Radial Turbo-compressor Modeling

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Abstract In order to utilize exhaust gas energy effectively, various engine systems equipped with turbochargers have been proposed based in matching techniques. The matching between internal combustion engines and turbochargers depends on the previous knowledge of flow maps of turbine and compressor. This work presents a radial turbocharger model based on modified Euler equation. This equation was obtained through mass, energy and angular moment balances for compressor and turbine, considering an ideal gas model with polytropics compression and expansion and using thermodynamics properties of stagnation. The Euler equation allows determination of the operation points of the ensemble engine-turbocharger through the knowledge of the thermodynamics properties of exhaustion gases and geometrical characteristics of turbocharger. It still allows the attainment of flow maps of compressors and turbines. The model was validated with experimental flow map of Garrett T2 turbocompressor that equips the FIAT Uno Turbo 1.4 i.e. Relative average error of the model for compression ratio, in relation to the experimental data, was of 4,16 %. The turbines T2 had presented relative average error for the expansion ratio of 2,10 %.

Keywords: Flow Maps, Turbocharger, Euler Equation, Internal Combustion Engine, Flux Test Stand.

1. Introduction

In order to utilize the exhaust gas energy in an efficient manner and increase the global engine efficiency, supercharging systems, especially turbochargers, have been proposed. Its known that transferred work to the cylinder, on each cycle, that controls the total power that the engine can provide, depends on how much fuel each cylinder can burn. The quantity of fuel even depends on how much air is introduced (Heywood, 1988). The purpose of a supercharging system is to enlarge the charge density that will be admitted in the cylinder, no matter if the charge is air or air-fuel mixture (Heisler, 1995). This increase on the air density, allows that more air and fuel are introduced in the cylinder and can be efficiently burned, so that the outlet power enlarge significantly when compared to the one obtained in natural atmosphere conditions.

2. Bibliographic Review

Kanamaru et al. (1994) show a turbocharger optimization model to piston engines. The optimization method used was the ascending steps with restrictions. The engine and the compressor are zero-dimensional. The engine is treated in a parametric form by the volumetric efficiency and the displaced volume as a rotation function. The turbine and the compressor are treated like a flow through orifices and the entail between them is established by the same rotation speed and balance equations. The turbine shows a relation between a transversal section area and the expansion ratio, with independence between these two parameters. This proposed model allows, at first, the calculation of flow maps for turbochargers presenting a great adjustment with the experimental data, as shown on Fig. 1.

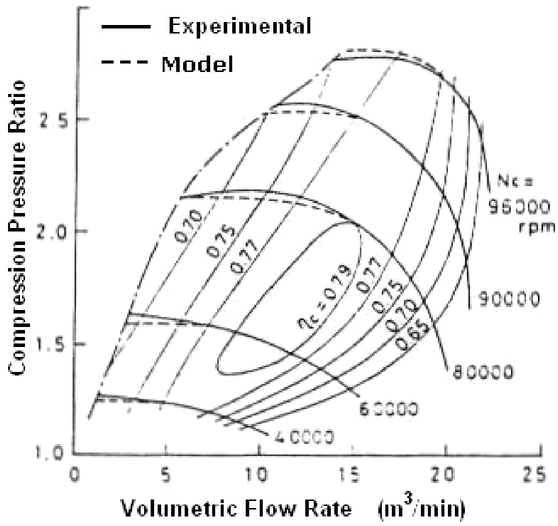

Figure 1- Compressor flow map proposed by Kanamaru et al. (1994)
in relation to the experimental data.

3. Objectives

The objective of the present work is to develop a semi-empiric mathematical model to turbo machinery, based on the Euler equation and the thermodynamic relations to ideal gases that permit obtain radials turbines and compressors flow maps with less experiments. The model will take into consideration the turbine and compressor geometrics characteristics and the exit engine conditions. This model will be also a software base that will be able to obtain centrifugal turbocharger flow maps, parting from the geometric characteristics and the necessary thermodynamic parameters.

4. Methodology

The compressor model is one-dimensional model developed based on the Euler equation and on the thermodynamic relations using stagnation properties to compressive flow. The polytropic efficiency was defined by Oates (1988), according to the equation:

$$e_{pc} = \lim_{\delta h \to 0}\left(\frac{\delta h_s}{\delta h}\right) = \frac{dh_s}{dh} = \frac{\upsilon\, dp}{C_p\, dT} \tag{1}$$

where h is the specific enthalpy, c_p is the specific heat at constant pressure, T is the temperature, p is the pressure, υ is the specific volume and the subscript “s” indicates a isentropic process.

The Euler equation on the stagnation enthalpy terms is given by the following equation:

$$h_{0_2} - h_{0_1} = \dot{m}\,(U_2\, V_{t2} - U_1\, V_{t1}) \tag{2}$$

In the Equation (2) U is the tangential velocity of the rotor, V_t is the flow tangential velocity and $\dot{m}$ is the mass flow rate (kg/s). The subscripts “0”, “1” and “2” mention to the stagnation conditions and to the sections of entrance and exit of the compressor, respectively.

The development of the semi-empiric model for the compressor starts basically on the Eq. (2) and considers the flow stagnation thermodynamic properties at the compressor entrance section, that is, P_{01} and T_{01} are considered before passing through the rotor. The continuity equation provides the mass flow rate on a transversal section considered starting from the α_1 angle projection, formed by the rotor absolute and tangential velocity, or the β_1 angle, formed by the rotor tangential and relative velocity, both angles projected on the radial direction. The β_1 angle is an entry parameter on the model, like area A_1, that is, the area comprehended between the smaller rotor radius and the shaft radius (crown area). The Figure 2 shows the velocity triangle on the rotor entrance.

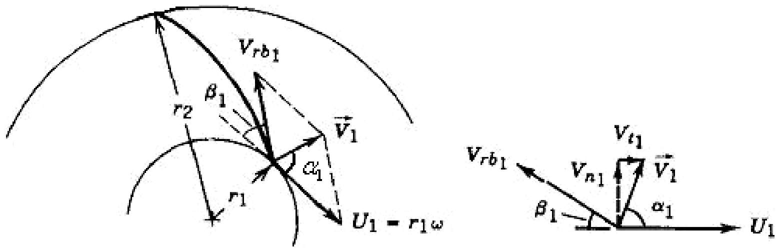

Figure 2 –Velocity triangle at the rotor entrance.

The Equation 3 is the result of the model obtained.

$$\pi_{CT} = \left\{ 1 + \frac{2\left[\left[\left((\omega r_2)\,(\omega r_2 - \frac{\dot{m}}{\rho_2 A_2}\cot g(\beta_2)\right)\varepsilon - \left((\omega r_1)\,(\omega r_1 - \frac{\dot{m}}{\rho_1 A_1}\cot g(\beta_1)\right)\right]\right](\gamma-1)\,(p_1^2 A_1^2\,\gamma)\left(\sin\beta_1\right)^2}{\dot{m}^2 \left(R T_1\right)^2 \gamma\,(\gamma-1) + 2(p_1^2 A_1^2\,\gamma^2 R T_1)\left(\sin\beta_1\right)^2} \right\}^{\frac{\gamma\, e_{pc}}{\gamma-1}} \tag{3}$$

where π is the compression ratio, ω is the angular velocity of the rotor (rad/s), r is the radius, ρ is the density, A is the area, γ is the specific heat ratio, R is the gas constant, ε is the slip factor, β is the blade angle and "e_{pc}" is the polytropic efficiency. The subscript "$_T$" refers to the total (or stagnation) compression ratio. The too many parameters already had been defined to the long one of the text.

The Table 1 summarizes the main geometric characteristics of the T2 compressor used for the model validation.

Table 1 – Important geometric parameters of the T2 compressor.

Compressor	$\mathbf{r_1}$ [m]	$\mathbf{r_2}$ [m]	**Z**	β_1	β_2	ε	$\mathbf{A_1}$ [m^2]	$\mathbf{A_2}$ [m^2]
T 2	0.017	0.024	12	41º	35º	0.865	8.523 x 10^{-4}	7.090 x 10^{-4}

The slip factor ε, on equation 3, is an empiric factor given by the relation proposed by Stanitz (1952) cited by Japikse and Baines (1994) given by the equation:

$$\varepsilon = 1 - 1.98\,\cos\beta_{2b}/Z \tag{4}$$

where Z is the number of blades and β_{2b} is the blade angle in the section of exit of the compressor.

This factor quantifies, in velocity terms, the difference between the blade angle and the flow angle at the exit of a radial section, therefore the flow does not follow exactly the section blade.

4.1. Determination of the Compressor Operational Limits

The model is based on a compressive flow through holes. Furthermore, it becomes necessary to verify if the flow does not reach the sonic limit at the smaller area considered and, to set up a criterion at the surge region, in case of a compressor.

4.1.1. Choking Criterion for the Compressor (*Stonewall* Limit)

The choking criterion is based on the compressive flow relations. In the compressive flow theory, the mass flow rate is related to the throat area, A*, in a nozzle or diffuser by the following relation (White, 2002):

$$\dot{m}_{\max} = \rho^* A^* V^* = \rho_0\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} A^*\left(\frac{2\gamma}{\gamma+1}RT_0\right)^{1/2} \tag{5}$$

One observation related to the Eq. (5) is that the maximum calculated mass flow rate is based on the isentropic flow and hence is an ideal flow. The introduction of a discharge coefficient becomes necessary. The value of C_d used to calculate the maximum mass flow rate in a determinate section was a value stipulated as 0.9, usually found in the literature to convergent nozzles (Zucrow, 1976).

Expressing the mass flow rate as a compression ratio function and deriving the expression obtained in relation to the radius verify the choking on the rotor. The mass flow rate expression as a compression ratio function is given by:

$$\dot{m} = \frac{p_1}{R T_1} A_1 \sin(\beta_1) \sqrt{\gamma R T_1} \left\{ \frac{2\left[\left((\omega r_2)(\omega r_2 - \frac{\dot{m}}{\rho_2 A_2} \cot g(\beta_2)\right)\varepsilon - \left((\omega r_1)(\omega r_1 - \frac{\dot{m}}{\rho_1 A_1} \cot g(\beta_1)\right)\right]}{(\pi_c^{\frac{\gamma-1}{\gamma e_c}} - 1)(\gamma R T_1)} - \frac{2}{\gamma - 1} \right\}^{\frac{1}{2}} \quad (6)$$

Nevertheless the derivation of the Eq. (6) is a non-linear derivate, because it will be shown that the efficiency is a polynomial function of the mass flow rate and analytically the derivate is too much complicated to solve. The rotor choking verification was done by a graph method, by fixing the smaller radius on the established value (0.00925m) and varying the rotor exit radius to many different compression ratios adopted. The geometric parameters adopted were the ones adopted to the T2 compressor, except for the exit area that was calculated in function of the exit radius considered. The polytropic efficiency in the equation 6 was considered constant with a 70 % value. The thermodynamic properties at the entrance section, specifically temperature and pressure, were fixed at 288.15K and 101325 Pa. The γ value adopted was 1,4 and C_p value was 1004.465 J/(kg K). The rotation also was fixed at 180000 rpm.

The Figure 3 illustrates the mass flow rate graphic as a compressor rotor exit radius function, for many compression ratios.

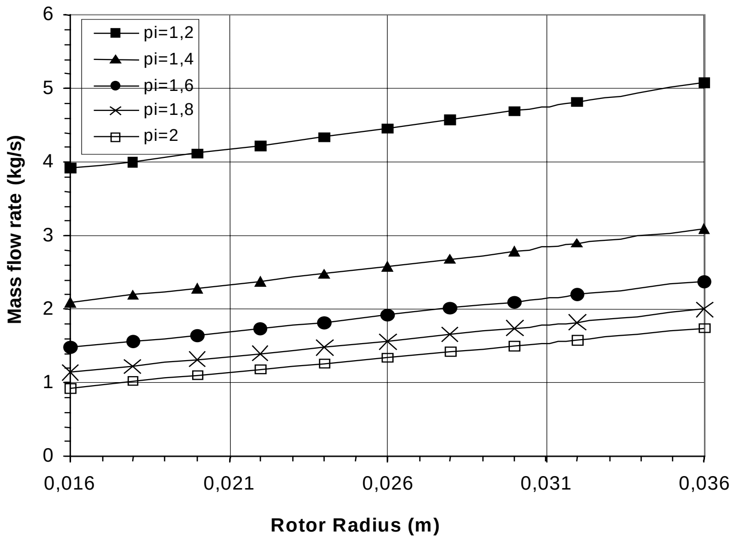

Figure 3 - Verification of the compressor rotor choking as function of exit radius and the compression ratio.

This graphic method allows detecting if the mass flow rate passes through an inflection point, where the presence of a maximum point at the mass flow rate curve as a radius function means that the rotor choking occurs.

4.1.2. Surge Criterion for the Compressor

The surge criterion was defined according to Rodrigues (1991) referencing the maximum point of the compression ratio or the maximum energy for each mass unit ("head") in function of mass flow rate. The criterion proposed considers the mass flow rate relative to the beginning of the surge, correspondent to a compressor relation lower than 1.5% of the maximum compression obtained. The Figure (4) illustrates graphically the surge criterion.

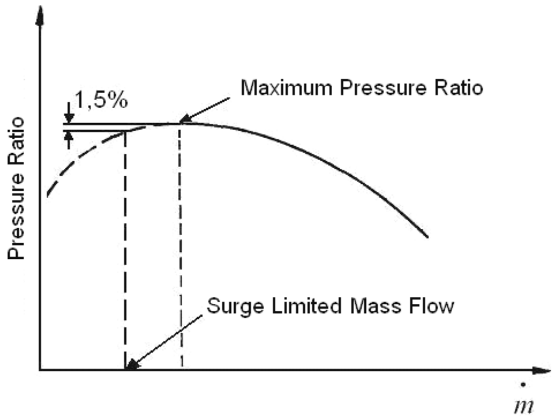

Figure 4 – Definition of the surge criterion for the compressor.

4.2. Determination of the Compression Efficiency

The polytropic efficiency was adjusted in function of the called non-dimensional flow coefficient defined by:

$$\phi = \frac{\dot{m}}{\rho\ N\ D^3} \tag{7}$$

The equation of the adjustment for the T2 compressor is given by the following relation:

$$e_{c\ T2} = -18478\ \phi^2 + 1383{,}3\phi + 52{,}429 \tag{8}$$

The determination coefficient R2 for the efficiency adjustment as an flow coefficient function was 0.9212 for the T2 compressor. Figure (5) illustrates the obtained adjustment.

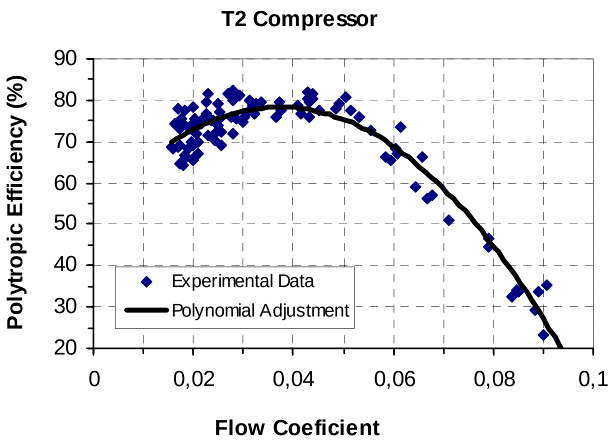

Figure 5 – Adjustment of the polytropic efficiency for the compressor T2.

4.3. Methodology for the Turbine

The turbine semi-empiric model was developed based on Euler equation and on the thermodynamic relations using stagnation properties to compressive flow. The polytropic efficiency to the turbine by Oates (1988) is so defined:

$$e_{pt} = \lim_{\delta h_s \to 0}\left(\frac{\delta\ h}{\delta\ h_s}\right) = \frac{dh}{dh_s} = \frac{C_p dT}{\upsilon\ dp} \tag{9}$$

The Euler equation, based on stagnation enthalpies is given by:

$$h_{04} - h_{03} = \dot{\mathrm{m}}\ (U_4\ V_{t_4} - U_3\ V_{t3}\ \varepsilon) \tag{10}$$

Parting from the Equation (10), the Euler equation in terms of the stagnation enthalpies can be rewrite as a function of the total temperature ratio (or stagnation) according to the equation:

$$\left(\frac{T_{04}}{T_{03}}\right) = 1 + \frac{(\omega r_4)\, V_{t4} - (\omega r_3)\, V_{t3}\ \varepsilon}{C_p T_{03}} \tag{11}$$

Where the subscripts 3 e 4 refers to the sections of entrance and exit of the turbine, respectively.

The semi-empiric model for the turbine was developed parting from the equation 11 considering the trigonometric relations of the entrance and exit velocity triangles.

The final equation for the turbine model that is given by:

$$\pi_{tT} = \left\{ 1 + \frac{2\left[\left((\omega r_4)\,(\omega r_4 - \frac{\dot{m}}{\rho_4 A_4}\cot g(\beta_4)\right) - \left((\omega r_3)(\omega r_3 + \frac{\dot{m}}{\rho_3 A_3}\cot g(\beta_3)\right)\ \varepsilon\right]\,(\gamma-1)\ p_3^2\ A_3^2\ \left(\sin\beta_3\right)^2}{\left(\dot{m}^2\ (RT_3)^2\ (\gamma-1) + 2\,\gamma\, RT_3\ p_3^2\ A_3^2 (\sin\beta_3)^2\right)} \right\}^{\frac{\gamma}{(\gamma-1)\ e_{pt}}} \tag{12}$$

The geometric characteristics of the T2 turbine are resumed at the Tab 2.

Table 2 - Geometric parameters of the T2 turbine.

Turbine	**r_3** [m]	**r_4** [m]	**Z**	**β_3**	**β_4**	**r_{eixo}** [m]	**A_3** [m^2]	**A_4** [m^2]
T 2	0.022	0.018	11	90º	38º	0.004	9.676 x 10^{-4}	9.680 x 10^{-4}

4.3.1. Turbine Choking Criterion

The turbine choking criterion is determined in a similar way that the compressor, using the relations for compressible flow. The rotor choking was verified graphically expressing the mass flow rate as an expansion ratio function at the Eq. (12) and deriving the obtained expression in relation to the radius. Graphically the choking wasn't detected through the turbine rotor.

4.3.2. Expansion Efficiency Determination

The efficiency for the T2 turbine was adjusted in function of the flow coefficient as shows the Fig. 6.

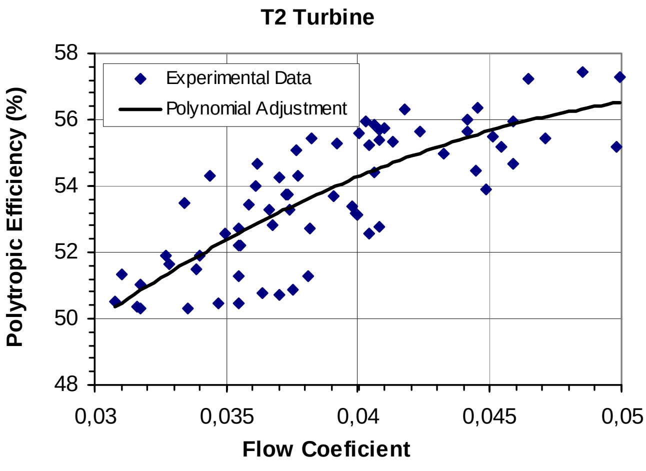

Figure 6 - Polytropic efficiency adjustment for the turbine T2.

The following equation represents the adjustment obtained for the T2 turbine:

$$e_{t\ T2} = -10481\,\phi^2 + 1167{,}9\,\phi + 24{,}333 \qquad (13)$$

The determination coefficient R^2 obtained was 0.6414 for the T2 turbine.

4.4. Methodology to Obtain the Flow Maps

Parting from the entrance thermodynamic conditions (temperature and pressure) we determine the correct mass flow rate and the correct rotation (in rpm) using the following relations:

$$N_{Corrected} = N\left(\frac{T_{ref}}{T_{in}}\right)^{1/2} \qquad (14)$$

$$\dot{m}_{Corrected} = \frac{\dot{m}\left(\frac{T_{in}}{T_{ref}}\right)^{1/2}}{\left(\frac{p_{in}}{p_{ref}}\right)} \qquad (15)$$

where T_{ref} and P_{ref} are the atmosphere temperature and pressure at the ISA aeronautic standard, that is, T_{ref} = 288,15 K e P_{ref} = 101325 Pa.

The attainment of the rotation isocurves is made by fixing a certain rotation and using many mass flow rate values provided by the engine, the static thermodynamic properties at the entrance and at the exit and the other parameters, we obtain the compression ratios values corresponded to the mass flow rate for that considered rotation, using the Eq. (3) and (12). The rotations and mass flow rate considered are the correct ones.

The attainment of efficiency isocurves starts from the Eq. (8) and (13) analyses. One efficiency value inside the operational range of the compressor is fixed and the flow coefficients (equation roots) are calculated. Parting from the definition of the flow coefficient given by the Eq. (7), fixes a determinate correct mass flow rate and calculate the two correct rotations that satisfy the efficiency equation Parting from the calculated rotations considering as roots the flow coefficients in the adjustment efficiency equation, for a certain correct efficiency and mass flow rate, we use the Eq. (3) and (12) to calculate the compression and expansion ratios correspondents to the considered efficiency and the stipulated mass flow rate. The procedure is repeated for many values of polytropic efficiency, calculating new flow coefficients as roots of the adjustment efficiency equation and fixing the corrects mass flow rate necessaries for the attainment of the correspondent correct rotations.

5. Results

The table 3 resumes the obtained errors with the semi-empiric model compared with the experimental data obtained by Rodrigues Filho (2003).

Table 3 - Medium and maximum relative errors for the T2 compressor and turbine.

Turbomachinery	**Relative Error (%)**			
	Compression / Expansion Ratio		**Temperature Ratio**	
	Medium	Maximum	Medium	Maximum
Compressor T2	4.16	7.44	1.44	4.11
Turbine T2	2.10	5.31	0.98	2.01

The flow-maps of the T2 compressor and turbine are shown on Fig. (7) and (8), respectively. This maps presents errors relatively small when compared with the flow-maps obtained by Rodrigues Filho, 2003, using the minimum square method.

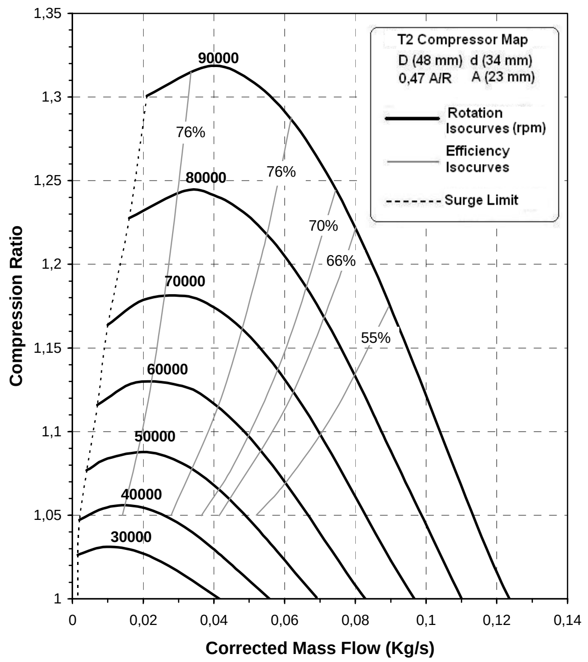

Figure 7 - Flow map of the compressor T2.

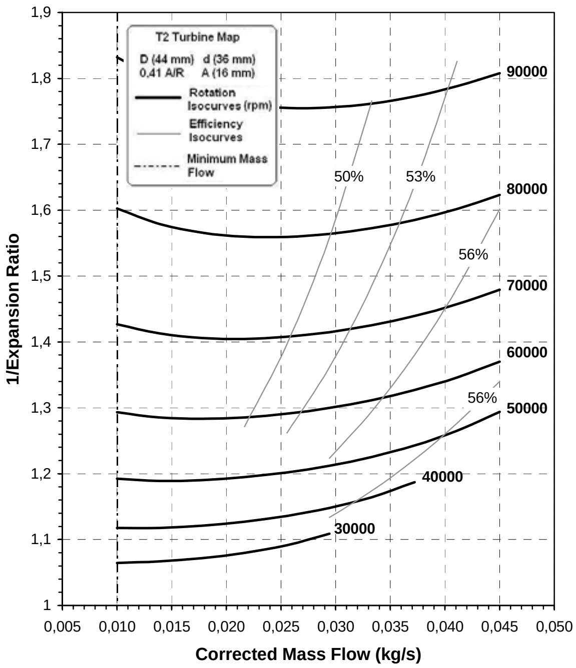

Figure 8 - Flow map of the turbine T2.

6. Conclusions

The little existing models in the literature for the attainment of flow maps are almost all numeric and highly empirics. The semi-empiric model for compressors and turbines show that is able to plot the flow maps analytically parting from the geometric characteristics and the polytropic efficiency with maximum errors around 7 %, with reduced use of empiric correction coefficients. The model further permit foresee the values of mass flow rate relatives to the surge region and to the sonic limit for the compressors and turbines. The model also permits the attainment of temperature ratios for compressors and turbines with a small error in around 4 %.

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8. Thanks

This work was realized with the support of the Conselho Nacional de Desenvolvimento Cientifico e Tecnológico CNPq – Brasil.

9. Responsibility notice

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