here the sensors embedded in the vehicle, which provide noisy measures. So, in order to compensate the not-perfectly modelled dynamics and the sensor noises, we will use a Kalman Filter, so as to estimate the state (position, velocity) of the aerospacial vehicle and use in the controller, and also obtain an estimate of the states of the measured vehicle.
So, we analyse the efficiency of the designed controller, and also its performance when using the estimated states.

Keywords: *aerospacial vehicle, optimal control, LQR, estimation, Kalman filter*

1. Introduction

This paper is about the analysis of the influence of an estimator on a controller's performance. This controller was designed in order to follow a trajectory in the state space, using the theory of Optimal Control. As the sensors embedded in the vehicle are not perfect, and as there are some uncertainties concerning the model of the vehicle, we cant't just use the sensors measures or propagate the states of the vehicles using its dynamic equations. So, there is a need of filtering the data given by the sensors in order to obtain the state values as close to the real states as possible. For this, we used the algorithm of the Kalman Filter. Most of the papers published in this area, as the works of Lin and Su [5] and Cheng and Gupta [1], have their focuses on controlling aerospatial vehicles, but do not consider noisy sensors embedded in it.

2. Problem Formulation

Here we consider a problem including three vehicles: the launcher, the aerospace vehicle to be studied and the tracked vehicle. We shall consider here the midcourse phase, when the aerospace vehicle is not subject to the laucher influences anymore and the mass change is not critical. This phase lasts about 90 seconds. The aerospace vehicle (V1) flies tracking a reference trajectory, while measures its own position and velocities (using an inercial platform) and the line sight angle (using a seeker) with a second vehicle (V2).

Figure 1 shows the problem scheme.

2.1 Plant

The plant to be considered here is the 2-D case of the plant found in [1]. This model is simplified, for not considering some effects present in this situation, as rotational dynamics, presence of wind, temperature effects, among others. For this application the rotational dynamics do not need to be considered, as we shall treat the vehicle as a punctual one, analysing its translational displacement and velocity only. The winds will not be considered in these equations, as we do not have a deterministic model for it. The temperature effects and other are so much smaller than the ones considered here, that it makes possible to neglect them. However, the non-deterministic events like wind we be considered in the Kalman Filter. The state equations, then, are those described in Eq. (1).

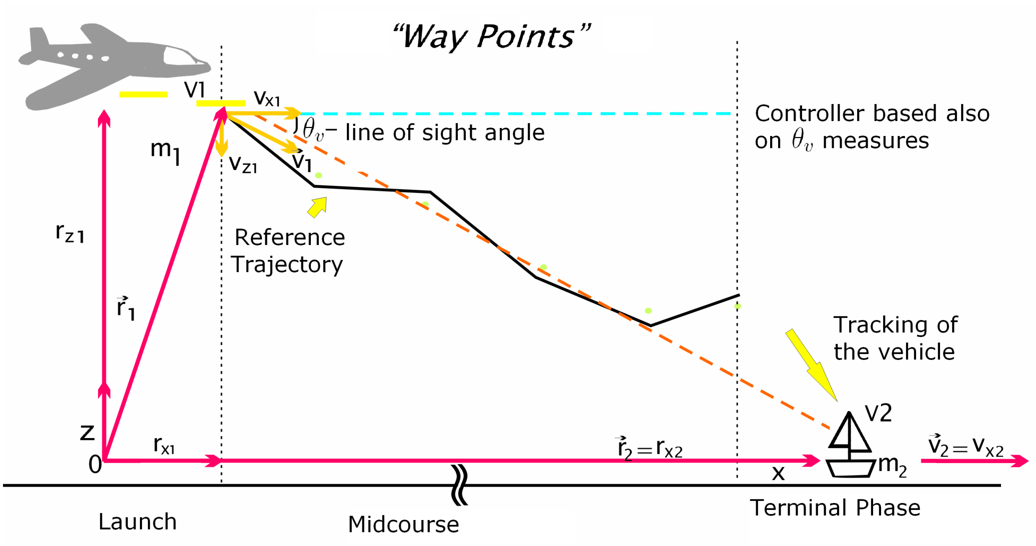

Figure 1. Problem Formulation

$$\begin{aligned}
\dot{r}_{x1}(t) &= V_1(t)\cos\gamma(t) \\
\dot{r}_{z1}(t) &= V_1(t)\sin\gamma(t) \\
\dot{V}_1(t) &= \frac{F_E(t)}{m(t)} - \frac{\rho S C_D V_1(t)^2}{2m(t)} - g\sin\gamma(t) \\
\dot{\gamma}(t) &= \frac{-g\cos\gamma(t)}{V_1(t)} + \frac{u(t)}{m(t)V_1(t)}
\end{aligned} \tag{1}$$

Where:

- $\rho = \rho_0 \exp\left(-\frac{r_z}{H_0}\right)$
- $S = 0,04m^2$
- $g = 9,8m/s^2$
- $\rho_0 = 1,752kg/m^3$
- $H_0 = 6700m$
- $m(t) = 200kg + m_a(t) + m_c(t)$

The values of the thrust $F_E(t)$, acceleration engine mass, m_a, midcourse engine mass, m_c and aerodynamic coefficient, C_D, were interpolated using Tabs. 1 , 2 and 3 .

The model of the velocity of the air is given as an approximatelly linear decay of $1m/s$ every $250m$ [2].

As this model is not continuous and the theory of the LQR is meant to be applied in linear systems, we shall proceed a linearisation of the model. For such, we will the Taylor Series theory, considering points of the reference trajectory, and the reference for control will be that to compensate the weight of the vehicle.

2.2 Optimal Control

We used in this work the theory of Optimal Control, that is, the control is designed in order to extremize one (or more) parameters of the system. This parameter can be related to fuel consumption, time of flight, among others. In this case we have chosen minimizing the difference between a reference trajectory and the real trajectory of the vehicle [3]. The theory used here is that of the Linear Quadratic Regulator.

Linear Quadratic Regulator theory was developed to be used in linear systems:

$$\underline{\dot{x}} = A(t)\underline{x}(t) + B(t)\underline{u}(t) \tag{2}$$

Table 1. Values of the acceleration engine thrust and mass *versus* time

t (s)	$F_E(N)$	$m_a\ (kg)$	t (s)	$F_E(N)$	$m_a\ (kg)$	t (s)	$F_E(N)$	$m_a\ (kg)$
0	104.52	13.961	1.9	9468.9	4.4143	3.9	540.35	0.21166
0.1	5415.3	14.205	2	9114.1	4.0129	4	437.12	0.14848
0.2	12224	13.804	2.1	8765.8	3.4913	4.1	347.61	0.14255
0.3	11854	13.494	2.2	8409.3	3.0968	4.2	284.9	0.12457
0.4	11844	12.845	2.3	8053.2	2.5793	4.3	230.05	0.074148
0.5	11828	12.081	2.4	7526.4	2.1981	4.4	191.84	0.046017
0.6	11844	11.561	2.6	6614.6	1.7179	4.5	155.69	0.061756
0.7	11833	10.932	2.7	5156.4	1.4407	4.6	127.38	0.046195
0.8	11826	10.289	2.8	3832.2	1.1165	4.7	103.61	0.027513
0.9	11817	9.7389	2.9	3117.9	0.91451	4.8	84.813	0.10397
1	11780	9.2036	3	2645.1	0.77081	4.9	68.251	0.13053
1.1	11760	8.5505	3.1	2266.7	0.67417	5.1	56.552	0.07614
1.2	11730	7.8892	3.2	1932	0.59574	5.2	0.12576	0.050367
1.3	11695	7.2777	3.3	1656.5	0.60684	200	0.066126	0.03553
1.4	11560	6.7276	3.4	1402.9	0.56661			
1.5	11175	6.2955	3.5	1186.2	0.4475			
1.6	10721	5.7696	3.6	989.28	0.3505			
1.7	10284	5.2558	3.7	821.06	0.32356			
1.8	9852	4.8786	3.8	671.52	0.25473			

Table 2. Values of the midcourse engine thrust and mass *versus* time

t (s)	$F_E(N)$	$m_c\ (Kg)$
0	1500	70
90	1500	0
90.1	0	0
200	0	0

Once the parameter to be extremized is chosen, we define a function called cost function, which represents mathematically this parameter:

$$J = \int_{t_0}^{t_f} f(\underline{x}, \underline{u}, t)dt \tag{3}$$

Where:

- J is the cost function
- t_0 and t_f are the initial and final time, respectivelly
- $\underline{x}$ is the state vector
- $\underline{u}$ is the control vector
- t is the time

In this particular case, where the state represents the difference between the real state and the reference, the cost function becomes:

$$J = \int_{t_0}^{t_f} f(\Delta\underline{x}, \Delta\underline{u}, t)dt \tag{4}$$

Where:

Table 3. C_D *versus Mach*

Mach	C_D	Mach	C_D
0	0.9	0.8	0.96
0.1	0.893	1	1.01
0.2	0.876	1.2	1.06
0.4	0.866	1.3	1.11
0.5	0.878	1.4	1.13
0.7	0.91	2.5	1.15

- $\Delta \underline{x} = \underline{x} - \underline{x}_r$
- $\Delta \underline{u} = \underline{u} - \underline{u}_r$
- $\underline{x}_r$ is the reference state
- $\underline{u}_r$ is the reference control

Considering that some states are to be prioritised and the control effort is limited, we use some matrices to weight the control law. The matrix Q_R is related to the weigths of the states to be tracked and the matrix R_R is the one responsible for limitting the control. So, the cost function becomes:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left[\|\underline{x}(t) - \underline{x}_r(t)\|^2_{Q_R(t)} + \|\underline{u}\|^2_{R_R(t)} \right] dt \tag{5}$$

Where Q_R is a positive semidefinite $n \times n$ matrix and $R_R(t)$ is a positive definite $m \times m$ matrix $\forall t \in [t_0, t_f]$.
The gain $K(t)$ of the controller will feed the plant in the form:

$$\underline{u} = u_{ref} - K(t)\Delta \underline{x} \tag{6}$$

2.2.1 Trajectory of Reference

The trajectory to be followed by the vehicle is given by:

$$r_{x_r}(t) = \begin{cases} 315t; & 0 \le t < 17,5 \\ 247t + 1185; & 17,5 \le t \le 90 \end{cases} \tag{7}$$

$$r_{z_r}(t) = \begin{cases} 3000; & 0 \le t \le 10 \\ 50t + 2500; & 10 < t \le 20 \\ 3500; & 20 < t \le 60 \\ -100t + 9500; & 60 < t \le 75 \\ 2000; & 75 < t \le 90 \end{cases} \tag{8}$$

$$V_r(t) = \begin{cases} -5t + 350; & 0 \le t \le 20 \\ 250; & 20 \le t \le 90 \end{cases} \tag{9}$$

$$\gamma_r(t) = \begin{cases} 0; & 0 \le t \le 10 \\ \frac{1}{300}t + \frac{4}{30}; & 12,5 < t \le 20 \\ 0; & 22,5 < t \le 60 \\ -0,4; & 60 < t \le 77,5 \\ 0; & 77,5 < t \le 90 \end{cases} \tag{10}$$

The units are those of the International System. This trajectory is considered to be known (itt's not an objective of this work to determine it) and it will be sampled in intervals of 2.5 seconds.

2.3 Estimation

The estimator chosen to be used here is the Kalman filter, with continuous dynamics and discrete measures.

The second vehicle, $V2$, had its dynamics simulated by:

$$
\begin{aligned}
r_{x_2}(t) &= r_{x_{20}} + v_{x_2}(t - t_0) && (11)\\
r_{z_2}(t) &= 0 && (12)\\
v_{x_2}(t) &= v_{x_{20}} && (13)\\
v_{z_2}(t) &= 0 && (14)
\end{aligned}
$$

For a system which dynamics is represented by:

$$\dot{\underline{x}} = \underline{f}(\underline{x}(t), \underline{u}(t), t) + G\underline{\omega}(t) \tag{15}$$

With measures:

$$\underline{z}_k = \underline{h}_k(\underline{x}, \underline{u}) + \underline{v}_k \tag{16}$$

Where:

- $\underline{f}$ is the function relating the states to their derivatives
- G is the matrix which adds the dynamic uncertainties
- $\underline{\omega} = N(0, Q_F)$
- $\underline{z}_k$ is the vector of measures in instant t_k
- $\underline{h}_k$ is the function relating the measures to the states and control in instant t_k
- $\underline{v}_k = N(0, R_F)$

The equations for the time update are given by [4]:

$$
\begin{aligned}
\dot{\overline{\underline{x}}}(t) &= \underline{f}(\overline{\underline{x}}(t), \underline{u}(t), t) && (17)\\
\dot{\overline{P}}_F(t) &= F(t)\overline{P}_F(t) + \overline{P}_F^T(t)F^T(t) + G(t)Q_F(t)G^T(t) && (18)
\end{aligned}
$$

Where:

- $F(t)$ is the linearisation of the function $\underline{f}(\underline{x}, \underline{u}, t)$
- $\overline{P}_F(t)$ is the state covariance

The disturbance in the plant was considered as being an uncertainty in the air density parameters. The air density model of the filter is:

$$\rho(r_z) = \rho_0 \exp\left(-\frac{r_z}{H_0}\right) \tag{19}$$

Where: $\rho_0 = 2kg/m^3$ and $H_0 = 7000m$
And the equations for measurement update are given by:

$$
\begin{aligned}
K_k &= \overline{P}_{Fk} H_{Fk}^T \left(H_{Fk}\overline{P}_{Fk}H_{Fk}^T + R_{Fk}\right)^{-1} && (20)\\
\hat{P}_{Fk} &= \left(I - K_k H_{Fk}\right)\overline{P}_{Fk} && (21)\\
\hat{x}_k &= \overline{\underline{x}}_k + K_k\left(\underline{z}_k - H_{Fk}\overline{\underline{x}}_k\right) && (22)
\end{aligned}
$$

Where:

- K_k is the Kalman gain
- H_{Fk} is the linearisation of $\underline{h}_k(\underline{x}, \underline{u}, t)$
- I is the Identity matrix
- $(\underline{z}_k - H_{Fk}\overline{\underline{x}}_k)$ is the residual, or the difference between the measure and the time updated value

The matrix R_F is related to the uncertainties of the sensors, and Q_F is to be adjusted to a better performance of the filter.

3. Simulations

The problem was simulated using MATRIXx. The controller was calculated using a pre-defined function of MATRIXx called *regulator*. The input of this function is an object representing a linear system, and the matrices Q_R and R_R.

So, in order to use the function, a linearisation of the system was performed, taking the points of the reference trajectory as the linearisation points. This linearisation was performed every 2.5 seconds. Then, after obtaining the matrices A and B, a linear system was created with the command *system*, taking as input a matrix $C = I_{4\times4}$ and a null matrix D.

The matrices Q_R and R_R had their values adjusted to:

$$Q_R = diag[\ 10 \quad 10 \quad 1 \quad 1 \quad 0 \quad 0\ ] \tag{23}$$

$$R_R = 0.1 \tag{24}$$

As there is no a theory for obtaining the best values for Q_R and R_R, theses values were achieved by trying some values, and choosing the best ones to use.

It's important to notice that, as the maximum acceleration supported by the actuators is $4g$, the control was limited to this value.

The initial conditions of the problem were given by:

- Initial state: $\underline{x} = \begin{bmatrix} r_{x_10} & r_{z_10} & V_{10} & \gamma_0 & r_{x_20} & v_{x_20} \end{bmatrix}^T = \begin{bmatrix} 0 & 2500 & 250 & 0 & 35000 & 15 \end{bmatrix}^T$; in S.I. units;
- Initial error of the estimate: $\Delta\underline{x} = \begin{bmatrix} 9.8 & 14 & 1{,}2 & -0{,}0015 & 39 & 13 \end{bmatrix}^T$, in S.I. units;
- Initial value of the covariance matrix: $P_{F0} = diag \begin{bmatrix} 30^2 & 30^2 & 3^2 & 0{,}03^2 & 30^2 & 3^2 \end{bmatrix}$, in S.I. units;
- Value of the proccess covariance: $Q_F = diag \begin{bmatrix} 0 & 0 & 3^2 & (5e-2)^2 & 0 & 3^2 \end{bmatrix}$, in S.I. units;
- Value of the noise covariance: $R_F = diag \begin{bmatrix} 30^2 & 30^2 & 3^2 & 3^2 & (2e-2)^2 \end{bmatrix}$, in S.I. units;

4. Results

The Kalman gains of the controller can be verified in Fig. 2.

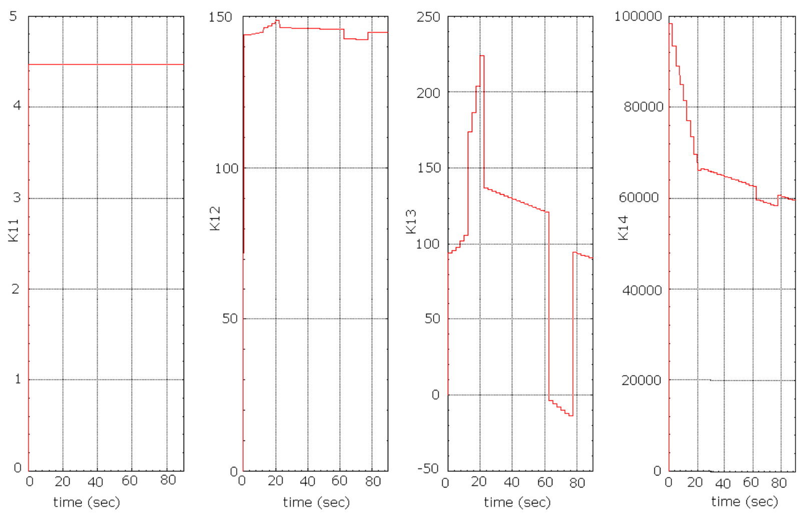

Figure 2. Kalman gains of the controller

As we see, the Kalman gains related to position are pratically constant. This fact can be considered while embedding the controller, for it can be considered to be constant, and the onboard processor will not need to compute this gains. The gains related to velocity and angle, however, have a significant variation and shall not be considerd constant in the controller.

The residuals of the measures are shown in Fig. 3

The errors of the estimatives are shown in Figs. 4 and 5.

The tracking is shown in Fig. 6.

As we can see, even if the vehicle is launched from a position and also with a velocity different from the reference one, the controller makes it possible to track the trajectory in a short period of time.

The control is shown in Fig. 7.

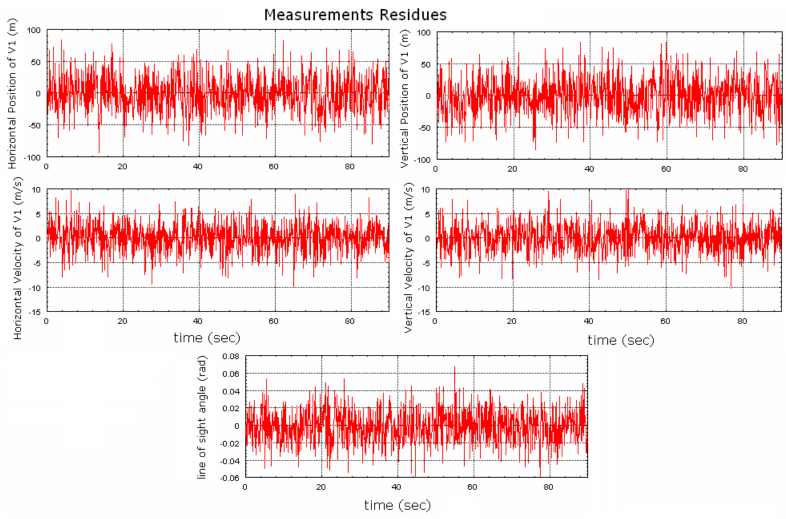

Figure 3. Residuals of the measures

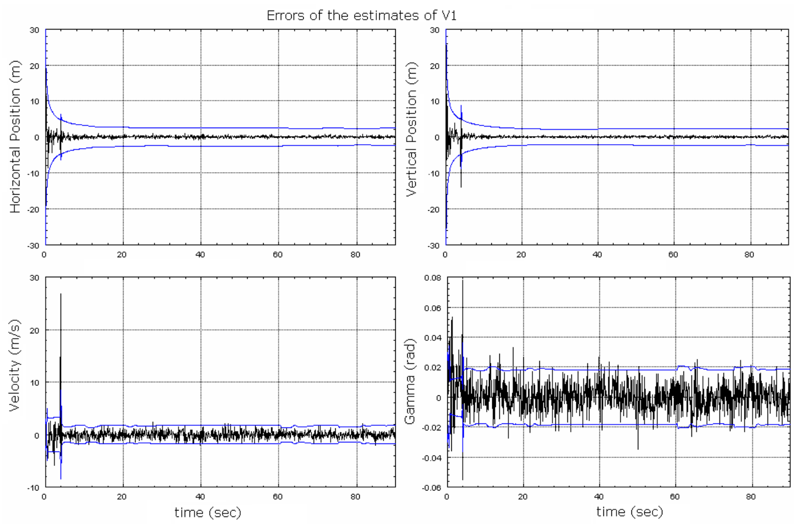

Figure 4. Errors of the estimatives of V1

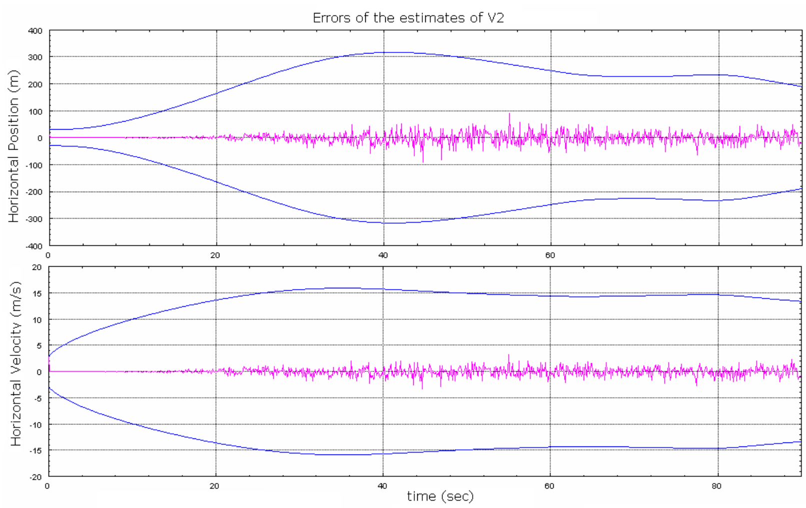

Figure 5. Errors of the estimatives of V2

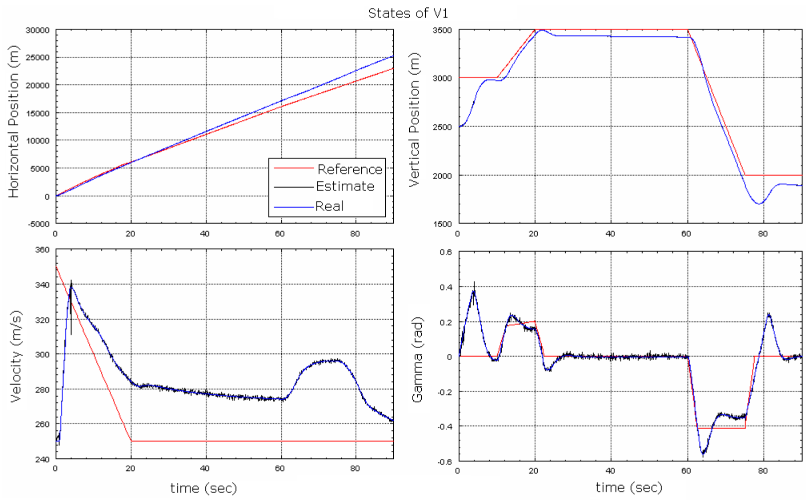

Figure 6. Trajectories of V1

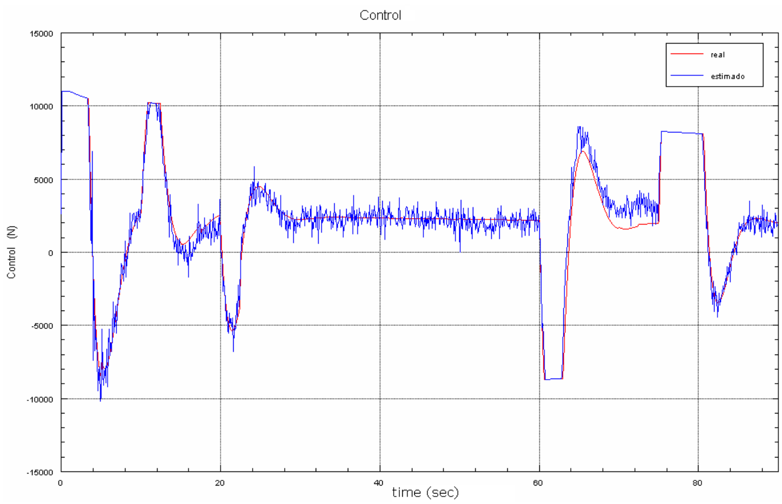

Figure 7. Control

5. Conclusions

As we can see in the graphics shown in last section, in despite of the uncertainties in the model of the vehicle V1, and the noisy sensors, the Kalman Filter was able to estimate the states of V1 with a small error. So, the controller was able to track the reference trajectory, even considering the uncertainties. But, as we can see in Fig. 5, the filter wasn't able to estimate correctly the states of V2. As we desired the acceleration not to be greater than $4g$, the control was limited to this value, but it didn't influence the tracking significantly. So, we conclude that the controller was robust enough to track the trajectory, in spite of the uncertainties.

6. Acknowledgments

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7. References

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