UNCERTAINTY ANALYSIS OF A LASER CALIBRATION SYSTEM USING GUM AND AN ALTERNATIVE METHOD

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Abstract. *A laser calibration system for evaluating the positioning accuracy of machine tools and coordinate measuring machines (CMM) under dynamic conditions has been developed. It is based on the Hewlett Packard (nowadays, "Agilent Technologies") 5529A laser interferometer that is capable of performing dynamic calibration. This paper deals with the evaluation of the measurement uncertainty of this laser calibration system using two methodologies. In order to assess the measurement uncertainty of this laser system, an analysis of the uncertainty components that make up the uncertainty budget of this laser system has been carried out. Two methods are used for evaluating the measurement uncertainty of this laser calibration system. The first method is based on GUM ("Guide to the Expression of Uncertainty in Measurement") and the second one is an alternative method. This uncertainty analysis was carried out when this laser calibration system was used to assess the positional errors of a moving bridge type CMM.*
Keywords: *positional error, laser interferometer, uncertainty budget, measurement uncertainty.*

1. INTRODUCTION

A calibration package has been developed for evaluating the positioning accuracy of machine tools and coordinate measuring machines (CMM) under dynamic conditions. The Hewlett Packard (nowadays, "Agilent Technologies") 5529A laser interferometer system has been utilized in this positional error calibrator as it is capable of making dynamic measurements of machine performance. The laser interferometer measures the position of the machine slide as it moves continuously along the axis under test. The data are collected on a position basis by triggering the laser interferometer system from a position-based reference signal. The position trigger signals are obtained directly from the machine encoder. A computational program has been developed using a microcomputer so that "on-the-fly" data acquisition can be made.

This paper presents a study of the individual uncertainties that affect the accuracy and repeatability of this laser calibration system based on two methodologies. The first method follows GUM ("Guide to the Expression of Uncertainty in Measurement") [1] and the second one is an alternative method. The uncertainty budget of this laser calibration system is determined using these two methods. For each uncertainty budget obtained from the above-mentioned methods, the measurement uncertainty of this calibrator is computed. This uncertainty analysis was carried out when this calibrator was applied to a moving bridge type CMM.

2. DESCRIPTION OF THE LASER CALIBRATION SYSTEM

Figure 1 shows the experimental set-up to measure the positional errors of a bridge type CMM dynamically. The measuring system consists of the following components: a HP 5519A laser head, a HP 10565B linear interferometer, a HP 10556A retroreflector, a cube-corner, a HP 10887A laser card, a microcomputer and a printer. The interferometer is placed on the granite table. The cube-corner is mounted to the end of the CMM probe holder. The HP 10556A retroreflector is affixed to the interferometer [2]. The laser interferometer measures the actual position of the probe along the axis. The reference position of the probe is measured by means of the optical grating (encoder of this axis). The A-quad-B pulses produced by such encoder are picked up and sent to the A-quad-B connector of the laser card. The data acquisition program decodes and processes these signals in order to calculate the reference position of the probe along the axis under test. The laser readings are acquired when the encoder counter reaches these target reference positions which were specified by the user via the software [2].

A coordinate measuring machine model G-90C manufactured by LK has been utilized in the tests.

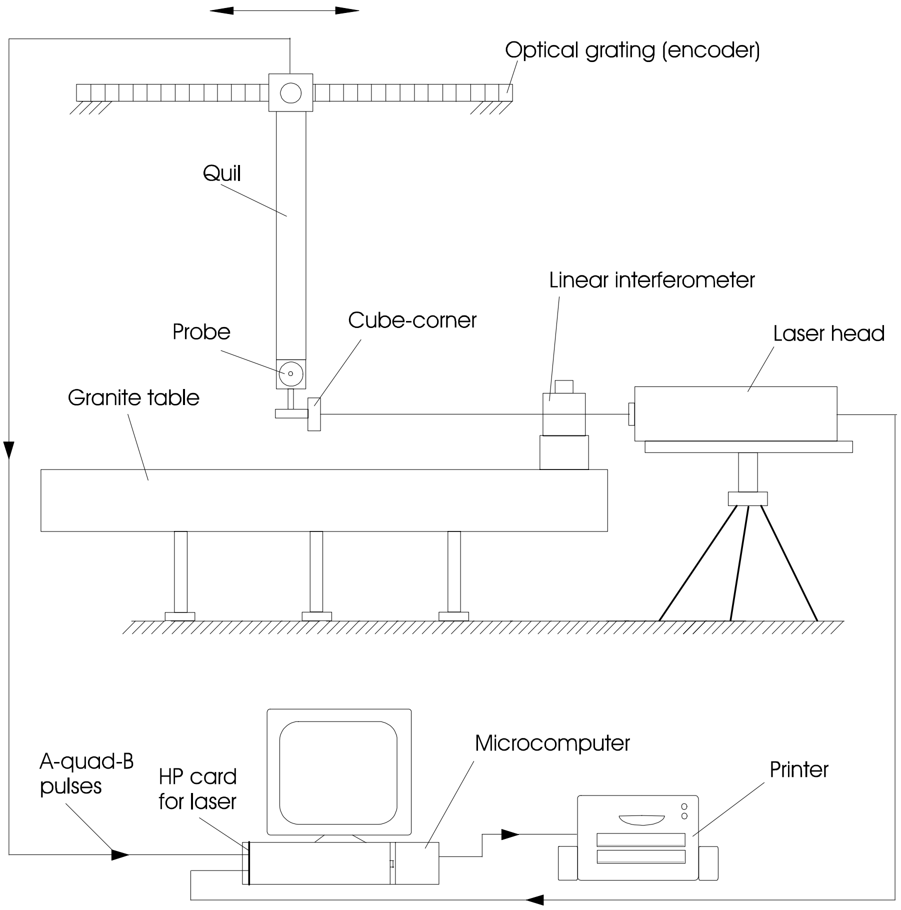

Figure 1. Schematic arrangement for evaluating the positioning accuracy of a CMM [2].

3. EVALUATION OF THE INDIVIDUAL MEASUREMENT UNCERTAINTIES

In order to evaluate the measurement uncertainty of the positional error calibrator, a study of the uncertainty components that make up the uncertainty budget of this calibrator is accomplished. These uncertainty components can be classified into three categories as follows: 1) uncertainties intrinsic to the laser system; 2) uncertainties due to environmental effects; 3) uncertainties due to the installation. These uncertainty components can be divided into proportional and fixed terms.

In this study, the method that follows GUM is designated as *Method I* whilst the alternative method is named *Method II*.

3.1 Uncertainties Intrinsic to the Laser System

Some sources of uncertainty of the laser interferometer system are intrinsic to this equipment and affect the measurement accuracy. They are considered in the following sections.

3.1.1 Timing Errors

A specific problem concerning this technique of evaluating the positional errors is the *timing* *errors*. It results from the time difference between when the laser interferometer records its readings and when the machine controller records the encoder readings. As the laser interferometer and machine controller readings do not happen necessarily at the same time, a measurement error may result. However, in the HP laser system, these errors tend to be negligible.

3.1.2 Laser Wavelength Uncertainty

An interferometer system generates fringes when displacement occurs between the measurement optics of the system. Each fringe generated is equivalent to a fraction of a wavelength of the laser. If the wavelength changes, fringes are generated, thereby giving an apparent distance measurement even without actual displacement. This apparent movement is a measurement error.

The wavelength uncertainty is specified in parts-per-million of the laser frequency. This implies that the wavelength error in the measurement is proportional to the distance measured. Lifetime wavelength uncertainty for the laser heads is ±0.1 ppm standard and ±0.02 ppm with optional calibration to MIL-STD 45662 [3]. In this analysis, the measured distance L is given in metres.

<u>Method I</u>

Assuming a rectangular probability distribution for the uncertainty in the laser wavelength, its standard uncertainty is calculated by

$$u_1 = \frac{L(\mathrm{m})(\pm 0.02 \times 10^{-6})}{\sqrt{3}} = \frac{0.02L}{\sqrt{3}} \ \mu\mathrm{m} \qquad (1)$$

<u>Method II</u>

The laser wavelength uncertainty (LWU) is computed by

$$\mathrm{LWU} = L(\mathrm{m})(\pm 0.02 \times 10^{-6}) = \pm 0.02L \ \mu\mathrm{m} \qquad (2)$$

3.1.3 Electronics Error

Electronics error stems from the method used to extend the basic optical measurement resolution in an interferometer system. The basic resolution in the laser system is wavelength/2 (when using cube-corner optics) and can be electronically or optically extended beyond that. In this research work, a linear interferometer has been employed. The measurement resolution is 1 nm in this case.

<u>Method I</u>

A rectangular distribution for the resolution of the laser interferometer system' s measurement display is assumed. This gives a standard uncertainty of

$$u_2 = \frac{0.001/2}{\sqrt{3}} \ \mu\mathrm{m} \qquad (3)$$

<u>Method II</u>

In the HP system, the electronics error is equal to the uncertainty of the least resolution count. That is, electronic error equals measurement resolution [3].

$$\text{Electronic error} = \pm 0.001 \ \mu\mathrm{m} \qquad (4)$$

3.1.4 Optics Non-linearity

The interferometer can contribute to measurement uncertainty because of its inability to separate perfectly the two laser beam components (vertical and horizontal polarisation). This error is referred to as optics non-linearity and occurs solely as a result of the optical leakage of one component into the other. This error is periodic and relates to a 360° phase shift between the reference and measurement frequencies. For a linear interferometer, the peak-to-peak phase error is 5.4°, corresponding to a distance of ±4.8 nm. Using a statistical model, this value is ±4.2 nm. This non-linearity error is a fixed term and is different for each interferometer [3].

Method I

As this error is periodic (sinusoidal), the "U"-shaped probability distribution is assumed [4]. The standard uncertainty of the optics non-linearity is computed by

$$u_3 = \frac{0.0042}{\sqrt{2}} \ \mu\text{m} \tag{5}$$

Method II

$$\text{Optics non-linearity error} = \pm 0.0042 \ \mu\text{m} \tag{6}$$

3.2 Uncertainties due to Environmental Effects

These uncertainties are related to the influence of the atmospheric conditions on the laser system accuracy and repeatability. The thermal properties of the machine under test and the temperature change of some optics during the measurement also contribute to the measurement uncertainty. The following sections deal with these sources of uncertainty.

3.2.1 Wavelength Compensation

The wavelength of the laser source is usually specified as the vacuum wavelength λ_v. In a vacuum the wavelength is constant, but in an atmosphere the wavelength depends on the index-of-refraction of this atmosphere. Since most laser interferometer systems operate in air, it is necessary to correct for the difference between λ_v and the wavelength in air λ_a. This correction is referred to as *atmospheric or wavelength compensation*. The index-of-refraction n of air is given by

$$n = \frac{\lambda_v}{\lambda_a} \tag{7}$$

The value of n is a function of air temperature, pressure, relative humidity and air composition.

The laser interferometer system counts the number of wavelengths of motion travelled. Therefore, the distance which is measured by the laser system can be determined as follows:

$$\text{Distance (measurement)} = \text{(wavelengths of motion)(WCN)}(\lambda_v) \tag{8}$$

where, WCN is named *Wavelength Compensation Number*. It is the inverse of the index-of-refraction n, that is,

$$\text{WCN} = \frac{\lambda_a}{\lambda_v} = \frac{1}{n} \tag{9}$$

The equation (8) shows that uncertainty in the WCN directly affects the interferometer measurement. The WCN has been computed in the software using the Edlén equation [5] which provides the index-of-refraction as a function of air temperature, relative humidity and barometric pressure. The value of WCN has been employed for compensating for the laser wavelength.

Without wavelength compensation, degradation in system accuracy and repeatability would occur. For example, using Edlén equation [5] and assuming a standard and homogeneous air composition, a 1 ppm error results from any one of the following conditions [3]:

- a 1°C change in air temperature;
- a 2.5 mm of mercury change in air pressure;
- an 80% change in relative humidity.

These atmospheric figures are used for determining the uncertainty of the wavelength compensation. It is calculated considering the instrumentation accuracy employed for measuring temperature, pressure and relative humidity. The thermometer and barometer utilized in the experiments have an uncertainty of ±0.2°C and ±2 mm Hg, respectively. The relative humidity was estimated at an uncertainty of ±20%. Hence, the uncertainties in temperature, pressure and humidity are, respectively, ±0.2 ppm, ±0.8 ppm and ±0.25 ppm.

Method I

Assuming a rectangular distribution, the standard uncertainties in the assessment of the air temperature, air pressure and relative humidity are, respectively, $0.2\ \text{ppm}/\sqrt{3}$, $0.8\ \text{ppm}/\sqrt{3}$ and $0.25\ \text{ppm}/\sqrt{3}$. The uncertainty in the wavelength compensation is a function of the uncertainties in the measuring of the air temperature, air pressure and humidity. Therefore, the standard uncertainty of the wavelength compensation in ppm u_W is calculated by appropriately combining the standard uncertainties of the temperature, pressure and humidity according to GUM [1]. Hence,

$$u_W = \sqrt{(\frac{0.2}{\sqrt{3}})^2 + (\frac{0.8}{\sqrt{3}})^2 + (\frac{0.25}{\sqrt{3}})^2} = \frac{0.862}{\sqrt{3}}\ \text{ppm} \tag{10}$$

Considering a length of L in metres, the standard uncertainty of the wavelength compensation in μm is given by

$$u_4 = u_W L(\text{m}) = \frac{0.862L}{\sqrt{3}}\ \mu\text{m} \tag{11}$$

Method II

The probable uncertainty of the laser wavelength compensation is calculated by

$$\text{Wavelength compensation uncertainty} = \sqrt{(\text{uncertainty in temperature})^2 + (\text{uncertainty in pressure})^2 + (\text{uncertanty in humidity})^2} \tag{12a}$$

$$\text{Wavelength compensation uncertainty} = \sqrt{(0.2)^2 + (0.8)^2 + (0.25)^2} = \pm 0.862\ \text{ppm} \tag{12b}$$

At a distance L, the position uncertainty due to wavelength compensation is

$$\text{Wavelength compensation error} = L(\text{m})(\pm 0.862 \text{ x } 10^{-6}) = \pm 0.862L\ \mu\text{m} \tag{13}$$

3.2.2 Material Thermal Expansion

Compensation for material thermal expansion has been implemented in the software. The method of correction is to change the laser wavelength compensation (ppm) by an amount sufficient to correct for thermal expansion. This correction is known as *material thermal compensation* (MTC) and is defined as

$$\text{MTC} = \alpha(T_\text{M} - 20) \tag{14}$$

where, α is the coefficient of material expansion (ppm/°C) and T_M is the temperature of the machine in Celsius (°C). MTC is given in ppm. The thermometer uncertainty is ±0.2°C. The material that should be considered in the thermal expansion compensation is the granite. This is the material of the support of the encoder scale and of the table of the G-90C CMM. For granite, α = 5.4 ppm/°C. It has been admitted that the uncertainty in the coefficient of material expansion is about 10% of α, i.e. 0.1α ppm for each °C from 20°C.

Method I

The uncertainty in the material thermal compensation depends on the uncertainties in the thermometer and coefficient of material expansion α. These uncertainties are assumed to have a rectangular probability distribution. Therefore, the standard uncertainty of the thermometer is found from

$$u_T = \frac{\alpha(\text{thermometer uncertainty})L(\text{m})}{\sqrt{3}}\ \mu\text{m} \quad (15a)$$

The standard uncertainty of the coefficient of material expansion α is given by

$$u_\alpha = \frac{|T_\text{M} - 20|(0.1\alpha)L(\text{m})}{\sqrt{3}}\ \mu\text{m} \quad (15b)$$

L is the measured distance in metres.

The standard uncertainty of the material thermal compensation u_5 is computed by combining the standard uncertainties u_T and u_α given by equations (15a) and (15b) in accordance with GUM [1]. Hence,

$$u_5 = \frac{\sqrt{(1.08L)^2 + [|T_\text{M} - 20|(0.54L)]^2}}{\sqrt{3}}\ \mu\text{m} \quad (16)$$

<u>Method II</u>

The position uncertainty due to the error in the material thermal compensation is as follows:

$$\text{Thermal compensation error} = (\alpha)(\text{thermometer uncertainty})(L(\text{m})) + |T_\text{M} - 20|(0.1\alpha)(L(\text{m})) \quad (17a)$$

Substituting the values of α and thermometer uncertainty, it obtains

$$\text{Thermal compensation error} = [1.08 + |T_\text{M} - 20|(0.54)]L\ \mu\text{m} \quad (17b)$$

3.2.3 Optics Thermal Drift

In a laser interferometer system, changes in temperature of some optical components during the measurement can cause measurement uncertainty. This occurs in the interferometer in the form of a change in optical path length with temperature. This change in optical path length appears as an apparent distance change. A typical value for optics thermal drift is 0.5 μm/°C [3].

The tests undertaken for evaluating the positional accuracy of the G-90C CMM were of short duration and the temperature practically did not change.

<u>Method I</u>

In this case, the correction for the optics thermal drift is negligible. Although this correction is null, the uncertainty of this correction should be accounted for [1]. The temperature of the linear interferometer during the tests was measured by a thermometer with uncertainty of ±0.2°C. Assuming a rectangular distribution for the thermometer uncertainty, the standard uncertainty of the optics thermal drift is calculated as follows:

$$u_6 = \frac{0.2}{\sqrt{3}}(0.5\ \mu\text{m/°C}) = \frac{0.1}{\sqrt{3}}\ \mu\text{m} \quad (18)$$

<u>Method II</u>

As the temperature did not vary during the tests, the optics thermal drift error is zero.

3.3 Measuring Uncertainties due to the Installation

These sources of uncertainty are related to the installation of the laser interferometer system.

3.3.1 Deadpath Error

Deadpath error is caused by an uncompensated length of the laser beam between the interferometer and retroreflector, with the machine stage at zero position. It appears as a shift in the zero position of the machine and occurs whenever environmental conditions change during the measurement time [3]. Deadpath error can be calculated as follows [3]:

$$\text{Deadpath error} = (\text{deadpath distance})(\Delta \text{WCN}) \tag{19}$$

where, ΔWCN is the change in wavelength compensation number during the measurement time.

As the tests carried out on the G-90C CMM were of short duration, the deadpath error (Eq. (19)) is negligible due to the fact that there were no significant changes in the atmospheric conditions, i.e $\Delta\text{WCN} = 0$.

Method I

Even though the deadpath correction is null, it is necessary to account for the uncertainty associated with the correction [1]. The uncertainty in the deadpath correction is a function of the uncertainty of the WCN. As seen in section 3.2.1, the latter is characterised by the standard uncertainty of the wavelength compensation u_W whose value is $0.862 \text{ ppm}/\sqrt{3}$ (Eq. (10)). Hence, the standard uncertainty of the deadpath correction is computed by the following expression:

$$u_7 = [\text{deadpath distance(m)}](u_W) = (0.3\text{m})(\frac{0.862 \text{ ppm}}{\sqrt{3}}) = \frac{0.2586}{\sqrt{3}} \ \mu\text{m} \tag{20}$$

where, deadpath distance = 0.3 m.

Method II

As the deadpath error is negligible, the deadpath compensation error is null.

3.3.2 Cosine Error

Misalignment of the laser beam to the mechanical axis of motion results in an error between the measured distance and the actual distance travelled. This is called cosine error, because its magnitude is proportional to the cosine of the angle of misalignment.

The cosine error in ppm, when using the cube-corner reflectors, is approximately equal to $S^2/8L^2_{\text{mm}}$, where L_{mm} is the measured distance in millimetre and S is the lateral offset of the returning beam in micrometer [3]. Since the proper alignment procedures were carefully followed, S was estimated to be approximately 300 µm. Since $L_{\text{mm}} = L(\text{m})(1000)$, the cosine error is given by

$$\text{Cosine error (ppm)} = \frac{S^2}{8L^2_{\text{mm}}} = \frac{(300)^2}{8(L1000)^2} = \frac{\pm 0.01125}{L^2} \ \text{ppm} \tag{21a}$$

$$\text{Cosine error } (\mu\text{m}) = (\frac{\pm 0.01125}{L^2})(L) = \frac{\pm 0.01125}{L} \ \mu\text{m} \tag{21b}$$

Method I

Cosine error is described by a rectangular distribution. Therefore, the standard uncertainty of the cosine error is calculated by

$$u_8 = \frac{0.01125}{L\sqrt{3}} \ \mu\text{m} \tag{22}$$

Method II

In this case, the cosine error is given by equation (21b).

4. MEASUREMENT UNCERTAINTY OF THE LASER CALIBRATION SYSTEM

In the following sections, the measurement uncertainty of this positional error calibrator is evaluated using the method based on GUM (Method I) and the alternative method (Method II).

4.1 Measurement Uncertainty According to Method I

The equations (11) and (20) show that the standard uncertainties of the wavelength compensation (u_4) and deadpath correction (u_7) are a function of the u_W This means that these input uncertainties are dependent or correlated. Assuming all of the other standard uncertainties are independent, the combined standard uncertainty u_C is calculated by [1]

$$u_C = \sqrt{u_1^2 + u_2^2 + u_3^2 + u_5^2 + u_6^2 + u_8^2 + (u_4 + u_7)^2} \tag{23a}$$

As shown in Eq. (23a) the standard uncertainties u_4 and u_7 are added before squaring them because they are correlated. Substituting the values of the standard uncertainties in Eq. (23a), it obtains

$$u_C = \sqrt{(\frac{0.02L}{\sqrt{3}})^2 + (\frac{0.001}{\sqrt{12}})^2 + (\frac{0.0042}{\sqrt{2}})^2 + \{\frac{(1.08L)^2 + [|T_{\mathrm{M}} - 20|(0.54L)]^2}{(\sqrt{3})^2}\} + }$$
$$\sqrt{+ (\frac{0.1}{\sqrt{3}})^2 + (\frac{0.01125}{L\sqrt{3}})^2 + (\frac{0.862L}{\sqrt{3}} + \frac{0.2586}{\sqrt{3}})^2} \tag{23b}$$

For L = 1 m and T_{M} = 20°C, the value of u_C is

$$u_C = 0.900 \ \mu\mathrm{m} \tag{23c}$$

The expanded uncertainty U_p is obtained by multiplying the combined standard uncertainty u_C by a coverage factor k_p [1]. In order to compute k_p, the effective degrees of freedom ν_{eff} should be obtained first. ν_{eff} is given by the Welch-Satterthwaite formula [1] as follows:

$$\nu_{\mathrm{eff}} = \frac{u_C^{\ 4}(y)}{\sum_{i=1}^{N} \frac{u_i^{\ 4}(y)}{\nu_i}} \tag{24}$$

where, $u_C^{\ 2}(y) = \sum_{i=1}^{N} u_i^2(y)$; $u_i(y) = c_i u_i(x_i)$, c_i is the sensitivity coefficient and $u(x_i)$ is the standard uncertainty of the estimate x_i. ν_i is the degrees of freedom of each uncertainty component. N is the number of uncertainty components. y is the estimate of the measurand Y.

The expanded uncertainty $U_p = k_p u_C(y) = t_p(\nu_{\mathrm{eff}}) u_C(y)$. t_p is the Student factor and $t_p(\nu_{\mathrm{eff}})$ is the value of t for a given value of the ν_{eff} with a level of confidence p [1]. As the all standard uncertainties u_i have been obtained from a Type B evaluation and are assumed as exactly known, the value of $\nu_i \to \infty$, for i varying from 1 to 8 [1]. As consequence, by the Eq. (24), the value of $\nu_{\mathrm{eff}} \to \infty$. As $\nu_{\mathrm{eff}} \to \infty$ the t-distribution (Student's distribution) approaches the normal probability distribution [1]. Thus, for a level of confidence of approximately 95%, the coverage factor $k_{95} = 2$, considering the normal distribution. Hence,

$$U_{95} = 2u_C = 2(0.900) = 1.800 \ \mu\mathrm{m} \tag{25}$$

The uncertainty budget of this calibrator is presented in Table 1. These results are for L = 1 m and T_{M} = 20°C.

Table 1. The analysis of uncertainty of the positional error calibrator (for L = 1 m and T_M = 20°C) conforming to Method I.

Source of uncertainty	Probability distribution	Divisor	Standard uncertainty (μm)
1) Laser wavelength uncertainty	rectangular	$\sqrt{3}$	$u_1 = 0.012$
2) Electronics error	rectangular	$\sqrt{3}$	$u_2 = 0.00029$
3) Optics non-linearity	"U" shaped	$\sqrt{2}$	$u_3 = 0.0030$
4) Wavelength compensation	rectangular	$\sqrt{3}$	$u_4 = 0.498$
5) Material thermal compensation	rectangular	$\sqrt{3}$	$u_5 = 0.624$
6) Optics thermal drift	rectangular	$\sqrt{3}$	$u_6 = 0.058$
7) Deadpath correction	rectangular	$\sqrt{3}$	$u_7 = 0.149$
8) Cosine error	rectangular	$\sqrt{3}$	$u_8 = 0.0065$
Combined standard uncertainty	Normal	–	$u_C = 0.900$
Expanded uncertainty	Normal; $k_{95} = 2$; $p = 95\%$	–	$U_{95} = 1.800$

4.2 Measurement Uncertainty According to Method II

The computation of the measurement uncertainty of this calibrator based on Method II is made in two parts as follows:

1) First, the probable uncertainty for errors that are constant or depend on the same parameters are calculated. The probable uncertainty is given by

$$\text{Probable uncertainty} = \sqrt{\sum_{i=1}^{i=N} (\text{uncertainty}_i)^2} \tag{26}$$

2) Second, the measurement uncertainty is the sum of the probable uncertainties obtained by Eq. (26). Hence,

$$\text{Measurement uncertainty} = \sum_{j=1}^{j=M} (\text{probable uncertainty})_j \tag{27}$$

The sources of uncertainty of this laser calibration system are given in Table 2. The probable uncertainty for the individual uncertainties that are constant is given by

$$(\text{probable uncertainty})_1 = \sqrt{(0.001)^2 + (0.0042)^2} = \pm 0.004317 \ \mu\text{m} \tag{28}$$

The probable uncertainty for the uncertainty components that are a function of L is

$$(\text{probable uncertainty})_2 = \sqrt{(0.02L)^2 + (0.862L)^2 + (1.08L)^2} = \pm 1.382L \ \mu\text{m} \tag{29}$$

The probable uncertainty for the individual uncertainty that is a function of T_M and L is

$$(\text{probable uncertainty})_3 = \sqrt{[|T_M - 20|(0.54)(L)]^2} = \pm |T_M - 20|(0.54)L \ \mu\text{m} \tag{30}$$

The probable uncertainty for the uncertainty component that is inversely proportional to L is

$$(\text{probable uncertainty})_4 = \sqrt{(\frac{0.01125}{L})^2} = \pm \frac{0.01125}{L} \ \mu\text{m} \tag{31}$$

Table 2. The individual uncertainties of the positional error calibrator conforming to Method II.

Source of uncertainty	Unit (± μm)
LWU	$0.02\,L$
Electronics error	0.001
Optics non-linearity	0.0042
Wavelength compensation error	$0.862\,L$
Thermal compensation error	$[1.08 + \lvert T_M - 20\rvert(0.54)]L$
Cosine error	$0.01125/L$

Now, the measurement uncertainty of this calibrator can be computed by means of Eq. (27):

$$\text{Measurement uncertainty} = 0.004317 + 1.382L + \lvert T_M - 20\rvert(0.54)L + \frac{0.01125}{L}\ \mu\text{m} \tag{32}$$

For L = 1 m and T_M = 20°C, the Eq. (32) results

$$\text{Measurement uncertainty} = \pm 1.398\ \mu\text{m} \tag{33}$$

5. CONCLUSIONS

1) In Method II, the normal distribution (divisor = 1; see Table 1) is assumed to all individual uncertainties. The uncertainties that depend mathematically on the same parameter are grouped in a set. The probable uncertainty of this set is calculated considering all uncertainties are independent. After, the probable uncertainty of each set is added in order to compute the measurement uncertainty. This is done to obtain the worst scenario of probability, i.e., as if the probable uncertainty of all sets were correlated. Also, in this method, the uncertainty associated with any type of correction is not accounted for in the calculation of the measurement uncertainty.

2) On the contrary, GUM Method considers a type of probability distribution for each source of uncertainty. Furthermore, the uncertainty associated with a given correction should be considered as an input quantity. The combined standard uncertainty u_C and the expanded uncertainty U_p are computed for a level of confidence p. In this method, it is necessary to verify which of uncertainties are independent. The uncertainties that are correlated should be added before squaring them in the computation of u_C [see Eq. (23a)]. By contrast, the Method II does not require analysis about correlation of uncertainties.

3) The measurement uncertainty of this calibrator computed by Method I is ±1.800 μm. This uncertainty is based on a combined standard uncertainty multiplied by a coverage factor $k_{95} = 2$, providing a level of confidence of approximately 95%. According to Method II, the measurement uncertainty is ±1.398 μm. These results are for a measured distance of 1 m at a machine temperature of 20°C. This uncertainty analysis was calculated when this calibrator was applied on a moving bridge type CMM model G-90C made by LK.

6. REFERENCES

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