

THE EFFECTS OF UNBALANCE AND CLEARANCE ON THE BEARINGS OF AN OVERHUNG ROTOR

Thiago Gambôa Ritto
Companhia Siderúrgica de Tubarão.
Av. Brigadeiro Eduardo Gomes, 930, Serra, ES, 29163-970, Brasil
tritto@cst.com.br

Rubens Sampaio
Department of Mechanical Engineer, Pontifćia Universidade Católica.
Rua Marquês de São Vicente, 225, Rio de Janeiro, RJ, 22453-900, Brasil
rsampaio@mec.puc-rio

Abstract.*This works aims to identify the effects of unbalance and clearance on the bearings of an overhung rotor. Rotating machines are very important on the productive processes and each day the processes demand machines to operate for longer periods, with greater loads and at higher speeds. Bearing is the component that supports all the energy of loads and impacts. The rotor-bearing system is modelled as a continuous system. Finite Element Method is used to discretize the system and Assumed Modes is used to reduce the system. The forces acting on the bearings of an overhung rotor and the Fast Fourier Transform (FFT) of the transient response are computed. Numerical results are qualitatively compared with real cases of machines showing good agreement.*
Keywords: *overhung rotor, reduced model, unbalance, clearance*

1. Introduction

Rotating machines are extensively used in diverse industries, such as electrical power, steel production, petroleum and paper. Nowadays, the processes demand machines to operate for longer periods, with greater loads and at higher speeds. The dynamic modelling of rotors is essential to the dynamic analysis and control of vibration in systems such as power station, automobile and aircraft.

Unbalance of rotating machinery parts is the most common cause of large vibrations of machines. There are many standards concerning balancing as, for example: ISO 2953 - Mechanical Vibration - Balancing machines - Description and evaluation; ISO 1940-1 - "Mechanical Vibration - Balance quality requirements for rotors in a constant (rigid) state"; ISO 11342 - "Methods and criteria for the mechanical balancing of flexible rotors".

Unbalance, misalignment and clearance/looseness are responsible for almost 90% of rotating machines vibration problems. This work deals with unbalance and clearance. Bearing is the component that stands the effects of these vibrations. The standard ISO 10816 - "Mechanical vibration - Evaluation of machine vibration by measurements on non-rotating parts" - specifies references to vibration levels measured at bearings for a great deal of machines.

Childs (1993) is one of the standard references in rotordynamics modelling and analysis. Rotor mathematical models are complex even in the linear case. Qiu and Rao (2005) uses a different approach, with fuzzy logic, to the problem. The parameters of a rotor bearing system exhibit considerable variation and uncertainties due to manufacturing, assemblage and operation conditions, but in this paper we will consider the problem as deterministic.

The effect of clearance and unbalance on the dynamics of a rotor is investigated in many recent papers. Concerning internal radial roller bearing clearance, Harsha (2005a and 2005b), Sinou and Thouverez (2004) and Tiwari et al. (2000) investigate this problem. All of them consider the nonlinearities caused by hertzian contact and develop lumped parameter system models. Karlberg and Aidanpää (2003 and 2004) and Vakakis and Azeez (1999) consider clearance between bearing and housing and clearance between bearing and shaft, respectively. All of these articles center their discussions around chaotic motion, Poincaré maps, bifurcation diagrams and Lyapunov exponents. In this paper, on the other hand, it is used the frequency domain to characterize some of the problems. It is acknowledge in this paper that the most used tool to identify the causes of rotating machine problems through vibration analysis is the Fast Fourier Transform (FFT).

The great limitation of lumped parameter system models is that there is no scheme to compute the error of the model besides the difficulty to choose the parameters. They certainly help to understand qualitatively the problems, but can not describe the effects quantitatively since there no scheme of approximation. Continuous models do not have this limitations; when disctretizations are made with finite elements the properties that appear are material properties and there is an intrinsic scheme of approximation.

Finite element rotor analysis usually results in large dimensionality problems what turns the transient solution very time-consuming. It might include many insignificant modes because of the widely spread eigenspectrum. Vakakis and Azeez (1999) use Assumed Modes and *Karhunen-Loève Decomposition* (KLD) to reduce the problem. KLD is a relatively new technique in vibration analysis that uses statistics to form a basis to project the dynamics that inherit most of its coherence. It works for both linear and non-linear systems, Wolter and Sampaio (2001).

Clearance, in this work, is defined as clearance between bearing and housing. Harmful clearance happens if there is wear of the parts or manufacture/assembly problems. In many machines, due to shaft thermal expansion, one bearing must have freedom to move in the axial direction, so there should be some clearance between bearing and housing. Bearing is the component that absorbs all the energy from impacts and loads having, as a consequence, its life reduced.

In this paper a continuous model of an unbalanced overhung rotor with clearance is simulated. Springs and dashpots symmetrically distributed on the shaft perimeter will introduce the non-linearities. A initial discretization is made with the Finite Element Method to compute the modes that will be used to project the dynamics and to reduce the continuous model, a technique known as Assumed Modes.

Due to the non-linearity the model is constructed in the time domain using a weak formulation. But, our aim is to calculate the FFT of the responses at some special locations and to use this information to evaluate the state of the machines.

The motivation for this study is to construct a model to correlate and better understand the data observed in many rotating machines in which unbalance and clearance affect the forces and shocks acting on the bearings. The FFT of the time signal captured at the bearings can be very distinct for different clearances and unbalances, and moreover they vary with time. In predictive maintenance the vibration analysis of rotating machines are made on the basis of the FFT that are observed, thus it is vital in a plant to understand how the FFT describes the state of a machine. To understand this problem, a model is construct so the system can be analyzed and its behavior be predicted. Balance quality grade, clearance and rotation are used as parameters.

2. Motivation

Two representative cases are discussed here, a recent and an older, both of them occurred in CST (Companhia Siderúrgica de Tubarão), Espírito Santo, Brasil. These cases serve as motivation and guidance to develop a model to better understand the dynamics of rotor-bearings systems.

Any vibration chart (see Bloch and Geitner - 1983) tells that when there is unbalance, the frequency spectrum presents a high peak at the machine speed frequency (X). In the same way, when there is looseness (due to clearance, for example), the frequency spectrum presents peaks at frequencies multiples of the machine speed frequency (2X, 3X,...).

2.1 Main Blower of Continuous Casting machine - nominal rotation speed 1185 RPM

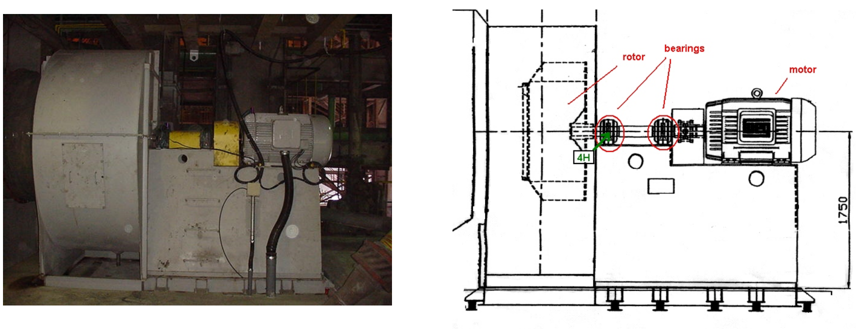


Figure 1. Main Blower of Continuous Casting Machine

The vibration was measured with an accelerometer at the position 4H (horizontal), Fig. 1. The velocity is obtained by integration. Figure 2 shows the frequency spectrum.

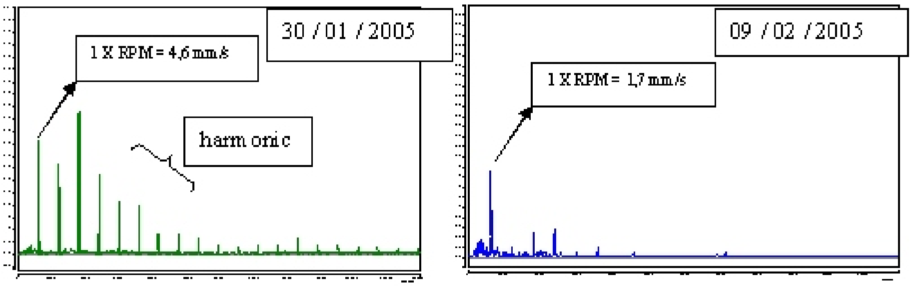
A piece of metal got stuck on a rotor blade at January 30th 2005 thus raising the unbalance. The effect of clearance is register in the frequency spectrum, Fig. 2a. One can see many harmonics (2X, 3X,..).

After the removal of the piece of metal from the blade and the performance of field balancing, the harmonics almost disappeared, Fig. 2b. The residual unbalance accomplished was 1,7 mm/s 0-Peak.

2.2 Dissulfuration Blower - nominal rotation speed 1800 RPM

The vibration was measured with an accelerometer at the position 3H (horizontal), Fig. 3. Figure 4 shows the frequency spectrum.

The clearance between bearing and housing was such that even a small unbalance provoked shocks. Field balancing of the rotor, a costly operation, was required almost every month. Figure 4a shows the frequency spectrum when the unbalance was small - no harmonics are observed. Figure 4b shows the frequency spectrum when there was a bigger unbalance - many harmonics are observed (2X, 3X,..).



(a) (b)

Figure 2. Frequency spectrum of velocity. (a) January 30th 2005, (b) February 9th 2005

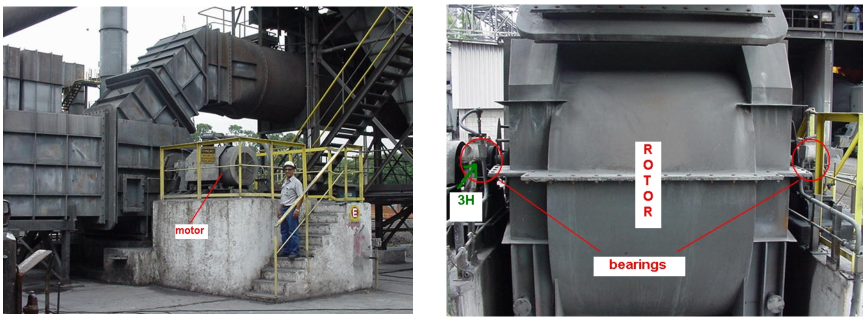
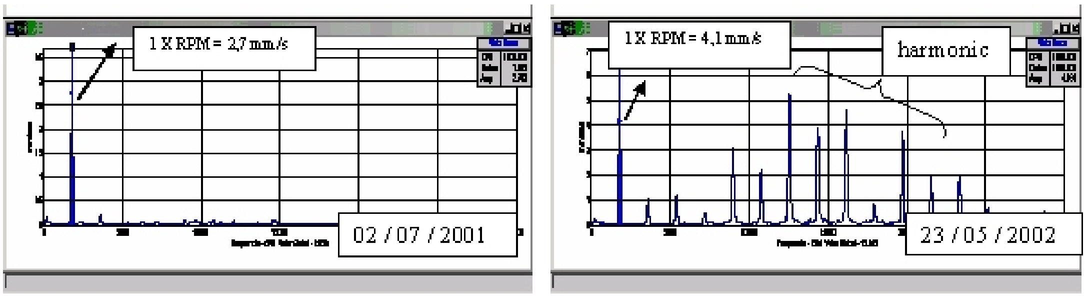


Figure 3. Dissulfuration Blower



(a) (b)

Figure 4. Frequency spectrum of velocity. (a) July 2nd 2001, (b) May 23th 2002

The specification of the bearing had to be changed, diminishing therefore the clearance and solving the problem.

2.3 Unbalance x Bearing Life

To determine the rotor unbalance one uses the balance quality grade (G), an index derived from accumulated practical experience with a large number of different rotors, see ISO 1940/1.

In general the permissible residual unbalance (U_{per} $[Kg \times m]$) for rotors is proportional to the rotor mass ($U_{per} \sim M_{rotor}$). So permissible specific residual unbalance is defined as: $e_{per} = U_{per}/M_{rotor}$. The experience shows that permissible specific residual unbalance varies inversely with the speed of the rotor. ($U_{per} \sim 1/\Omega$). Finally:

$$G = e_{per}\Omega 1000 \;\; [mm/s] \tag{1}$$

Where Ω is the rotation speed in rad/s and e_{per}=$[Kg \times m/Kg]$.

In the numerical analysis performed in the following sections G will be chosen and the unbalance is calculated:

$$U = \frac{GM_{rotor}}{\Omega 1000} \;\; [Kg \times m] \tag{2}$$

Mass unbalance in a rotating system often produces excessive synchronous forces that reduce the life span of various mechanical elements.

For example, consider a machine with balance quality grade $G = 6,3$ and speed of $2000\ RPM$, with spherical roller bearings (SKF 6917). The bearing life can be calculated with or without the extra load due to unbalance.

Considering a load of 1757 Newtons the bearing life will be 21,4 years. The centrifugal force due to unbalance for this case is 132 Newtons. The additional load due to unbalance reduces bearing life by 27% (15,6 years). This calculation is not accounting for excessive clearance and shock. Depending on the machine characteristics and speed this reduction can be smaller or bigger.

3. Formulation

Figure 5 shows the scheme of a continuous rotor bearing system.

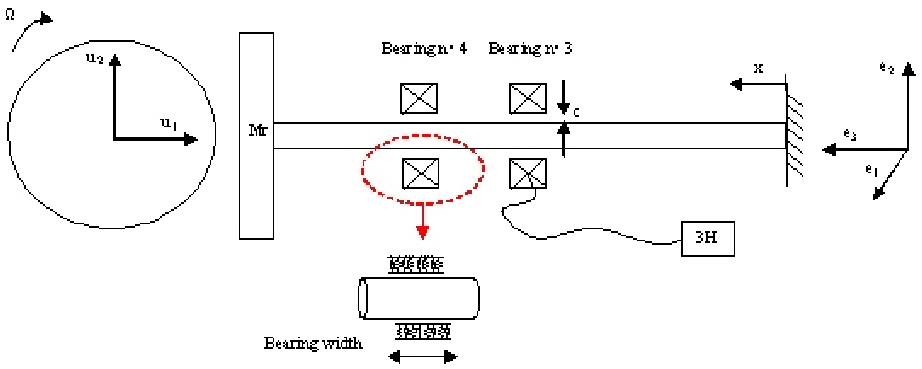


Figure 5. Rotor system considered

The two directions are coupled due to gyroscopic forces. One assumes that rotary inertia has very small effect on the dynamics for the system parameters under consideration.

The Euler-Bernoulli beam model is used. The external forces acting on the system are: centrifugal forces due to unbalance; force of the rotor weight due to gravity; and non-linear forces due to impact.

Unbalance forces ($Q_{1,2}$) and impact forces ($P_{1,2}$) are written:

$$Q_1 = U\Omega^2 cos(\Omega t) \qquad Q_2 = U\Omega^2 sen(\Omega t) - M_r g$$
$$P_1 = -\xi\left(k_h\frac{(r-c)u_1}{r} + d_h\frac{(\dot{u}_1u_1 + \dot{u}_2u_2)u_1}{r^2}\right) \quad P_2 = -\xi\left(k_h\frac{(r-c)u_2}{r} + d_h\frac{(\dot{u}_1u_1 + \dot{u}_2u_2)u_2}{r^2}\right) \tag{3}$$

Where:

$r = \sqrt{u_1^2 + u_2^2}$ $\quad$ k_h and d_h = housing stiffness and damping, respectively $\quad$ U = unbalance
c = clearance $\quad$ $\xi = 1$ if r $\geq c$, and $\xi = 0$ if $r < c$ $\quad$ g = acceleration of gravity

The system is discretized by the Finite Element Method to compute the vibration modes to a prescribed approximation error. The precision specified was 0,0005% of error in the 40th vibration mode. To reach this precision it was necessary 450 elements. The system is reduced by the projection of the dynamics in the vibration modes calculated. The weak formulation of the beam equation accounting for the external forces and the boundary conditions can be written:

$$\begin{aligned}
&\int_0^L \left(EI\frac{\partial^4 u_1}{\partial x^4} + m\frac{\partial^2 u_1}{\partial t^2} + M_r\delta(x-L)\frac{\partial^2 u_1}{\partial t^2} + 2\Omega I_r\frac{\partial^2 u_2}{\partial x\partial t}\delta'(x-L) + d_1\frac{\partial u_1}{\partial t}\right)\phi_j dx \\
&\qquad = \int_0^L (P_1(t)\delta(x-x_{B3})) + \int_0^L (P_1(t)\delta(x-x_{B4}) + Q_1(t)\delta(x-L))\,\phi_j dx \\
&\int_0^L \left(EI\frac{\partial^4 u_2}{\partial x^4} + m\frac{\partial^2 u_2}{\partial t^2} + M_r\delta(x-L)\frac{\partial^2 u_2}{\partial t^2} - 2\Omega I_r\frac{\partial^2 u_1}{\partial x\partial t}\delta'(x-L) + d_2\frac{\partial u_2}{\partial t}\right)\phi_j dx \\
&\qquad = \int_0^L (P_2(t)\delta(x-x_{B3}))\int_0^L (P_2(t)\delta(x-x_{B4}) + Q_2(t)\delta(x-L))\,\phi_j dx
\end{aligned} \tag{4}$$

Where:

$u_1 \rightarrow$ displacement at e_1 direction	$u_2 \rightarrow$ displacement at e_2 direction	$\phi \rightarrow$ vibration modes
$E \rightarrow$ elasticity module	$I \rightarrow$ shaft inertia momentum	$m \rightarrow$ density times section area (ρA
$M_r \rightarrow$ rotor mass	$I_r \rightarrow$ rotor inertia momentum	$L \rightarrow$ shaft length
$\Omega \rightarrow$ rotation speed	d_1 and $d_2 \rightarrow$ damping coefficients	x_{B3} and $x_{B4} \rightarrow$ bearing positions

In the Galerkin method the error of the approximation is orthogonal to the projection space. The trial functions are the vibration modes themselves so this method is called Assumed Modes. The approximation is:

$$u_1 = \sum_{i=1}^{N} a_i \phi_i \quad ; \quad u_2 = \sum_{i=1}^{N} b_i \phi_i \tag{5}$$

Where N is the number of modes used in the approximation. The basis functions ϕ are the eigenfunctions of a clamped beam with a mass at the extremity that was computed by FE.

After integration by parts, using the boundary conditions, and substituting u_1 and u_2 by there approximation one gets the following discrete set of equations:

$$\begin{aligned}
&\ddot{a}_i \int_0^L m\phi_i\phi_j dx + a_i \int_0^L EI\phi_i''\phi_j'' dx + \ddot{a}_i M_r \phi_i(L)\phi_j(L) + \dot{b}_i 2\Omega I_r \phi_i''(L)\phi_j'(L) \\
&\qquad + \dot{a}_i \int_0^L d_1 \phi_i\phi_j dx = P_1(t)\phi_j(x_{B3}) + P_1(t)\phi_j(x_{B4}) + Q_1(t)\phi_j(L) \\
&\ddot{b}_i \int_0^L m\phi_i\phi_j dx + b_i \int_0^L EI\phi_i''\phi_j'' dx + \ddot{b}_i M_r \phi_i(L)\phi_j(L) - \dot{a}_i 2\Omega I_r \phi_i''(L)\phi_j'(L) \\
&\qquad + \dot{b}_i \int_0^L d_2 \phi_i\phi_j dx = P_2(t)\phi_j(x_{B3}) + P_2(t)\phi_j(x_{B4}) + Q_2(t)\phi_j(L)
\end{aligned} \tag{6}$$

The weak formulation is, by Galerkin method, transformed into to a system of ordinary differential equations.

4. Numerical Simulation

The set of ordinary differential equations obtained in the previous section are solved by the Matlab subroutine *ode45*. The parameters used for the standard simulation are:

Shaft length, $L = 3053\ mm$	Shaft diameter, $D_s = 110\ mm$	Rotor mass, $M_r = 600\ Kg$
Elasticity modulus, $E = 193\ GPa$	Density, $\rho = 8000\ Kg/m^3$	Damping, $d_1 = d_2 = 1000\ Ns/m^2$
Balance quality grade, $G = 6\ mm/s$	clearance, $c = 7,5\ \mu m$	Bearing stiffness, $k_h = 1$ GN/m
Bearing 3 position, $x_{B3} = 1,692\ m$	Bearing 4 position, $x_{B4} = 2,302\ m$	Bearing width, $B_w = 58\ mm$
Rotation speed, $\Omega = 124\ rd/s = 1185\ RPM$		

The dimensions and the material properties are the same as the blower of section 2.1, except for system damping and bearing stiffness. The system damping was chosen big on purpose, it is due, unfortunately, to the lack of computer processing availability. The goal here is to have a qualitative investigation on how the forces acting on the bearings and how the FFT curve behave by changing other parameters such as unbalance and clearance.

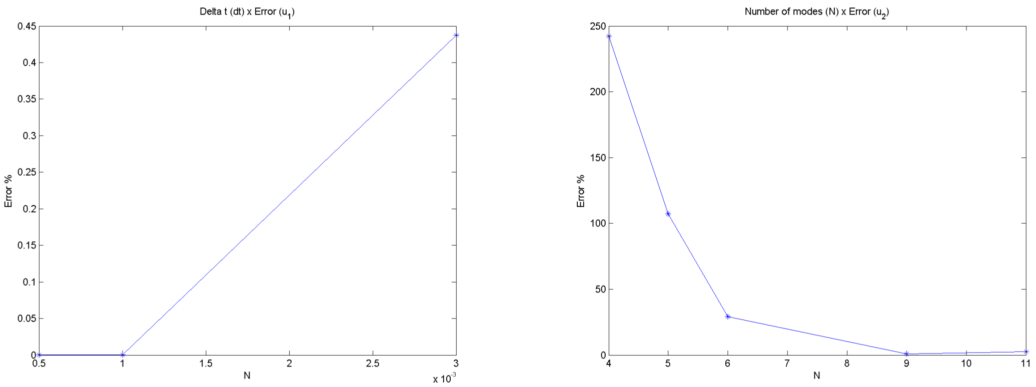


Figure 6. Left: Convergence of the error, varying dt; Right: Convergence of the error, varying N

Rotor mass was included in the simulation as a force acting on u_2 direction at the shaft extremity. The term of inertia due to the rotor mass was not included in the mass matrix because it is too big and it was leading to results far from reality.

In order to select the proper time steps and the number of normal modes used to generate the dynamics, the error of the dynamics were accounted for different time steps and number of modes of the approximation.

Figure 6 shows the percentual error in the dynamics for different number of modes and for different time steps. The error decreases increasing the number of modes and decreasing dt, respectively. The optimum point found in the simulations is: $dt = 0,001$ and $N = 10$, Henceforth, all simulation were done with the parameters fixed in the best values.

Figure 7 shows the shape of the shaft when the machine is in operation. The load zone of bearing number four is at the bottom part of the housing while the load zone of bearing number three is at the top.

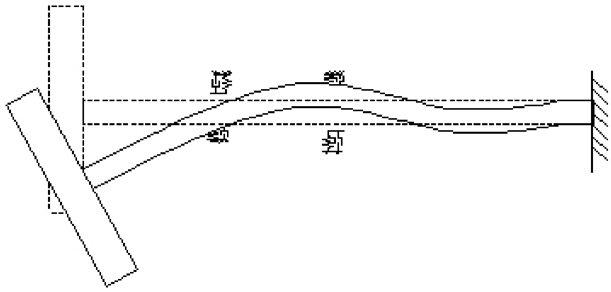


Figure 7. Shape of the shaft under operation condition

Simulations were done for different bearing stiffness. It was noticed that the mean force acting on the bearing number four increases with the stiffness. However, there is a point ($k_h = 10\ GN/m$ in this simulation) beyond which the forces remain constant, for the chosen level of accuracy. The inverse occurs with the mean force acting on the bearing number three: it decreases until a certain point.

Simulations were done varying the bearing damping from 1 to $1000\ Ns/m^2$. There was no difference in the results (errors below 0,01%). The bearing damping has no influence in the dynamics for this model.

For different rotation speeds it was noticed that the mean force acting on the bearings gets higher when the rotation speed increases. The relationship is almost quadratic, what is compatible with the relation $F_c = U\Omega^2$.

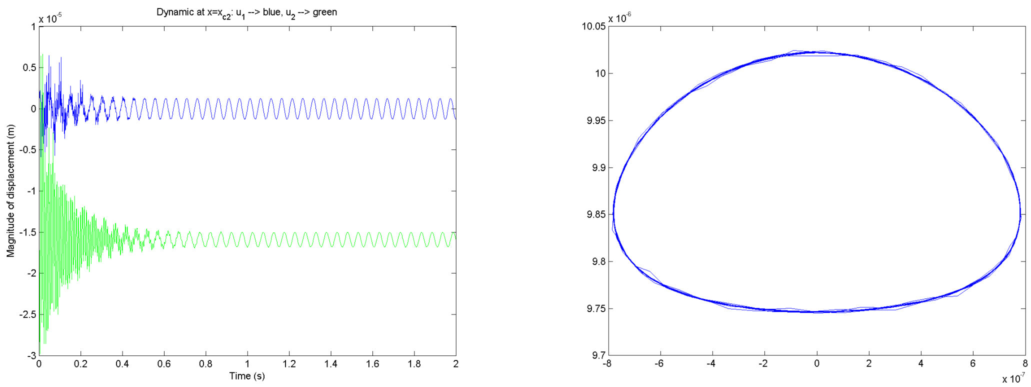


Figure 8. Response at bearing number 3. Left: Dynamic response - u_1 (blue) and u_2 (green); Right: Orbit after the transient

Figure 8 shows the dynamic response and the orbit at bearing number three. The results were saved after the system stabilization.

The focus of our investigation is the influence of unbalance and clearance on the system.

Varying the balance quality grade one can see that the mean force acting on bearing number four increases with the unbalance , Fig 9. The relation is almost linear, what is compatible with the relation $F_c = U\Omega^2$. On the other hand, the mean force acting on bearing number three do not follow this pattern. Figure 9 shows that the mean force acting on bearing number four is much higher than the one acting on bearing number three. This is due to the effect of the rotor mass.

Figure 10 shows two FFT curves for $G = 2$ and $G = 20$ [mm/s]. One can observe multiples of the fundamental frequency (2X, 3X,...) in the frequency spectrum for high levels of unbalance. If the unbalance is small, $G = 2$ for example, one can barely see the harmonics. If the unbalance is big, $G = 20$ for example, the harmonics are evident.

One can conclude that the unbalance influences the state of the system. This is the most important result because it agrees with the observations made in rotating machines (see section 2.1 and 2.2).

The results obtained by measuring the vibration at the bearings of the Main Blower of the Continuous Casting Machine

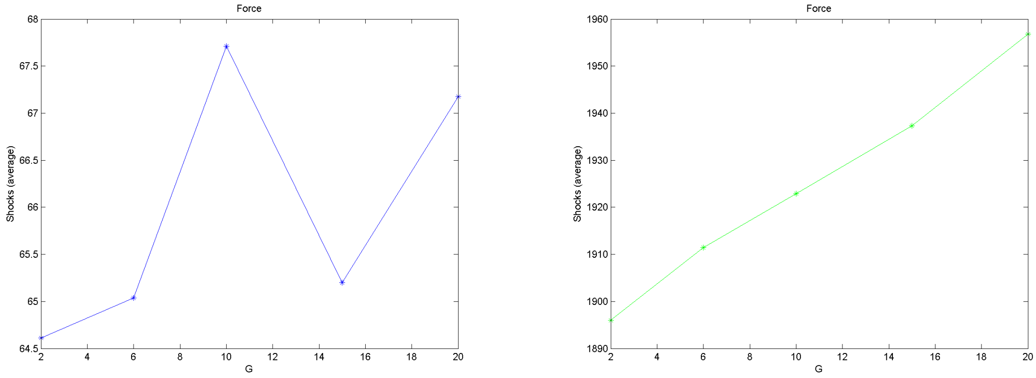


Figure 9. Left: Mean force at bearing number 3 versus G; Right: Mean force at bearing number 4 versus G

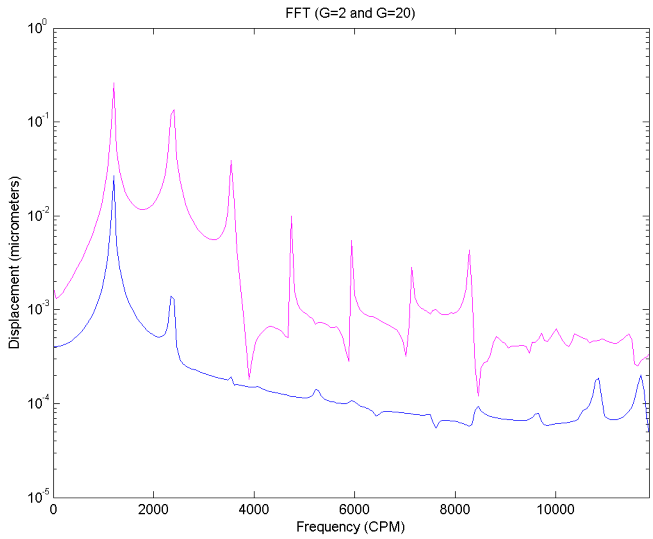


Figure 10. FFT of the displacement at bearing number 3: $G = 2$ (blue) and $G = 20$ (magenta)

(section 2.1) by means of an accelerometer are qualitative the same as the results computed in the simulations, Fig. 10. Another important agreement between numerical and experimental analysis is with respect to the shape of the shaft, Fig. 7. Indeed, the phase lag measured at bearings number three and number four of the main blower is $\sim 180^0$.

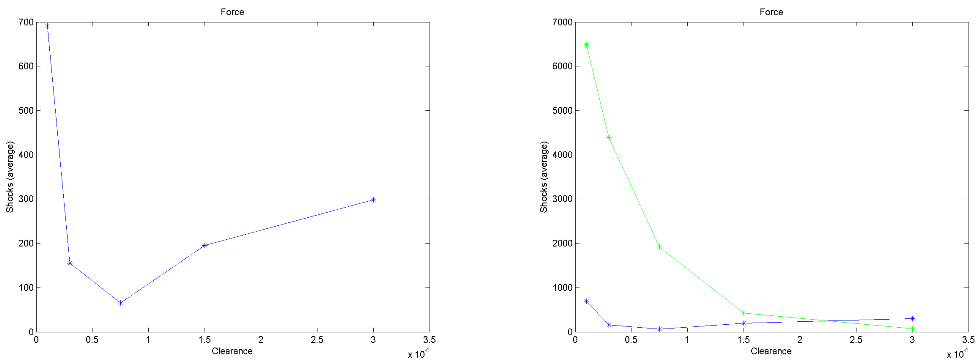


Figure 11. Left: Mean force at bearing 3 versus clearance; Right: Mean force at bearing 3 (blue) and 4 (green) versus clearance

The other important parameter is clearance. Figure 11 shows the mean forces acting on the bearings keeping all parameters the same and changing clearance.

Mean force acting on bearing number four gets smaller when clearance increases. It happens due to the effect of the clamped beam. The shaft elastic force against the shaft motion gets bigger when the clearance increases. However, the mean force at bearing number three presents a curious behavior, it passes through a minimum. This same effect was detected by Sampaio and Soize (2005).

5. Concluding Remarks

A rotor-bearing system model was proposed in this work. Numerical results showed good qualitative agreement with real rotating machine behavior. The model can thus help the understanding of the behavior of a rotor system.

Analyzing the FFT of the time signal computed at the bearings, it was observed that high unbalance of an overhung rotor with clearance cause the appearance of many harmonics in the frequency spectrum. The predictions corroborated the results obtained in the plant. These agreements motivate further studies to get quantitative results.

A study is now been carried out to find a more adequate projecting basis, a Karhunem-Loève Decomposition (KLD), to this model. Instead of projecting the dynamics in the space generated by the normal modes, it will be projected in the space generated by the KL-basis (Proper Orthogonal Modes - POM). Since the problem is strongly non-linear, it seems that to get a better description of the dynamics one should search for a smart base that takes into account the non-linearities (the normal modes do not). KLD aims to describe the observed phenomenon in a reduced dimension and it is capable to capture the most interesting features of the dynamics. A rotor-bearing system is very complex. Several other aspects of the rotor system should be taken into account in future works, such as: a better description of the damping, the influence of lubricant and the control of the dynamics.

6. References

Bloch, H.P. and Geitner, F.K., 1983, "Machine Failure Analysis and Troubleshooting - Practical Management Process Plants - Volume 2", Gulf Plublish Company.

Childs, D., 1993, "Turbomachinery Rotordynamics: Phenomena, Modeling, and Analysis", Wiley-Interscience.

Harsha, S.P., 2005, "Nonlinear Dynamic Analysis of an Unbalanced Rotor Supported by Roller Bearing", Chaos Solutions and Fractals, Vol.26, pp. 47-63.

Harsha, S.P., 2005, "Nonlinear dynamic response of a balanced rotor supported on rolling element bearing", Mechanical System and Signal Processing, Vol.19, pp. 551-578.

Karlberg, M. and Aidanpaa, J.O., 2003, "Numerical Investigation of unbalanced rotor system with bearing clearance", Chaos Solutions and Fractals, Vol.1, pp.653-664.

Qiu, Y. and Rao, S.S., 2005, "A fuzzy approach for the analysis of unbalanced nonlinear rotor systems", Journal of Sounds and Vibration, Vol.284, pp.299-323.

Sampaio, R. and Soize, C., 2005, "On measures of nonlinearities for uncertain dynamical systems", submitted to publication.

Sinou, J.J. and Thouverez, F., 2004, "Non-linear dynamic of rotor-stator system with non-linear bearing clearance", CR Mecanique, Vol.332, pp.743-750.

Tiwari, M., Gupta and K., Prakash, O., 2000, "Dynamic response of an unbalanced rotor supported on ball bearings", Journal of Sounds and Vibration, Vol.238, pp.757-779.

Vakakis, A.F. and Azeez, A.M.F., 1999, "Numerical and experimental analysis of a continuos overhung rotor undergoing vibro-impacts", Non-Linear Mechanics, Vol.34, pp.415-435.

Wolter, C. and Sampaio, R., 2001, "Bases de Karhunen-Loève: Aplicações à Mecânica dos Sólidos" Aplicon 2001, EEUSP São Carlos, 30/julho–03/agosto de 2001 available at http://www.mec.puc-rio.br/prof/rsampaio/rsampaio.html

7. Responsibility notice

The authors are the only responsible for the printed material included in this paper.